Trento 03/06

- Flux loops in hot gluodynamics

Casimir scaling

and

- precise quasi-particle description

lattice
M. Teper et al

P. de Forcrand et al

H. Meyer

theory
H. Meyer, CPKA
hep-ph/0509138

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hep-ph/9909135

Forces and Fluxes

heavy quark: $Q = \sum_c Q_c$

Correlation of $Q\bar{Q} = \exp -\frac{1}{T} F_e(T,r)$

$$F_e(T,r) = \sigma(T)\,r \qquad T < T_c$$

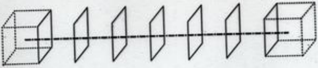
$$= \frac{1}{4\pi r}\exp - m_D(T)\,r \qquad T > T_c$$

$\sigma(T)$ string tension: constant force

$m_D(T)$ screening mass: force $\downarrow$

$W(L_{\tau x})$ gives same information

Now rotate τ in y:



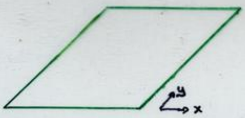
7: Monopole antimonopole pair induced by twisting the plaquettes pierced by Dirac string.

$$\exp -F_M(r)/T = \left(\int DA \exp -S_{(k)}(A)\right) / \int DA \exp -S(A).$$

The action $S_{(k)}$ is the usual action, except for those plaquettes pier the Dirac string. Those plaquettes are multiplied by a factor $\exp ik\frac{2\pi}{N}$

Screening is expected in both confined and deconfined phases:

$$F_M(r) = F_{M0} - c_M \frac{\exp -m_M r}{r}.$$

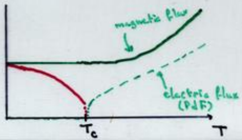


Get spatial Wilsonloop:

$$W(L_{xy}) = \mathrm{Tr}\, P \exp ig \int_{L_{xy}} \vec{A}\cdot d\vec{l}$$

$$\langle W(L_{xy})\rangle_T = \exp - \sigma_s(T) L_x L_y$$

Stokes: $W(L_{xy})$ is magnetic flux loop



— = string tension

— = $\sigma_s(T)$

--- = $\rho_s(T)$.

- Asymptotic freedom : $g^2(T) \sim \frac{1}{\log T/\Lambda_T} \ll 1$

- In plasma of bosons (glue)

$$g^2_{\mathrm{dim}} \equiv g^2(T)\, n_{BE}(T,p) \;\; \begin{cases} = g^2(T) & \text{if } p\sim T \\ = g(T) & \text{if } p\sim gT \\ = O(1) & \text{if } p\sim g^2T \end{cases}$$

  - despite AF : $g^2_{\mathrm{dim}} \sim O(1)$ if $p\sim g^2T$!

  - effect is due to soft tail of n_{BE}, where population is high.

- Quantitavely:

integrate out $p\sim T$ modes: expansion in $g^2(T)$

" " " " $p\sim gT$ " " : " " " in $g(T)$

$p\sim g^2T$: LATTICE

· Doing these integrations in loop expansion

· Euclidean, BE $\leftrightarrow$ periodic T, $p_0 \leftrightarrow 2\pi n T$

$$\times\ L_{QCD}(g\ A_r) \xrightarrow[n\neq 0]{} L_{EQCD}(\underset{\substack{\uparrow \\ \text{static}}}{A_{n=0}}\ \text{only}, \ldots)$$

$$3d:\ L_{EQCD} = (D_i A_0)^2 + m_E^2\ A_0^2 + \lambda_E A_0^4 + F_{ij}^2 + \delta L_E$$

$$D_i A_0 = \partial_i A_0 + i g_E [A_i, A_0]$$

$$g_E^2 = g^2 T\ (1 + O(g^2))$$

$$m_E = g T\ (1 + O(g^2))$$

$$\vdots$$

Physics at $p \lesssim gT$, cut-off $2\pi T$

$$\therefore$$

Reduction limited to $g \lesssim 2\pi$

$$g^2(2T_c) = 2.7$$

· Depends on observable

If $m_E = gT \gg g_E^2 = g^2 T$ then

- integrate out gT scales in loop expansion
- expansion parameter is g (not g^2)

$$\mathcal{L}_{EQCD}(A_0\, \vec{A}) \longrightarrow \mathcal{L}_{MQCD}(\vec{A}) = F_{ij}^2 + \delta L_M$$

$$F_{ij} = \partial_i A_j - \partial_j A_i + g_M [A_i\, A_j]$$

$$g_M^2 = g_E^2 [1 + \cdots g + \cdots g^2 + \cdots]$$

magnetic coupling g_M in L_{MQCD} is computed in terms of L_{EQCD} parameters by integrating out the m_E scales perturbatively:

$$g_M^2 = g_E^2 \left[1 - \underbrace{\frac{1}{48} \frac{g_E^2 N}{\pi m_E}}_{1\text{ loop}} - \underbrace{\frac{19}{4608} \left(\frac{g_E^2 N}{\pi m_E} \right)^2}_{2\text{ loop}} + \dots \right]$$

- corrections $\lesssim$ % even as $T \sim 2T_c$

- same applies to coefficients in HiDe terms in:

$$L_{MQCD} = \frac{1}{2} \mathrm{Tr} F_{ij}^2 + \delta L_M$$

. P. Giovannangeli
hep-ph/0312307
/0506318

3 d YM non-perturbative as $\frac{g_M^2}{p} \sim O(1)$
or $p \sim g_M^2 \approx g^2(T) T$

Spatial Wilson loop

- Exact opposite of pressure: leading term is 3d NP!

$$\langle W_f(L)\rangle_T = \int D\vec{A}\, \mathrm{Tr}\,P \exp ig\oint d\vec{l}\cdot\vec{A}\, \exp(-F_{ij}^2 + \delta\mathcal{L}_M)$$

× hard modes → only perimeter L: QCD → EQCD

× soft modes only in action: EQCD → MQCD

× only modes ~ g^2T left: non perturbative job!

Write $\langle W_f(L)\rangle_T \sim \exp(-\sigma_s \text{Area})$

Dimensional argument:

$$\sigma_s = c\,(g_M^2)^2$$

$c = 0.5530(20)$ Karsch, Teper (3d)

! The 3d number c and 1+2 loop running through hard modes determine data from $1.1\,T_c$!

M. Laine, Y. Schröder, hep-ph/0503061

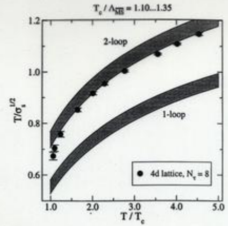


Figure 4: *We compare 4d lattice data for the spatial string tension, taken from Ref. [22], with expressions obtained by combining 1-loop and 2-loop results for g_E^2 together with Eq. (4.5) and the non-perturbative value of the string tension of 3d SU(3) gauge theory, Eq. (4.4). The upper edges of the bands correspond to $T_c/\Lambda_{\overline{MS}} = 1.35$, the lower edges to $T_c/\Lambda_{\overline{MS}} = 1.10$.*

- Similar for magnetic screening.

- Similar for electric flux loop ("t Hooft loop)

Electric fluxloop in SU(N) gauge theory

In QED: $V(L) = \exp i \frac{2\pi}{e} \int d\vec{S}.\vec{E}$

In QCD: $V_k(L) = \exp i \frac{2\pi}{g} \int d\vec{S}.\mathrm{Tr}\,\vec{E}\,Y_k$

N×N matrix $\quad Y_k \doteq \frac{1}{N} \begin{pmatrix} N-k & & & & 0 \\ & N-k & & & \\ & & N-k & & \\ & & & -k & \\ 0 & & & & \ddots -k \end{pmatrix} \qquad \exp i 2\pi Y_k = e^{-ik\frac{2\pi}{N}} \mathbb{1}$

- $V_k(L)$ creates $Z(N)$ fluxloop of strength $e^{-ik\frac{2\pi}{N}}$
- gauge invariant in physical Hilbert space ($Y_k \to \Omega Y_k \Omega^\dagger$ invariant)
- Y_1 hypercharge: $2(N-1)$ gluons have charge $\pm g$, remaining ones charge $= 0$

$\boxed{Y_k: \quad 2k(N-k) \text{ gluons have charge } \pm g}$

- So one single gluon shines a flux $\pm \frac{1}{2} g$ through L: $\quad V_k(L)\Big|_{1\text{ gluon}} = \exp\left(i \frac{2\pi}{g} \cdot \pm \frac{1}{2} g\right) = -1$

- assume : i) gluons are independent

  ii) have a Poisson distribution around mean number $\bar{l}$ in the slab

- Then for one species with charge $\pm g$:

  × $\langle V_k \rangle_{1\,\text{species}} = \sum_l P(l)(-)^l = \exp -2\bar{l}$

  × independence:

  $$\langle V_k \rangle_{\text{all species}} = \exp - 4\bar{l}\, k(N-k)$$

  $$= \exp(-\underbrace{8\, l_D\, n\, k(N-k)}_{=\,\rho_k}\, \text{Area})$$

  l_D is Debye screening length $\left(\frac{\lambda}{3}T^2\right)^{-\frac{1}{2}}$

  n is single gluon density $\frac{\zeta(3)}{\pi^2}T^3$

  × $\rho_k = \#\, k\,(N-k)\, \frac{1}{\sqrt{\lambda}}\, T^2$

  Same parametric dependence as from 1 and 2 loop calculation

This k dependence is precisely that of the 2^{nd} order Casimir in

$$\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 3 \\ \hline \cdot \\ \hline \cdot \\ \hline k \\ \hline \end{array}$$

$$C_2 = k(N-k)\frac{C_F}{2N}$$

$$C_F = \frac{N^2-1}{N}$$

Totally a.s. Young tableaux with k boxes

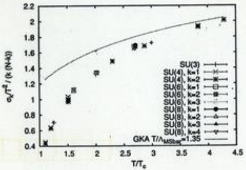
"Casimir scaling"

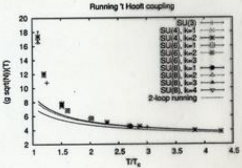
For k finite, $N \to \infty$:

$$C_2 = (kN - k^2)\underbrace{\left(1 - \frac{1}{N^2}\right)}_{\to\, O\left(\frac{1}{N}\right)}$$

How to rhyme with 't Hooft diagr. argument : "double line" argument

- Propagator has background colour field dependence, so no "double line".


SU(3)
SU(4), k=1
SU(4), k=2
SU(6), k=1
SU(6), k=2
SU(6), k=3
SU(8), k=1
SU(8), k=2
SU(8), k=3
SU(8), k=4
T/T_c


Running 't Hooft coupling
SU(3)
SU(4), k=1
SU(4), k=2
SU(6), k=1
SU(6), k=2
SU(6), k=3
SU(8), k=1
SU(8), k=2
SU(8), k=3
SU(8), k=4
2-loop running
(g sqrt(N))(T)
T/T_c

1 - 2 loop result

$$\rho_k(T) = \rho_k^{(1)}(T)\left(1 - \frac{\alpha_s}{4\pi}N\left(15.2785\ldots - \frac{11}{3}\left(\gamma_E + \frac{1}{22}\right)\right) + \underbrace{O(g^3)}_{k\text{-dependence}}\right)$$

$$\rho_k^{(1)}(T) = k\,(N-k)\,\frac{4\pi^2}{3\sqrt{3}\,(g^2N)^{1/2}}\,T^2$$

Compare to $\rho^{qp}(T) = \underbrace{2k\,(N-k)}_{\text{multiplicity}}\, \ell_D\, n(T)\, 6.568\ldots$

$n(T)$ density of one single gluon species

- $O(g^4)$: in progress (only hard $\sim T$ contributions). PG / CPKR.

from lattice it seems: perturbation theory unreliable

YET: the Casimir scaling from the low orders is still valid!

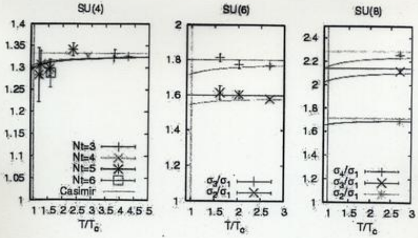
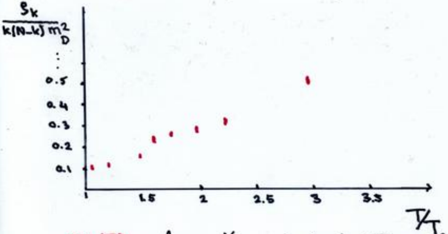


Figure 5: Ratios of interface tensions $\tilde{\sigma}_k/\tilde{\sigma}_1$, as a function of temperature, for $SU(4)$ (*left*), $SU(6)$ (*middle*) and $SU(8)$ (*right*). The horizontal lines mark the Casimir values $\frac{k(N-k)}{N-1}$. The curves show the $\mathcal{O}(g^3)$ perturbative prediction of [7]. The 2 curves for $SU(4)$ correspond to $T_c/\Lambda_{\overline{MS}} = 1.10$ and 1.35.

Diluteness of electric quasi-particles



$m_D(T)$ from Kaczmarek et al, LAT 2005, hep-lat/0503017

NB : $1/g^3(T) \sim (\log T)^{3/2}$ behaviour only at $\gtrsim 10^6\, T_c$

Wilsonloops at high T

- High T = 3d for W-loop

Model for 3d YM.

① magnetic gluons form lumps of size $\ell_M = m_M^{-1}$

② lumps are dilute : $\ell_M^3 n_M \equiv \delta \ll 1$

③ lumps are monopoles

- GNO classification of monopoles in unbroken gauge theory:

monopoles are in multiplets of magnetic global $SU(N)$.

④ Choose the adjoint

- provides the N^2-1 factor for 3d pressure
- compatible with quarks (GNO).

- Wilsonloop given by IRREP R

$$W_R(L) = \frac{1}{d_R} \mathrm{Tr} \exp ig \oint_L d\vec{l} \cdot \vec{A}_R$$

Need a flux representation!

$$W_R(L) = \int D\Omega \exp\left[ig \int d\vec{S}\, \mathrm{Tr}\, \vec{B}\, \Omega H_R \Omega^\dagger\right]$$

DP

H_R highest weight of R.

If R is totally AS with k boxes:

$$\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline \vdots \\ \hline k \\ \hline \end{array} \qquad H_R = y_k$$

- Compute for k-AS the average in the gas, with adjoint monopoles.

$$\langle W_{k\text{-}AS}(L) \rangle = \langle \exp ig \int d\vec{S} \cdot \mathrm{Tr}\, \vec{B}\, y_k \rangle$$

- Since thermal de Broglie length $\sim \frac{1}{T} \ll \frac{1}{g^2 T} \sim \ell_M$

we can take the gas classical, so

Poisson distribution for ℓ lumps inside

our slab of thickness ℓ_M around the loop.

* Like in the gluoncase:

$2k(N-k)$ monopoles have charge $\pm \frac{2\pi}{g}$

* one charged monopole shines $\pm \frac{1}{2}\frac{2\pi}{g}$ through loop

$$\exp i\, g \cdot 1 \tfrac{1}{2} \tfrac{2\pi}{g} = -1$$

* one charged species averages loop to:

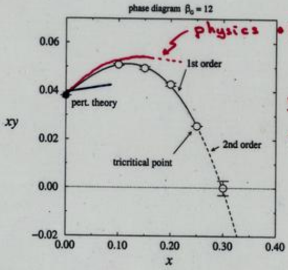
$$\langle W_k(L) \rangle_{1\,\text{species}} = \sum_\ell P(\ell)(-)^\ell\, e^{-\bar{\ell}} = e^{-2\bar{\ell}}$$

* independence of the $2k(N-k)$ species:

$$\langle W_k(L) \rangle_{\text{all}} = e^{-4\bar{\ell}\,k(N-k)}$$

$$\sigma_k = 4\,\ell_M\, n_M\, k(N-k)$$

Physics line to second order coincides with transition line calculated to same order! Not with the one from MC data!

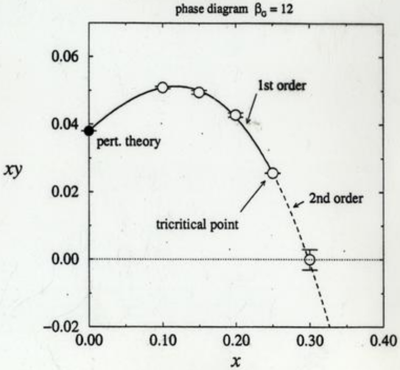


∴

no Higgs phase

no 't Hooft Pol. monopoles

Figure 6: The phase diagram of the 3d SU(3) + adjoint Higgs theory. The open symbols are results from the simulations, and the filled circle is the perturbative result in Eq. (17). The transition line is a polynomial fit to the data.



phase diagram $\beta_0 = 12$
0.06
0.04
0.02
0.00
−0.02
0.00
0.10
0.20
0.30
0.40
xy
x
1st order
pert. theory
2nd order
tricritical point

Pressure

Up to overall factor N^2-1:

$$P_{hard} = P_{Stefan-Boltzmann} + \cdots$$

$$P_{soft} = \frac{T}{12\pi}(m_D)^3 + \cdots$$

$$P_{ultrasoft} = \frac{T}{12\pi}(m_M)^3 + cT(g_M N^{1/2})^6 \log\frac{\Lambda_M}{d\, g_M^2 N}$$

$$c \cong \frac{1.6}{(2\pi)^4}$$

$$m_M^3 = (0.801\ldots)^3 (g_M N^{1/2})^6$$

- relative coefficient is small! so ignorance of d in the log may be unimportant!

6 Comparison to lattice simulations

We have been discussing a model at very high temperature. Hence it is tested in 3d lattice simulations. The ratios found [14] for the totally antisymmetric irreps are close — within a percent for the central value — as far as the adjoint multiplet of magnetic quasi-particles is concerned :

$SU(4) : \sigma_2/\sigma_1$	=	1.3548 ± 0.0064	adjoint : 1.3333	fundamental : 1.8182
$SU(6) : \sigma_2/\sigma_1$	=	1.6160 ± 0.0086	adjoint : 1.6000	fundamental : 1.9686
σ_3/σ_1	=	1.808 ± 0.025	adjoint : 1.8000	fundamental : 2.3635

The results are that precise, that you see a two standard deviation from the adjoint, except for the second ratio of $SU(6)$. This deviation is natural, since the diluteness of the magnetic quasi-particles is small, on the order of a couple of percent, as we will explain at the end of this subsection. So we expect corrections on that order to our ratios.

There is a less precise determination of the ratio $\sigma_2/\sigma_1 = 1.52 \pm 0.15$ in $SU(5)$ [20]. But the central value is within 1 to 2% of the predicted value 3/2 from the adjoint. The fundamental gives a ratio 1.8231.

The $SU(8)$ ratios are known on a rather course lattice [20] and using a different algorithm:

σ_2/σ_1	=	1.692(29)	adjoint : 1.714	fundamental : 2.106
σ_3/σ_1	=	2.160(64)	adjoint : 2.143	fundamental : 2.958
σ_4/σ_1	=	2.26(12)	adjoint : 2.286	fundamental : 3.256

In conclusion: the seven measured ratios are consistent with the quasi-particles being independent, as in a dilute gas and in the adjoint representation. The number of quasi-particle species contributing to the k-tension is $2k(N-k)$. This number happens to coincide with the quadratic Casimir operator of the anti-symmetric representation.

The fundamental monopoles are clearly disfavoured by the data.

— = multilevel algorithm (Harvey Meyer)

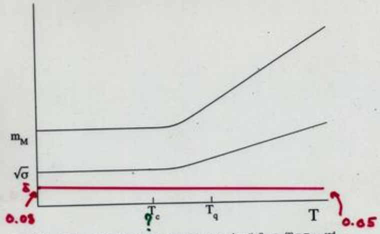


Figure 2: Magnetic mass m_M and tension σ as function of temperature, schematically. $m_M(0) = m_{0^{++}}$ and from ref. (13): $T_c = 0.174\ m_{0^{++}}$. The temperature $T_q \leq m_{0^{++}}$ is where the de Broglie thermal wave length becomes equal to the magnetic screening length. For the calculation of the tension it is below T_q that quantum statistics applies, above classical statistics applies as in section 5.

. For $T \gtrsim T_c$ both $\sqrt{\sigma}$ and m_M scale like $g^2(T)T$, so $\delta(T)$ is constant with known value at $T = \infty$ $(d=3)$

? For $T \lesssim T_c$ we need data on m_M and $\sqrt{\sigma}$. At $T=0$ $\delta =$ known.

Teper, Lucini, hep-lat/0502003

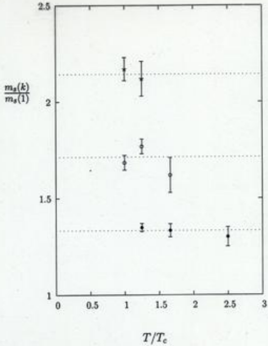


Figure 9: The mass ratio of the $k = 2$ to $k = 1$ spatial loops in SU(4) (•) and in SU(8) (∘), and of the $k = 3$ to $k = 1$ loop in SU(8) (⋆). All in the deconfined phase and for $a \simeq 1/5T_c$. High T Casimir scaling predictions are shown for comparison.

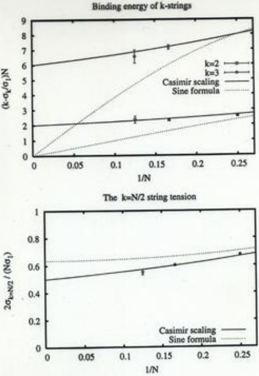


Figure 7: Top: the binding energy of k-strings per unit length, in units of σ_1 and rescaled by a factor N, as a function of N. Bottom: the string tension ratio $\frac{\sigma_{k=N/2}}{\sigma_1}$, rescaled by a factor $\frac{2}{N}$. The lattice data for SU(4) and SU(6) is taken from[25].

Epilogue

① adjoint multiplet of monopoles gives 1 to 2 % deviation from lattice results. WHY so small?

$$\frac{\sigma_k}{\sigma_1} = k(N-k)\,(1 + O(\delta))$$

② Our model gives generically

$$\sigma \sim l_M n_M$$

hence

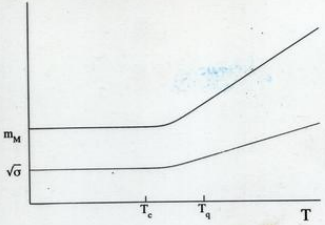
$$\delta = l_M^3 n_M = l_M^2 \sigma = \frac{\sigma}{m_M^2} \cong 0.05 \quad \text{Teper } /98$$

③ ratio's sensitive to choice of monopole rep.. Fundamental rep. off by $\gtrsim 30\%$.

④ $n_M \sim (g^2 T)^3 \sim \left(\frac{T}{\log T}\right)^3$ so SB limit is recovered.

⑤ 3d calculation of C. Herzog gives Casimir scaling as well, using AdS_CFT, hep-th/020564

Perspective for future:



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$$\delta(T) = \frac{\sigma}{m_M^2}(T) = \begin{matrix} 0.05 & T = \infty \\ 0.09 & T = 0 \end{matrix}$$

Is our monopole gas a dilute Bose gas, with BE condensation at T_c?

To check $\delta(T)$, 4d calculations are needed.