* Flux loops in hot gluodynamics

* Casimir scaling

* Precise quasi-particle description

References:
- P. Giannotti et al., CPK, hep-ph/0007252
Forces and Fluxes

heavy quark: $\Delta \phi_e$

Correlation of $Q \bar{Q} = \exp -\frac{1}{T} F_2(T, r)$

$F_2(T, r) = \sigma(T) r \quad T < T_c$

$= \frac{1}{4\pi} \exp -\frac{m_0(T)}{T} r \quad T > T_c$

$\sigma(T)$ string tension: constant force

$m_0(T)$ screening mass: force

$W(\Delta T, \omega)$ gives same information

Now rotate $\tau$ in $\chi$: 
2: Monopole anti-monopole pair induced by twisting the plaquette pierced by Dirac string.

\[ \exp \left( \frac{F_0}{T} \right) = \left( \frac{\int D A \exp \left( -S_0(A) / T \right)}{\int D A \exp \left( -S(A) \right)} \right) \]

The action \( S_0 \) is the usual action, except for those plaquettes pierced by the Dirac string. Those plaquettes are multiplied by a factor \( \exp \left( \frac{F_0}{T} \right) \).

Scattering is expected in both confined and deconfined phases:

\[ F_0 \exp \left( \frac{F_0}{T} \right) = F_{00} \exp \left( \frac{F_0}{T} \right) \]
Get spatial Wilson loop:

$$W(L_{xy}) = \text{Tr} \exp \left( \oint_{L_{xy}} A \right)$$

$$\langle W(L_{xy}) \rangle_T = \exp \left( -\phi_2(T) L_x L_y \right)$$

Stokes: $W(L_{xy})$ is magnetic flux loop

- $\phi_2(T)$
- $\phi_1(T)$
Asymptotic freedom: \( g_\text{as}(T) \sim \frac{1}{\log T/M} \ll 1 \)

- In plasma of bosons (glue):
  \[ g_\text{as}^2 \equiv g_\text{as}(T) \eta_{\text{bg}}(T) = g(T) \text{ if } p \gg T \]
  \[ g_\text{as}^2 = O(1) \text{ if } p \ll T \]
  
  - despite RH: \( g_\text{as}^2 = O(1) \) if \( p \ll T \)
  - effect is due to soft tail of \( \eta_{\text{bg}} \), where population is high.

Quantitatively:
integrate out p\(\times\)T modes: expansion in \( g_\text{as}(T) \)

- \( p \gg T \Rightarrow \cdots \cdots \text{ in } g(T) \)
- \( p \ll T \Rightarrow \text{ LATTICE} \)
Doing these integrations in loop expansion

Euclidean, $BE \rightarrow$ periodic $T$, $P_0 \rightarrow 2\pi T$

$xL_{\text{QCD}}(a, P_0^0) \rightarrow L_{\text{QCD}}(a, P_0^0\text{ only}, ...)$

$\rightarrow \langle \rangle$

$L_{\text{QCD}} = \langle D, R_0^2 + m^2 \rangle + \lambda e^2 R_0^2 + \lambda e^2 R_0 T + \delta e$

$D_0 R_0 = g_\lambda R_0 + g_E [R_0, R_0]$

$g^2 = g^2 T (1 + O(g^2))$

$m_\lambda = a T (1 + O(g))$

Physics at $pT$, cut-off $2\pi T$

Reduction limited to $g \leq 2\pi$

$g^2 (a T_c) = 2.7$

Depends on observable
If $m_e = gT \gg \bar{g}_e = g^*T$ then
- integrate out $gT$ scales in loop expansion
- expansion parameter is $g$ (not $g^*$)

$$L_{eGCD} (\phi_0, \bar{\phi}) \rightarrow L_{M(\phi)} = \tilde{F}_{ii} \tilde{S}_{ii}$$

$$\tilde{F}_{ii} = \tilde{B}_i \tilde{B}_i - \tilde{B}_i \tilde{B}_i + s_m [\tilde{B}_i \tilde{B}_i]$$

$$\tilde{S}_{ii} = g^*_e [1 + \ldots g + \ldots g^* \ldots]$$
magnetic coupling $g_M$ in $\tilde{L}_{MQCD}$

is computed in terms of $L_{QCD}$ parameters by integrating out the

$\mu$ scales perturbatively:

$$g_M^2 = g_\mu^2 \left[ 1 - \frac{1}{4 \pi} \frac{g_\mu^4}{\pi} \right]$$

corrections $\leq 1\%$ even for $\arctan 2$

same applies to coefficients in $\Pi_{QD}$

forms in:

$$L_{MQCD} = \frac{1}{2} \Pi_{QD} + g_{LM}$$

P. Giovannangeli
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3 & YM non-perturbative as $\frac{g_{YM}}{\pi} \sim 0.67$

or $2 - 3\%$
Spatial Wilson loop

Exact opposite of pressure: leading term is 3d NP!

\[ \langle W_2 (L) \rangle_T = \int \mathcal{D} \phi \exp \{ i \int d^4 x \left(\mathcal{L} + g \phi \right) + S_\gamma \} \]

- Hard modes → only perimeter: L̂: QCD → EGCD
- Soft modes only in action: EGCD → HGCD
- Only modes \( g^T \): non-perturbative job!

Write \( \langle W_2 (L) \rangle_T = \exp \left( - \frac{\sigma_3}{2} \text{Area} \right) \)

Dimensional argument:

\[ \sigma_3 = c \left( \frac{\sigma_3}{a} \right)^2 \]

\( c \approx 0.5 \) (3d) Karsch, Teper (3d)

The 3d number \( c \) and 1+1 loop running through hard modes determine data from 1+1.
Figure 6: We compare 1d lattice data for the spatial string tension, taken from Ref. [32], with expressions obtained by combining 1-loop and 5-loop results for $\kappa_5$ together with $\kappa_3$ (4.3) and the non-perturbative value of the string tension of 3d SU(N) gauge theory, Eq. (1.4). The upper edges of the bands correspond to $\kappa_5/\kappa_3 = 1.25$, the lower edges to $\kappa_5/\kappa_3 = 1.10$.

- Similar for magnetic screening.
- Similar for electric flux loop (Hessert loop).
Electric flux loop in SU(N) gauge theory

\[ \text{In QED: } V(N) = \exp \left( i \frac{e}{g} \Phi_0 \right) \]

\[ \text{In QCD: } V(N) = \exp \left( i \frac{e}{g} F_{\mu \nu} \right) \]

\[ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ e^{-ik \Phi_0} \]

\[ V(N) \] creates a (N) flux loop of strength \[ e^{ik \Phi_0} \]

- Gauge invariant to physical Hilbert space (\[ g \rightarrow g, B^0 \text{ invariant} \])
- \[ A_\mu \] hyper-charges \[ Z(N) \] gluons have charge \[ Z \]
- Remaining ones charge \[ A_\mu \] \[ Z(N) \] gluons have charge \[ \frac{Z}{2} \]
- So one single gluon shrinks a flux \[ \frac{\pi}{2} \]

through \[ L : V(N)_{\text{gluon}} = \exp(i \frac{e}{g} \frac{\pi}{2}) = -i \]
4. Assume: 1) gasses are independent
   2) have a Poisson distribution around mean number $\bar{n}$ in the cell

Then for one species with charge $e$:

$$x \langle N_x \rangle_{\text{species}} = \sum_i P_i \langle N_i \rangle_{\text{species}} = \exp(-\bar{n})$$

By independence:

$$\langle N_x \rangle_{\text{all species}} = \exp(-e^2 \kappa (N_x - \bar{n}))$$

$$= \exp(-e^2 \kappa (N_x - \bar{n})) \text{ Here}$$

$$= \frac{\phi}{\mathcal{P}}$$

$\phi$ is Debye screening length $\left(\frac{e^2 \kappa}{\pi} \right)^{\frac{1}{2}}$

$\mathcal{P}$ is single gass density $\frac{\bar{n}}{2 \pi}$

$$x \phi = \# k (N_x) \frac{1}{\mathcal{P}}$$

Same parameter dependence as from 1 and 2 above calculations.
This $k$ dependence is precisely that of the $2\text{nd}$ order Casimir in

- $C_2 = \begin{bmatrix} (N \pm k) \end{bmatrix} C_{2\text{nd}}$
- $C_2 \approx \frac{N^2}{N}$

Totally $\pm$ Young Tableaux with $k$ boxes

"Casimir scaling"

For $k$ finite, $N \to \infty$:

- $C_2 \approx (kN - k^2)(1 - \frac{\phi}{N})$
- $L \approx o(\frac{1}{N})$

How to rhyme, with 't Hooft diverg.
argument: "double line" argument

Propagator has background colour
field dependence, so no "double line".
1 - 2 loop result

\[ g_k(T) = g_k^I(T) \left( 1 - \frac{N(12.2755 \cdots - 1/2)}{N} \right) + O(g^3) \]

\[ \frac{1}{g^2} \text{ dependence} \]

\[ g_k^I(T) = k (N/M) \frac{m^2}{35 (N/M)^2} T^2 \]

Compare to \[ g^I_{\text{lo}} \text{ with } N(12.2755 \cdots - 1/2) \]

\[ n(0) \text{ density of one single gluon species} \]

- \( O(g^4) \): in progress (only hard-\( T \) contributions). PG/CPK

From what it seems: Perturbative theory unreliable

Yet: The Casimir scaling from the low orders is still valid!
Figure 5: Ratios of interface tensions $\sigma / R$, as a function of temperature, for SU(4) (left), SU(6) (middle) and SU(8) (right). The horizontal lines mark the Casimir values $\sigma_c / R$. The curves show the $\delta C^2$ perturbative prediction of [7]. The 2 curves for SU(6) correspond to $\delta C^2 = 1.00$ and 1.55.
Diluteness of electric quasi-particles

\[ m_0(T) \text{ from Kurnick et al., LNT 2007, } \text{ref. 1}\text{/05/2007} \]

\[ \frac{\Delta T}{T} \text{ at } T<T_c \]
Wilson loops at high $T$:

- High $T$ used for Wilson loops.
  - Model for 2d YM.
  - Magnetic glue form lumps of size $L = m^t$
  - Lumps are dilute: $L^3 \ll N^t$
  - Lumps are monopoles.

- Gauge theory: monopoles are in multiplets of magnetic global $SU(N)$.
  - Choose the adjoint:
    - provides the $N^t$ factor for ad pressure,
    - compatible with quarks (GNO).
Wilson loop given by \( W_R \):

\[
W_R(L) = \text{Tr} \exp \int_{L} g d\gamma \cdot T_R
\]

Need a flux representation!

\[
W_R(L) = \text{Flux} \exp \left[ \int_{L} g d\gamma \cdot T_R \cdot H_R \right]
\]

\( H_R \) highest weight of \( R \).

If \( R \) is totally \( AS \) with \( k \) boxes:

\[
H_R = Y_k
\]

Compute for \( k \) AS the average in the gas, with adjoint monopoles:

\[
\langle W_{kAS}(L) \rangle = \langle \exp \int_{L} g d\gamma \cdot T_B \cdot Y_k \rangle
\]
Since the thermal de Broglie length \( \xi = \frac{\hbar}{mS} \) we can take the gas classical, so

Poisson distribution for |\( \lambda \rangle \) jumps inside our slab of thickness \( L \) around the loop.

Like in the quenched case:

\( 2k(N-k) \) monopoles have charge \( \frac{2e}{3} \)

a one-charged monopole,
\( \delta \exp \left( \frac{i}{\lambda} \right) = \frac{1}{2} \frac{2e}{3} \)

\( \lambda \exp \left( \frac{i}{\lambda} \right) = -1 \)

The one-charged species averages loop to:

\( \langle \psi(k) \rangle \) species

\( \sum_i \mathcal{P}(N) \delta \lambda \exp \left( \frac{i}{\lambda} \right) \)

\( \gamma \) independent of the \( 2k(N-k) \) species.

\( \langle \omega(k) \rangle \) = \( -2 \pi \delta_{\mathcal{P}(N)} \)

\( \delta \exp \left( \frac{i}{\lambda} \right) = \frac{1}{3} \Pi S \langle k \rangle \)
Physics line to second order coincides with transition line calculated to same order! Not with the one from MC data!
Pressure

up to overall factor $N^2$:

$P_{hard} = P_{Stefan-Boltzmann} + \cdots$

$P_{soft} = \frac{T}{12\pi} (mD)^3 + \cdots$

$P_{turbulent} = \frac{T}{12\pi} (m^3) + a(mH, MW)^6 \log \frac{\Delta m}{\Delta t}$

$\alpha = 1.6 \left(\frac{\Delta t}{2\pi}\right)^4$

$m^3 = (0.801 \cdots) (\Delta m \Delta t)^3$

relative coefficient is small so ignorance of $\alpha$ in the log may be unimportant!
6. Comparison to lattice calculations:

We have been running a model at very high temperatures because it is needed in 3d lattice calculations. The ratio (H) for the model autoconstrains to zero — within a percent for the central value — as far as the adjoint multiplies of magnetic spin-particles is concerned:

\[
\begin{align*}
\text{SYG } & & \text{ST} \\
\text{H} & & \text{H} \\
\text{ST} & & \text{ST} \\
\text{H} & & \text{H}
\end{align*}
\]

The results are that percent, thus we see a two standard deviation from the adjoint, except for the second ratio of SYG. This deviation is incorrect, since the absence of the magnetic spin-particle is small, as the ratio of a couple of percent, as we can see at the end of this subsection. So we added corrections on that value to our results.

There is a fine precision determination of the ratio \(\frac{c}{v} = 1.00 \pm 0.01\) for SYG and ST. The central value is within 1 to 2% of the predicted value \(\Delta\) from the adjoint. The fundamental gives a ratio 1.00.

The SYG values are known on a lattice gauge lattice and using a different algorithm:

\[
\begin{align*}
\text{H} & & \text{ST} \\
\text{H} & & \text{ST} \\
\text{ST} & & \text{ST} \\
\text{H} & & \text{H}
\end{align*}
\]

In conclusion, the error estimated values are consistent with the quasi-particles being independent, as it is a gauge field in the adjoint representation. The number of quasi-particle species contributing to the bound is \(\Delta\). This number happens to coincide with the quark QCD quark of the adjoint representation.

The fundamental amplitudes are directly determined by the data.

--- Multilevel algorithm (Harvey Hayes)
For $T > T_c$, both $F_0$ and $m_0$ scale like $g^2(T)T$, so $S(T)$ is constant with known value at $T > T_c$.

For $T < T_c$, we need data on $m_0$ and $F_0$. At $T = T_c$, $S$ is known.
Figure 9. The mass ratio of the $k=2$ to $k=1$ spatial loops in $3$SU (a) and in $8$SU (b), and of the $k=3$ to $k=1$ loop in $2$SU (c). All in the deconfined phase and for $u = 1/2$. High-$T$ Catterall scaling predictions are shown for comparison.
Figure 5: Top: the binding energy of k-string per unit length, in units of \(\gamma\), as a function of \(H\). Bottom: the string tension ratio, \(\frac{T_{c}}{T_{k}}\), as a function of \(H\). The latter data for Eq.(1) and Ref.(3) is taken from Ref.(5).
Epilogue

adjoint multiplet of monopoles give 1 to 2% deviation from lattice results. Why so small?

\[ V_0 = V (N x) \left( 1 + o(x) \right) \]

Our model gives generally

\[ \delta = \frac{m}{V} \]

hence

\[ \frac{\delta}{m} = \frac{V}{m} = \frac{V}{m} = 0.05 \text{ Torr} \]

ratio's sensitivity to choice of monopole rep. Fundamental rep. of hydrogen.

\[ \left( \frac{\delta}{m} \right)^3 \sim \left( \frac{T}{1 \text{ Torr}} \right)^3 \text{ so SB limit is recovered.} \]

\[ \text{as calculation of C. Horowitz gives Cassini scaling as well, using RDS-CFT, hypm/\Delta x} \]
Perspective for future:

\[
\delta(T) = \frac{\delta(T)}{\delta_0} = 0.05 \quad T = 0
\]

As our monopole gas a diluted Bose gas, with condensation at \( T_c \)?

To check \( \delta(T) \), HD calculations are needed.