Matching Fermion Actions

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Outline:

- Philosophy of matching
- Free fermions
- 2D Schwinger model:

  overlap, staggered, Wilson

Philosophy of matching

Simulations frequently use different sea / valence quarks

-actions

- flavor number

- quark mass

the error introduced is corrected perturbatively

Is there a direct non-perturbative matching between different fermionic actions?

Large part of the fermion effect is pure gauge

- shift of β can be predicted even for small masses

In what coupling range (a,ma) can the ratio of two fermion determinants be described by local gauge loops?

How good is such a matching?

Take two fermion actions

D_1+m_1 and D_2+m_2

and relate their determinants:

$$\det(D_1+m_1) \simeq \int \mathrm{D}\psi\, e^{-\overline{\psi}(D_1+m_1)\psi}$$

$$\simeq \int \mathrm{D}\psi\, e^{-\overline{\psi}(D_2+m_2)\psi} \int \mathrm{D}\Phi\Psi\, e^{-\overline{\Psi}(D_1+m_1)\Psi - \Phi^+(D_2+m_2)\Phi}$$

$$\simeq \det(D_2+m_2)\, e^{-S_{\mathrm{correction}}}$$

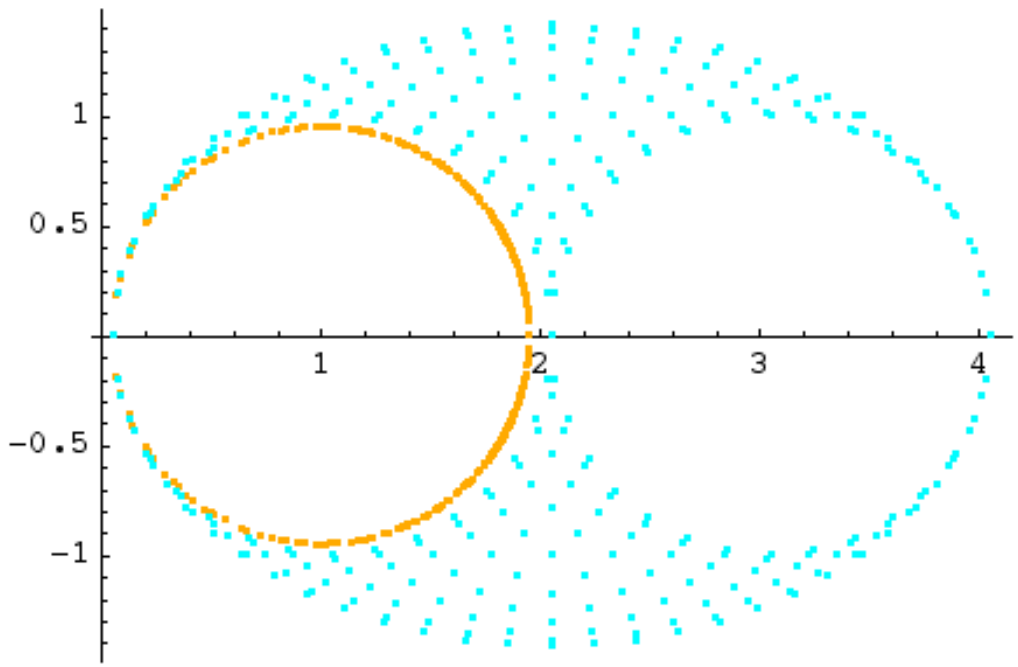
S_{corr} is the combination of light fermions and light ghosts; there is not much to do about it -

Or is there?

Eigenvalue spectrum of fermions

Wilson, staggered and overlap fermions are

- very different in the UV
- only O(a) different in the IR



Take the ratio $R = \frac{D_1 + m_1}{D_2 + m_2}$ for ordered eigenvalues

Eigenvalue spectrum of fermions

Wilson, staggered and overlap fermions are

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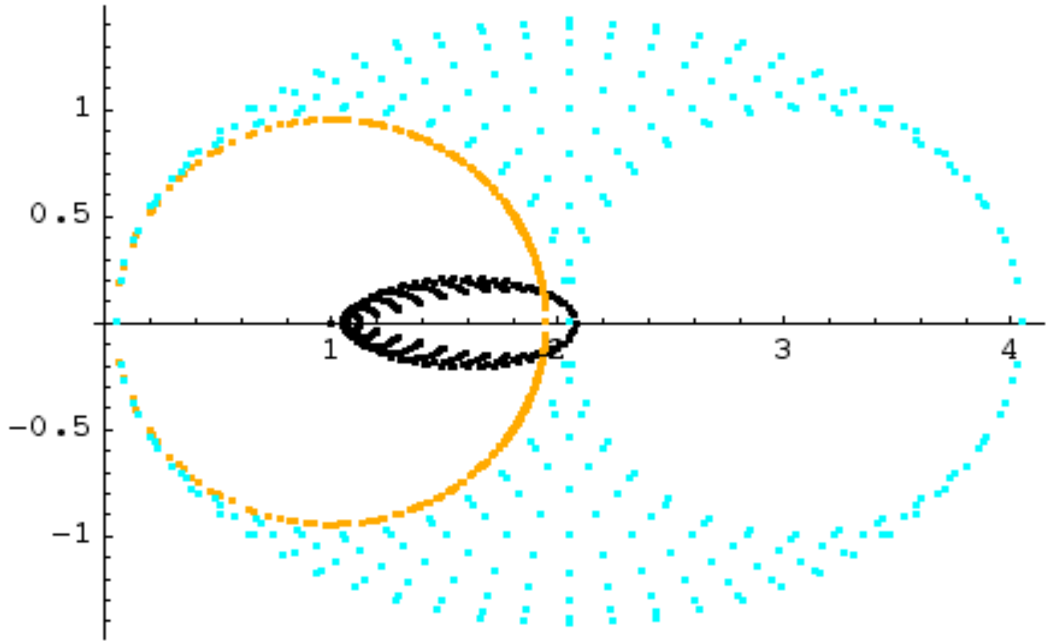
- only O(a) different in the IR

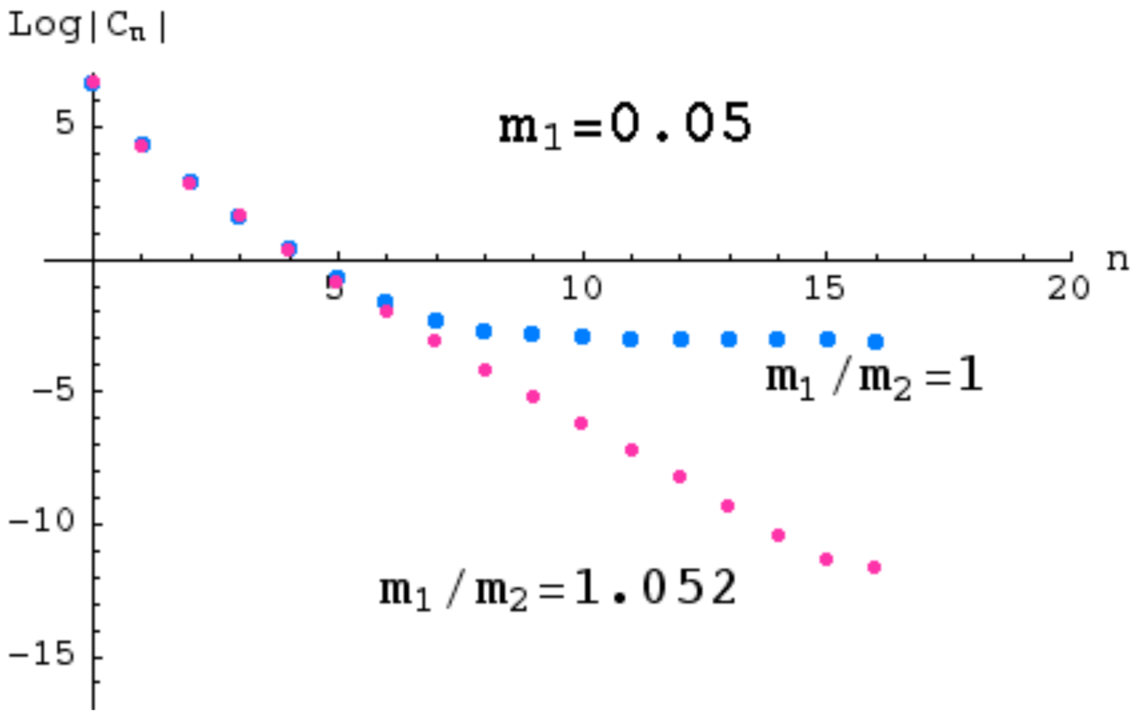


The ratio looks like a heavy fermion spectrum - equivalent to pure gauge ?

Is the ratio local?

Range of

$$R = (D_{overlap} + m_1)/(D_{Wilson} + m_2)$$



Free fermions

The ratio decays exponentially but m_2 has to be adjusted

Interacting case:

If the ratio corresponds to heavy fermions, it is equivalent to pure gauge operators.

Fit R with gauge loops on a set of typical configurations and find the best match in m_2.

Residue:

$$r(m_2) = \left\langle \left(\log\left[\frac{D_1 + m_1}{D_2 + m_2} \right] - \sum_{i=0}^{n} \alpha_i \,\mathrm{Tr}\, U_i \right) \right\rangle^{1/2}$$

In the fit n=3-8 loops up to length 8

Alternative : replace Tr U_i $\longrightarrow$ Tr D^i

Interacting case:

o Matching has a chance to work if

- m_1 m_2 >> chiral symmetry violations
- topology agrees: smeared and/or fine lattices

o Matching masses m_1 and m_2 correspond to physically equivalent systems

o $r(m_2)$ vs n measures the locality of the ratio

Numerical tests:

2-D Schwinger model with $N_f=2$

- Overlap
- Wilson
- staggered fermions

Simulations on fixed physical volume: $z = L/\sqrt{\beta} = 5 - 6$

$L = 12 - 24$

Scale: $a^2 g^2 = 1/\beta$

Simulations: pure gauge configurations reweighted with $\det(D_1+m_1)$, matched with $\det(D_2+m_2)$

Reference action (D_1):

- smeared overlap (mostly)
- staggered (some)

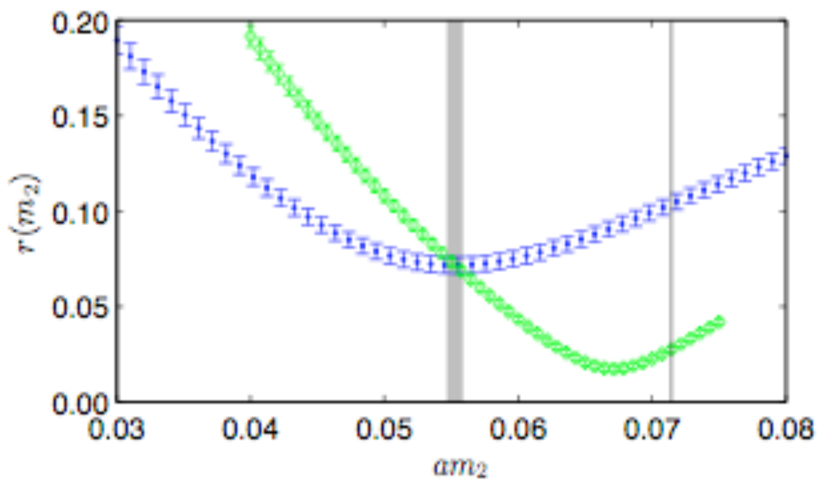
Typical matching:

dynamical overlap matched to staggered/Wilson

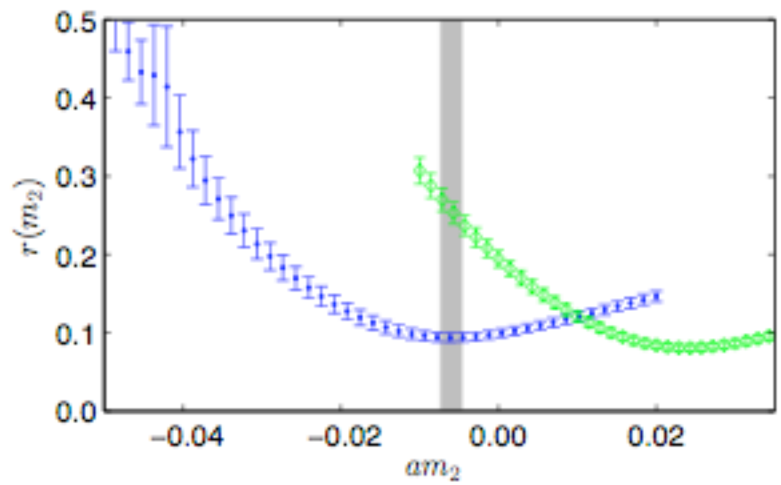
z=6, L=14 m_1=0.07

Find the minimum of the residue r(m2)

Staggered



Wilson



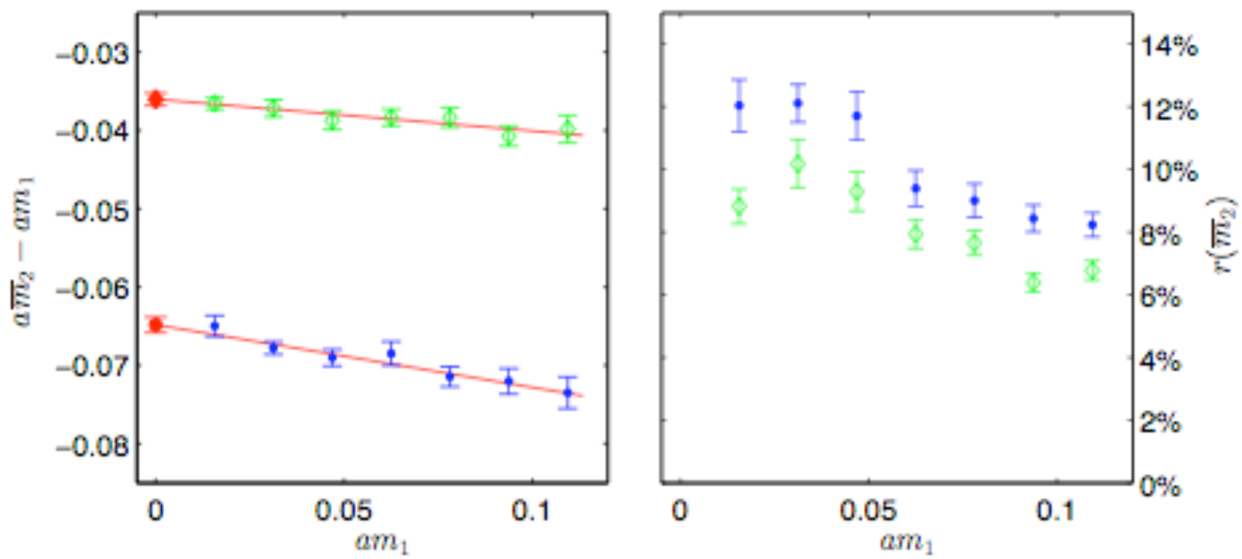
Blue : standard, green: smeared

- Definite mass shift at the minimum
- Smeared overlap has <1% residue

Wilson Action: we know what mass shift to expect

z=6, L=16 , fixed β

m_2-m_1 vs m_1 r(m_2) vs m_1



Blue : standard
green:smeared

$m_1 \longrightarrow 0$ predicts the critical point

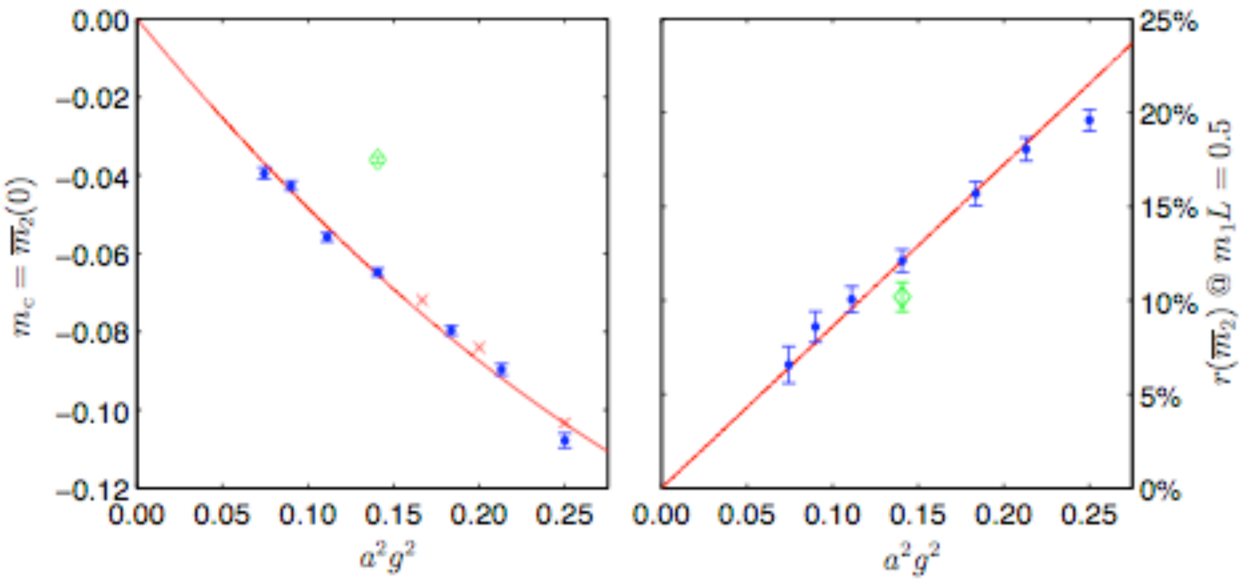
Continuum limit $z=6$, $m_1L=0.5$

Matched mass agrees with m_{crit} from spectroscopy

Residue ~ a^2

not a very good match - spread of real modes?

m_{crit} vs a^2 $r(m_2)$ vs a^2



Blue:standard, green:smeared action

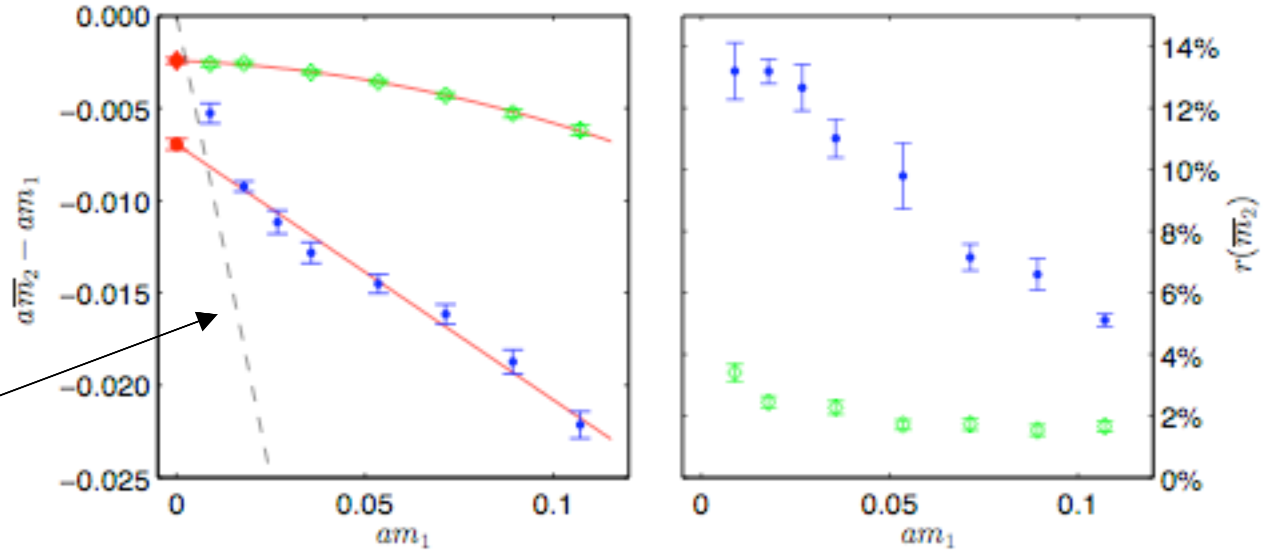
red cross:critical mass from spectroscopy

Staggered action

dynamical overlap matched with staggered;

z=6, L=14 ⟶ $(a^2g^2)=0.18$

m_2-m_1 vs m_1 $\qquad$ $r(m_2)$ vs m_1



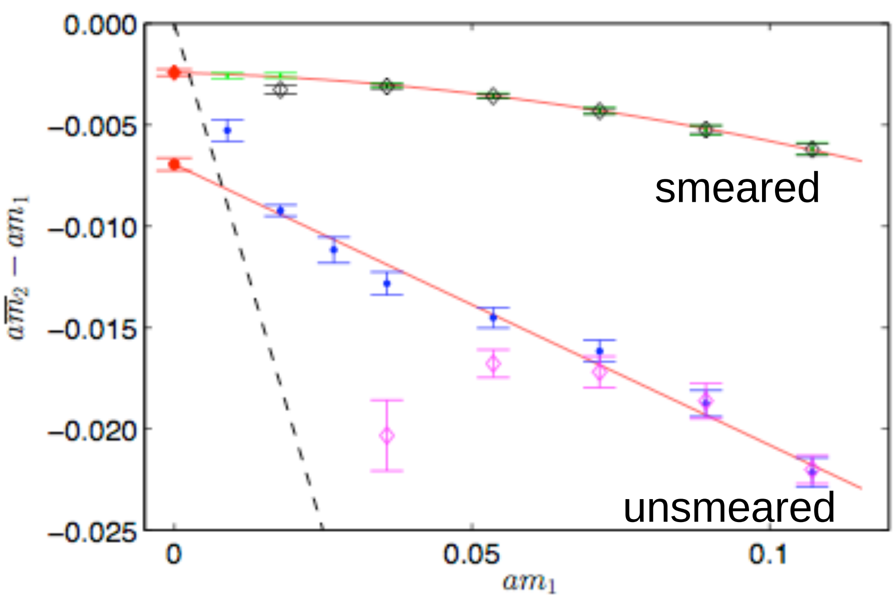
m_{st}=0

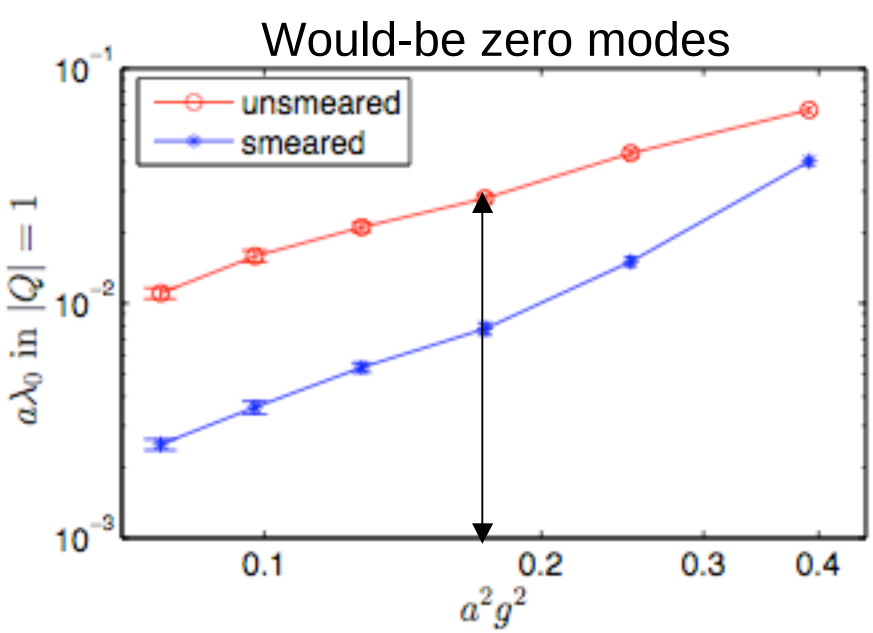
At the smaller masses the matching breaks down;

m_{ov}=0 cannot be reached

Residue for smeared action < 2%

Reverse: dynamical staggered matched with overlap





$a^2g^2 = 0.18$

Unsmeared:

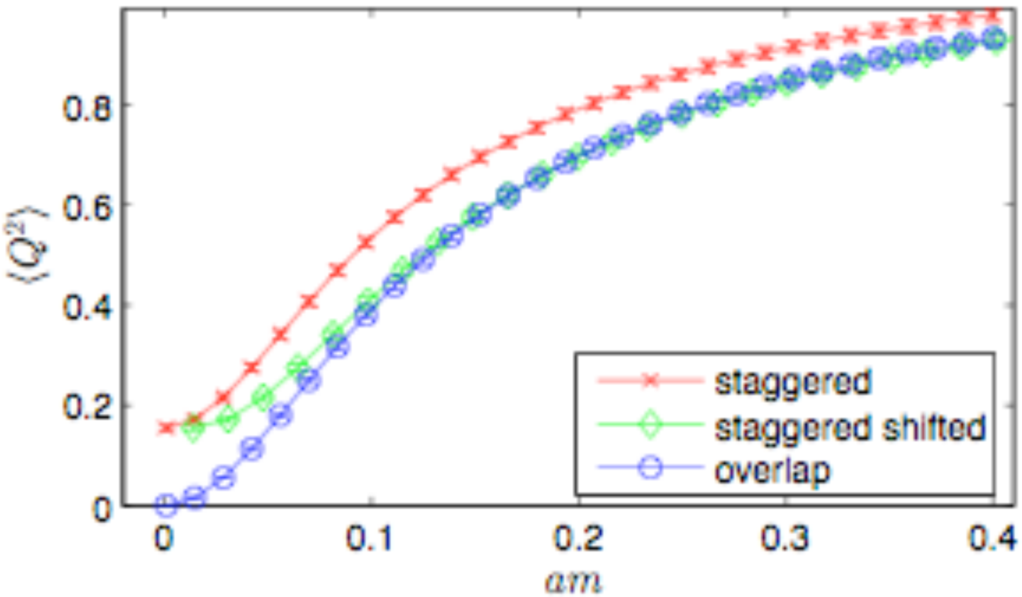
ov → stag (purple) matching breaks down at $m_1 \sim 0.05$

Would-be zero modes at $a^2g^2 = 0.18$ are $\lambda_0 \sim 0.035$

When the would-be zero modes are comparable to m_1 matching is not possible

Smeared: same relation

Topology shows the same (Duerr at al)



Unsmeared data

Staggered data agrees with overlap after shift $m \to Z\,m + \delta\,m$

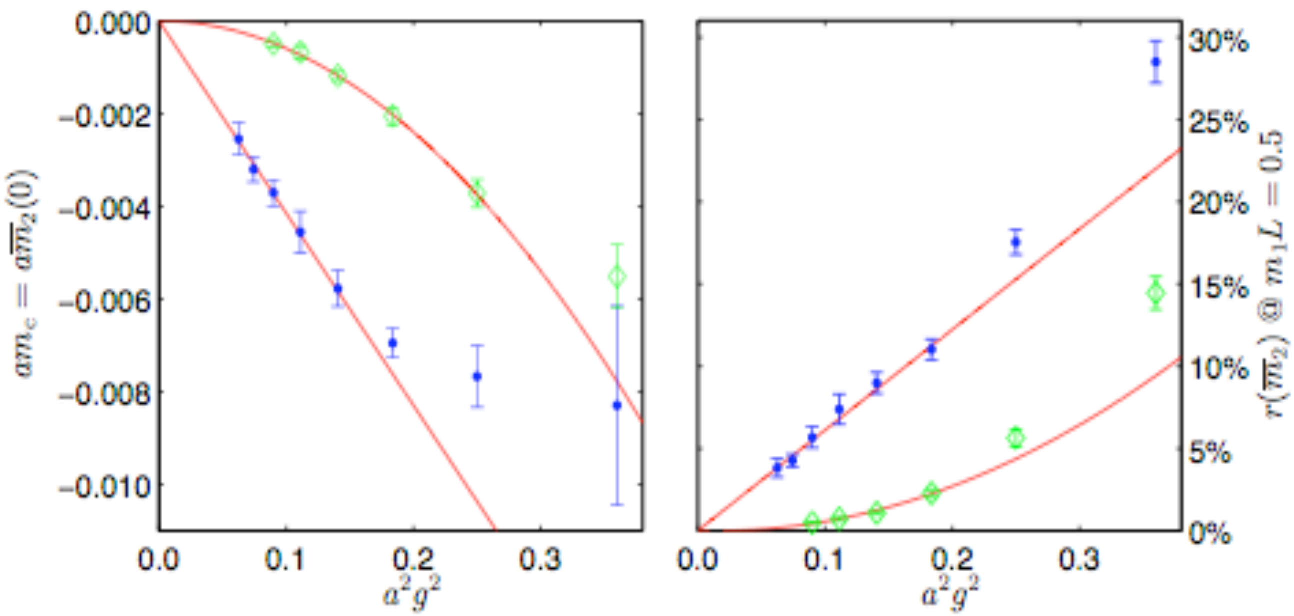
But only for $m > 0.05$

Continuum limit $z=6$, $m_1L=0.5$

Dynamical overlap matched with staggered

Smeared staggered residue, mass shift ~ $O(a^4)$

m_{crit} vs a^2 | $r(m_2)$ vs a^2



Blue : standard, green:smeared

Summary

- The ratio of fermion determinants for free fermions seem to correspond to local heavy fermions if the matching mass is tuned
- When this holds in the interacting case the determinant ratio is equivalent to a local gauge action
  - We studied 2D Schwinger model with overlap, staggered, Wilson actions
  - fitted determinant ratios with gauge loops
  - found matching masses
- Matching is possible for masses large compared to chiral symmetry breaking effects
- The smeared staggered action can be matched to overlap at small lattice spacing
  - In the matching region 4th root trick is justified
  - Matched staggered mass has an additive term
- Future: generalize to 4D, find a simpler way to identify matching masses, map the (a, am) region where matching is meaningful