Transverse spin densities from lattice QCD

Philipp Hägler

TECHNISCHE UNIVERSITÄT MÜNCHEN

In collaboration with

M. Göckeler, A. Schäfer (Regensburg U.)

R. Horsley, J. Zanotti (Edinburgh U.)
P. Rakow (Liverpool U.)

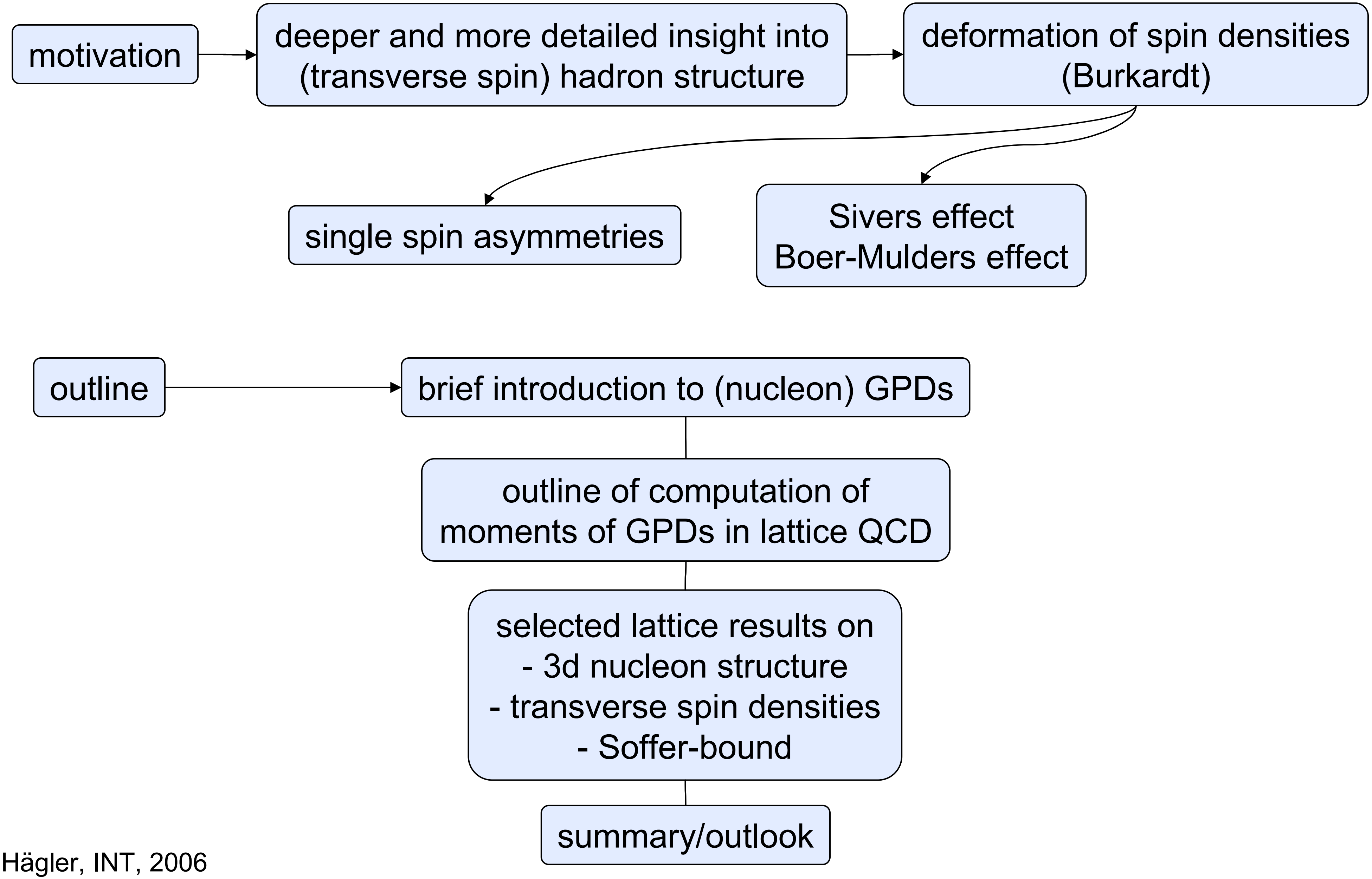
D. Pleiter, G. Schierholz
(DESY Zeuthen)

M. Diehl (DESY)

(QCDSF/UKQCD-collaborations)

supported by
Deutsche Forschungsgemeinschaft DFG

Overview



motivation

deeper and more detailed insight into (transverse spin) hadron structure

deformation of spin densities (Burkardt)

single spin asymmetries

Sivers effect
Boer-Mulders effect

outline

brief introduction to (nucleon) GPDs

outline of computation of moments of GPDs in lattice QCD

selected lattice results on
- 3d nucleon structure
- transverse spin densities
- Soffer-bound

summary/outlook

Probing the internal structure of hadrons

light quark hadron structure

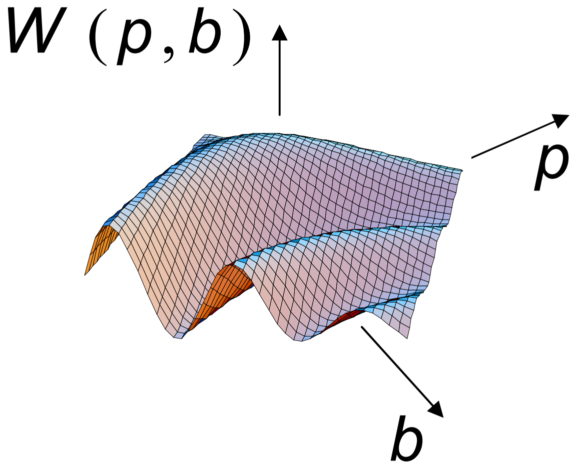
(probably) most general probe: Wigner phase-space-distributions

X. Ji, PRL 2003

A. Belitsky, X. Ji, F. Yuan, PRD 2004

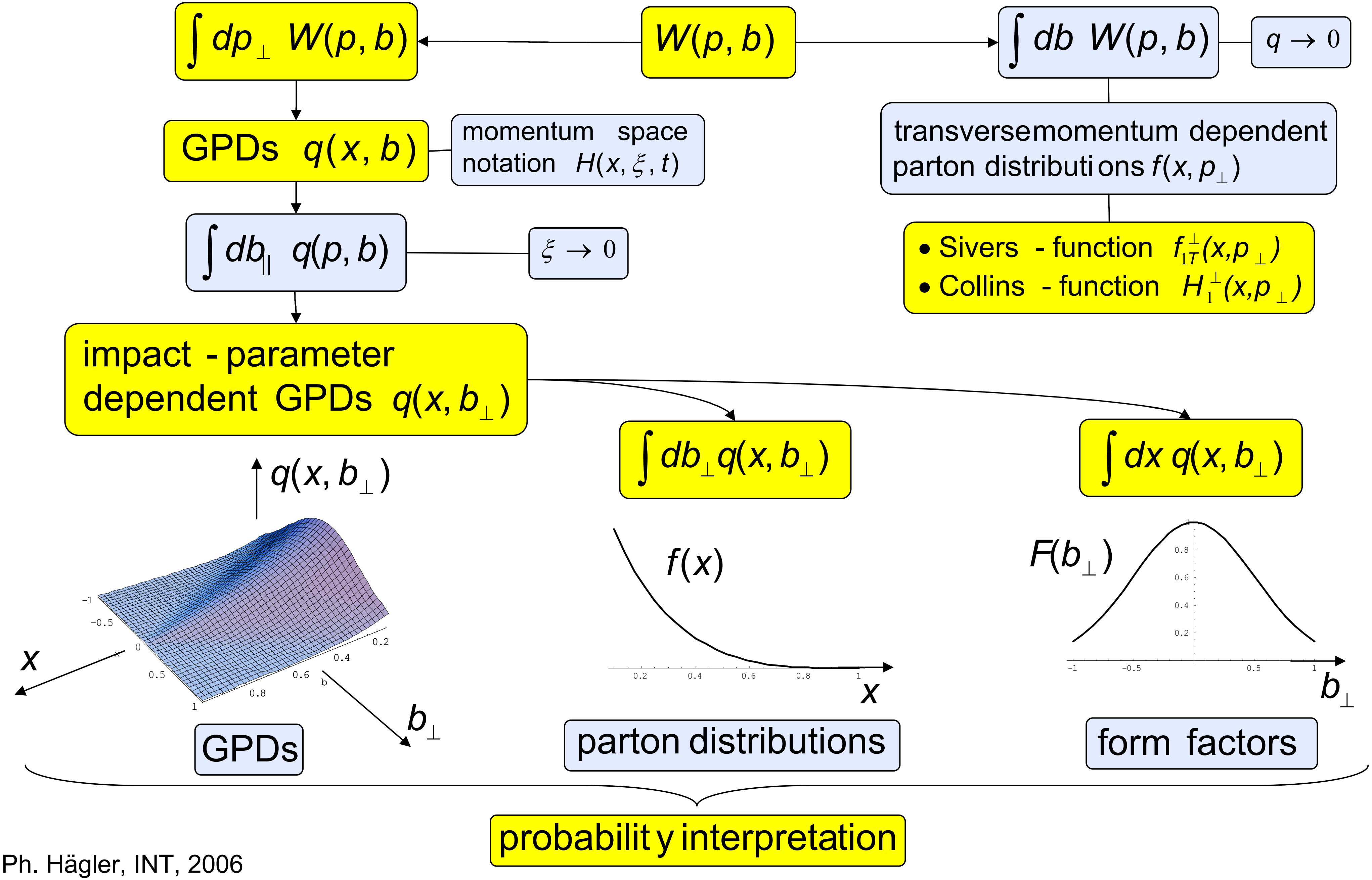
$$W(p,b) \propto \int dr\ dq\ e^{ip\cdot r - iq\cdot b} \left\langle P + \frac{q}{2} \right| \psi^{\dagger}\left(\frac{r}{2}\right) \psi\left(-\frac{r}{2}\right) \left| P - \frac{q}{2} \right\rangle$$

$p \hat{=}$ intrinsic quark momentum
$b \hat{=}$ impact parameter
$q \hat{=}$ momentum transfer



- highly interesting but complex objects
- simplify the job → projections
  ($\hat{=}$ integration over p and/or b)

Momentum/coordinate-projections



$\int dp_\perp \; W(p,b)$

$W(p,b)$

$\int db \; W(p,b)$

$q \to 0$

GPDs $q(x,b)$

momentum space notation $H(x,\xi,t)$

transversemomentum dependent parton distributions $f(x,p_\perp)$

$\int db_{||} \; q(p,b)$

$\xi \to 0$

- Sivers - function $f_{1T}^{\perp}(x,p_\perp)$
- Collins - function $H_1^{\perp}(x,p_\perp)$

impact - parameter dependent GPDs $q(x,b_\perp)$

$\int db_\perp q(x,b_\perp)$

$\int dx \; q(x,b_\perp)$

$q(x,b_\perp)$

$f(x)$

$F(b_\perp)$

GPDs

parton distributions

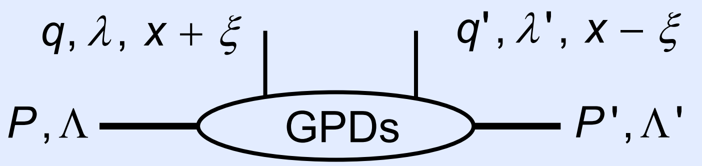
form factors

probability interpretation

Ph. Hägler, INT, 2006

Brief introduction to GPDs

Müller,Robaschik,Geyer,
Dittes,Horejsi,1994
Ji, 1997, Radyushkin, 1997



$t = (\Delta \equiv P' - P)^2 \,\hat{=}$ momentum transfer squared

$\xi = -n \cdot \Delta / 2 \,\hat{=}$ longitudinal momentum transfer

8 real functions needed for a complete description of the nucleon quark structure at twist 2
(M. Diehl, EPJ C19, 2001)

$H(x,\xi,t), E$
$\tilde{H}, \tilde{E}$
$H_T, \bar{E}_T, \tilde{H}_T, \tilde{E}_T$

$$\int \frac{d\eta}{4\pi} e^{i\eta x} \langle P' | \bar{q}(-\frac{\eta n}{2}) \gamma^\mu \mathcal{U} q(\frac{\eta n}{2}) | P \rangle = \bar{U}(P') \left(\gamma^\mu H(x,\xi,\Delta^2) + i \frac{\sigma^{\mu\nu}\Delta_\nu}{2M} E(x,\xi,\Delta^2) \right) U(P)$$

vector

$$\int \frac{d\eta}{4\pi} e^{i\eta x} \langle P' | \bar{q}(-\frac{\eta n}{2}) \gamma^\mu \gamma^5 \mathcal{U} q(\frac{\eta n}{2}) | P \rangle = \bar{U}(P') \left(\gamma^\mu \gamma^5 \tilde{H}(x,\xi,\Delta^2) + \frac{\gamma^5 \Delta^\mu}{2M} \tilde{E}(x,\xi,\Delta^2) \right) U(P)$$

axial-vector

$$\int \frac{d\eta}{4\pi} e^{i\eta x} \langle P' | \bar{q}(-\frac{\eta n}{2}) \sigma^{\mu\nu} \gamma_5 \mathcal{U} q(\frac{\eta n}{2}) | P \rangle = \bar{U}(P') \left\{ \sigma^{\mu\nu} \gamma_5 \left(H_T(x,\xi,\Delta^2) - \frac{t}{2m^2} \tilde{H}_T(x,\xi,\Delta^2) \right) \right.$$
$$\left. + \frac{\epsilon^{\mu\nu\alpha\beta} \Delta_\alpha \gamma_\beta}{2m} \bar{E}_T(x,\xi,\Delta^2) + \frac{\Delta^{[\mu} \sigma^{\nu]\alpha} \gamma_5 \Delta_\alpha}{2m^2} \tilde{H}_T(x,\xi,\Delta^2) + \frac{\epsilon^{\mu\nu\alpha\beta} \bar{P}_\alpha \gamma_\beta}{m} \tilde{E}_T(x,\xi,\Delta^2) \right\} U(P)$$

tensor

Some basic properties of GPDs

forward limit

$$H(x,0,0) = q(x) \hat{=} 1/2\left(\textcircled{\rightarrow}\rightarrow + \textcircled{\leftarrow}\rightarrow \right)$$

$$\tilde{H}(x,0,0) = \Delta q(x) \hat{=} \textcircled{\rightarrow}\rightarrow - \textcircled{\leftarrow}\rightarrow$$

$$H_T(x,0,0) = \delta q(x) = h_1(x) \hat{=} \overset{\uparrow}{\textcircled{\uparrow}} - \overset{\uparrow}{\textcircled{\downarrow}}$$

„local“ limit

$$\int dx H(x,\xi,t) = F_1(t),$$

$$\int dx \tilde{H}(x,\xi,t) = g_A(t),$$

$$\int dx H_T(x,\xi,t) = g_T(t) \text{ etc.}$$

---

decomposition of the nucleon total spin

(O)AM of quarks in the nucleon

$$\frac{1}{2} = J_N = \frac{1}{2}\left(< x >_{q+g} + E^{n=2}(0) \right) = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

X. Ji

GPDs $H, E, \tilde{H}$

Quark densities in the transverse plane

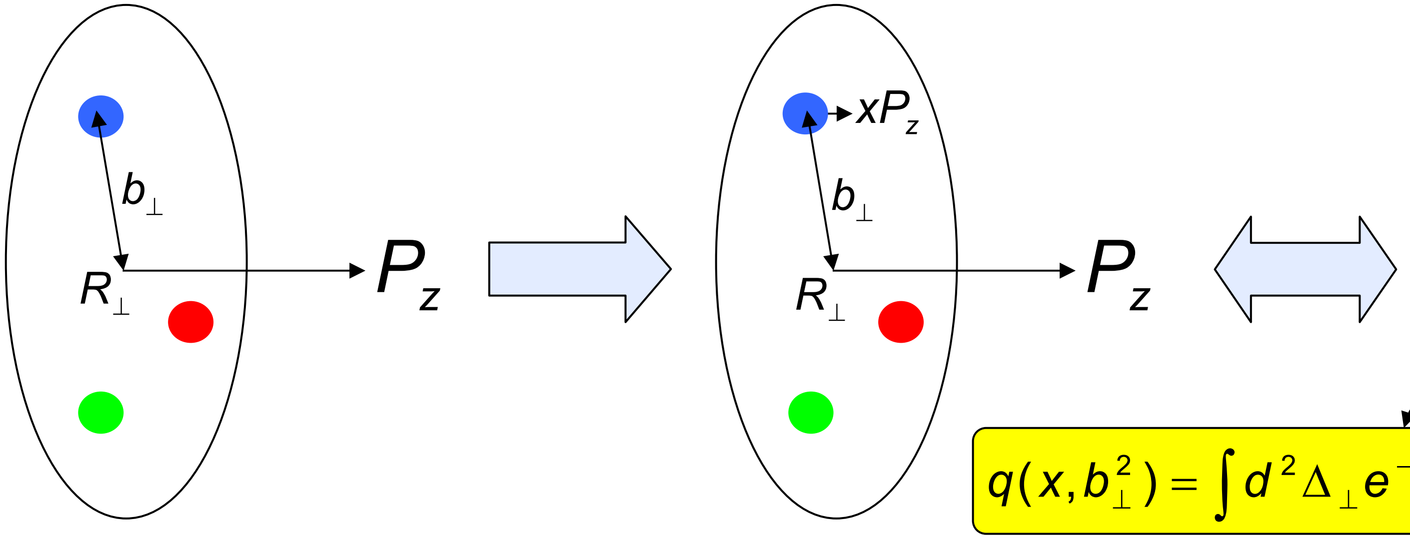
Fourier-transformation of (generalized) form factors gives probability densities!

M. Burkardt, 2000

$$q(b_\perp^2) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} F_1(\Delta^2)$$

probability interpretation
of quark distributions
in transverse coordinate space

$b_\perp \hat{=}$ distance of (active) quark to
the center of momentum $R_\perp$
in a nucleon with large P_z



$$q(x, b_\perp^2)$$

$$q(x, b_\perp^2) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H(x, \xi = 0, \Delta_\perp^2)$$

Parametrization in terms of e.g. tensor GFFS

lattice calculation requires Mellin moments $\int_{-1}^{1} dx\, x^n$ of matrix elements → local op's

$2\lfloor n/2 \rfloor + n + 3$ GFFs ✓

$$A_{\mu\nu} S_{\nu\mu_1\ldots\mu_n} \langle P'|\bar{q}(0)\sigma^{\mu\nu}\gamma_5 iD^{\mu_1}\ldots iD^{\mu_n} q(0)|P\rangle =$$

$$A_{\mu\nu} S_{\nu\mu_1\ldots\mu_n} \bar{U}(P')\Big\{\sum_{i=0,\text{even}}^{n}\Big(\sigma^{\mu\nu}\gamma_5\Delta^{\mu_1}\ldots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\ldots\bar{P}^{\mu_n}\Big[A_{\mathrm{T}n+1i}(\Delta^2)-\frac{t}{2m^2}\tilde{A}_{\mathrm{T}n+1i}(\Delta^2)\Big]$$

$$+\frac{\in^{\mu\nu\alpha\beta}\Delta_\alpha\gamma_\beta}{2m}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\ldots\bar{P}^{\mu_n}B_{\mathrm{T}n+1i}(\Delta^2)$$

$$+\frac{\Delta^{[\mu}\sigma^{\nu]\alpha}\gamma_5\Delta_\alpha}{2m^2}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\ldots\bar{P}^{\mu_n}\tilde{A}_{\mathrm{T}n+1i}(\Delta^2)\Big)$$

$$+\sum_{i=0,\text{odd}}^{n}\frac{\in^{\mu\nu\alpha\beta}\bar{P}_\alpha\gamma_\beta}{m}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\ldots\bar{P}^{\mu_n}\tilde{B}_{\mathrm{T}n+1i}(\Delta^2)\Big\}U(P)$$

Ph.H., PLB 594(2004),
see also Chen/Ji, PRD 71(2005)

how to get back the moments of the GPDs from the GFFs? →

$$H_T{}^n(\xi,\Delta^2) = A_{\mathrm{T}n+10}(\Delta^2) + \sum_{i=2,\text{even}}^{n}(-2\xi)^i A_{\mathrm{T}n+1i}(\Delta^2),$$

$$\bar{E}_T{}^n = \bar{B}_{\mathrm{T}n+10}(\Delta^2) + \sum_{i=2,\text{even}}^{n}(-2\xi)^i \bar{B}_{\mathrm{T}n+1i}(\Delta^2) \quad \text{etc.}$$

Computation of moments of GPDs on the lattice

(1)

$$C_{3pt}^{\mu_1\mu_2\ldots}(\tau,P',P) = Tr\left\{\tilde{\Gamma}\left\langle N(\tau_{snk},P')\mathcal{O}_{\Gamma}(\tau,\Delta)\overline{N}(\tau_{src},P)\right\rangle\right\}$$

$$\mathcal{O}_{\Gamma}(\tau,\Delta) = \sum_{x} e^{ix\Delta}\overline{q}(\tau,x)\Gamma iD^{\mu_1}\ldots iD^{\mu_n}q(\tau,x)$$

define 3pt-function and choose operator

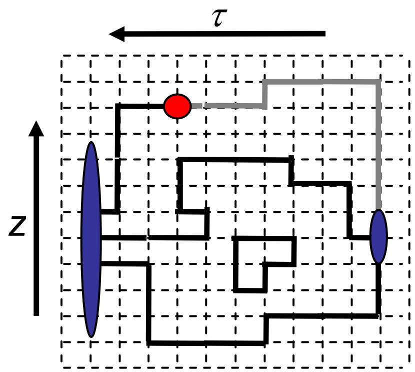
(2)

$$N_i(t,P) = \sum_{x} e^{-ixP} \in_{abc} u_i^a(t,x)Tr\left\{\left(u^b(t,x)\right)^T C\gamma_5 d^c(t,x)\right\}$$

choose sink/source operators

(3)

$$C_{3pt}^{\mu_1\mu_2\ldots}(\tau,P',P=P'-\Delta) \propto \sum_{x',x'',y,y'} e^{i\Delta x'}e^{iP'x''}$$
$$Tr\left\{\tilde{\Gamma}G(x'',y')K_{\Gamma}^{\mu_1\mu_2\ldots}(y',y,x')G(y,x=0)\right\}$$
$$Tr\left\{\tilde{G}(x'',x=0)G(x'',x=0)\right\}+\ldots$$



$$C_{3pt}^{\mu_1\mu_2\ldots}(\tau,P',P=P'-\Delta) \propto \sum_{x',y,y'} e^{i\Delta x'}$$
$$Tr\left\{\Omega_{\tilde{\Gamma}}(P';x=0,y')K_{\Gamma}^{\mu_1\mu_2\ldots}(y',y,x')G(y,x=0)\right\}+\ldots$$

get rid of all-to-all propagators

(4)

1. first inversion $\rightarrow G(z, x=0), \forall z$
2. second inversion $\rightarrow \Omega_{\tilde\Gamma}(P'; x=0, y')$ for fixed $P', \tilde\Gamma$
3. glue together with the operator $K_\Gamma^{\mu_1\mu_2\cdots}(y', y, x')$

$\rightarrow C_{3\,pt\,,\mathrm{lattice}}^{\mu_1\mu_2\cdots}(\tau, P', P)$

(5)

renormalize

$C_{3\,pt\,,\mathrm{lattice}}^{\mu_1\mu_2\cdots}(\tau, P', P) \rightarrow C_{3\,pt\,,\,\overline{\mathrm{MS}}}^{\mu_1\mu_2\cdots}(\tau, P', P)$

(6)

„transfer matrix formalism“

parametrize matrix element of local ops in terms of generalized form factors

$$C_{3\,pt}^{\mu_1\mu_2\cdots}(\tau, P', P) \propto Z e^{-E(P')(t_{\mathrm{snk}}-\tau)} e^{-E(P)\tau}\, Tr\left\{\tilde\Gamma(\not P'+m)\left(a_\Gamma^{\mu_1\mu_2\cdots}A(t)+b_\Gamma^{\mu_1\mu_2\cdots}B(t)+\ldots\right)(\not P+m)\right\}+\ldots$$

(7)

$$\frac{\langle N(P')|\mathcal{O}_\Gamma|N(P)\rangle}{\sqrt{E(P')E(P)}} \,\hat{=}\, \frac{C_{3pt}(\tau,P,P')}{C_{2pt}(\tau_{snk},P')}\left(\frac{C_{2pt}(\tau_{snk}-\tau,P)C_{2pt}(\tau,P')C_{2pt}(\tau_{snk},P')}{C_{2pt}(\tau_{snk}-\tau,P')C_{2pt}(\tau,P)C_{2pt}(\tau_{snk},P)}\right)^{\frac{1}{2}} \equiv \frac{R(\tau,P',P)}{\sqrt{E(P')E(P)}}$$

(8)

finally, equate the lattice ratio 3pt/2pts
and the continuum parametrization for
different momenta, indices and projectors
simultaneously (for a given momentum transfer^2 t)

extract the GFFs from
overdetermined
set of equations

enjoy

H(4) tensor operator index combinations on the lattice for n=1,2

$n=1$ → $\tau_1^{(6)}: O^{n=1}_{T\mu\nu},\ \mu<\nu$ →

$\#$ of Ops $=6$

$\#$ of FFs $=3$

$n=2$ →

$$\tau_1^{(8)}: O^{n=2}_{T1} = \frac{1}{2^{1/2}}\left(O_{T122} - O_{T133}\right),\ O^{n=2}_{T2} = \frac{1}{6^{1/2}}\left(O_{T122} + O_{T133} - 2O_{T144}\right)$$

$$O^{n=2}_{T3} = \frac{1}{2^{1/2}}\left(O_{T211} - O_{T233}\right),\ O^{n=2}_{T4} = \frac{1}{6^{1/2}}\left(O_{T211} + O_{T233} - 2O_{T244}\right)$$

$$O^{n=2}_{T5} = \frac{1}{2^{1/2}}\left(O_{T311} - O_{T322}\right),\ O^{n=2}_{T6} = \frac{1}{6^{1/2}}\left(O_{T311} + O_{T322} - 2O_{T344}\right)$$

$$O^{n=2}_{T7} = \frac{1}{2^{1/2}}\left(O_{T411} - O_{T422}\right),\ O^{n=2}_{T8} = \frac{1}{6^{1/2}}\left(O_{T411} + O_{T422} - 2O_{T433}\right)$$

$$\tau_2^{(8)}: O^{n=2}_{T1} = \left(\frac{2}{3}\right)^{1/2}\left(2O_{T\mu\{\nu\rho\}} + O_{T\nu\{\mu\rho\}}\right),\ O^{n=2}_{T2} = 2^{1/2}O_{T\nu\{\mu\rho\}},\quad \mu<\nu<\rho$$

→

$\#$ of Ops $=16$

$\#$ of GFFs $=4$

Göckeler, Horsley, Ilgenfritz,
Perlt, Rakow, Schierholz, Schiller,
Phys.Rev.D54:5705-5714,1996
Göckeler, 2005

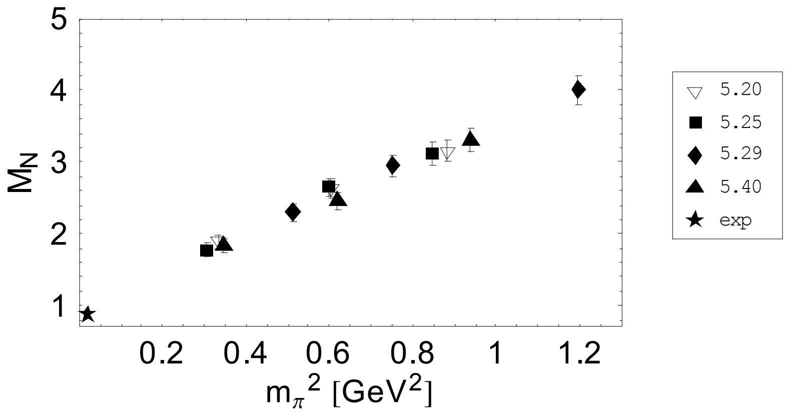
be aware of operator mixing for n>2:
Göckeler, Horsley, Perlt, Rakow, Schäfer,
Schierholz, Schiller, NPB 717, 2005

NP renormalization constants (M. Göckeler) for n=1,2 ✓

Lattice parameters – QCDSF/UKQCD

QCDSF/UKQCD - lattice - parameters :

- Wilson - fermions with (NP) clover - improvement
- unquenched calculation, but only connected contributions
- inverse lattice - spacing is $a^{-1} \approx 2$ GeV
- pretty large # of different β and κ available
- up to 1200 configurations for some $\beta,\kappa -$ combinations
- data for finite volume analysis available
- lattice spacing fixed using $r_0 = 0.467 fm$
- three projectors $\tilde{\Gamma}_{unpol} = \frac{1}{2}(1+\gamma_0), \tilde{\Gamma}_{1,2} = \frac{1}{2}(1+\gamma_0)\gamma_5\gamma_{1,2}$
- three sink - momenta $p' = (0,0,0),(1,0,0),(0,1,0)$



#	1	2	3	4	5	6	7	8	9	10	11	12
β	5.20	5.20	5.20	5.25	5.25	5.25	5.29	5.29	5.29	5.40	5.40	5.40
κ	.13420	.13500	.13550	.13460	.13520	.13575	.13400	.13500	.13550	.13500	.13560	.13610
a[fm]	0.12	0.11	0.099	0.11	0.097	0.091	0.1	0.096	0.09	0.082	0.079	0.075
L[fm]	1.96	1.68	1.59	1.69	1.56	2.19	1.66	1.53	2.16	1.97	1.88	1.79
m_π[Gev]	0.94	0.777	0.578	0.92	0.774	0.553	1.09	0.867	0.716	0.969	0.788	0.588
$m_\pi \times$ L	9.36	6.64	4.66	7.89	6.11	6.14	9.23	6.74	7.85	9.68	7.48	5.3

The p-pole ansatz

p - pole ansatz $A(t) = \frac{A(0)}{\left(1 - t/m_p^2\right)^p}$ → FT → $A(b^2) = C(A(0),p)\, b^{p-1} K_{p-1}(m_p b)$

pro :

- describes almost all GFFs reasonably well over the whole range of momentum transfer
- simple relation to charge radius
- only 2(3) parameters
- simple Fourier - transform

contra :

- no sound theoretical foundation, purely phenomenological
- it is hard to determine m_p and p simultaneously from fit

(Mellin moments of) quark densities should be well behaved for $b \to 0$ and $b \to \infty$ →

for finite densities, we need

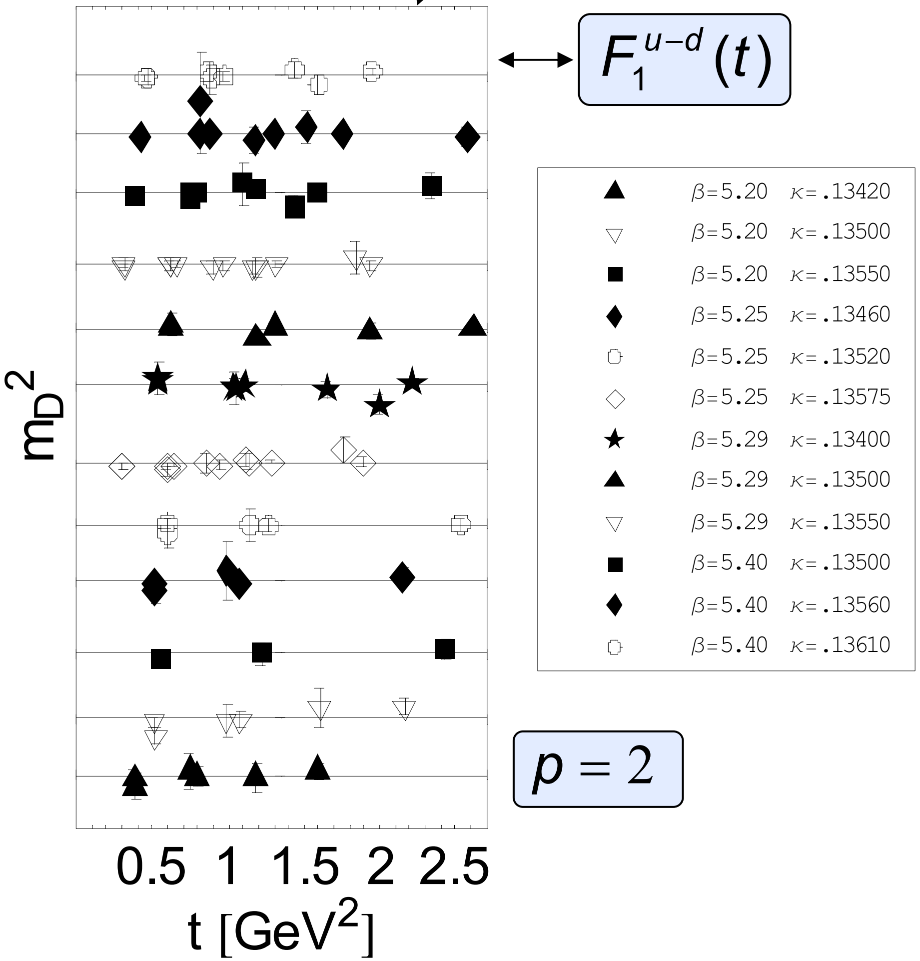
$H, \tilde{H}, H_T : \quad p > 1$

$E, \bar{E}_T : \quad p > \frac{3}{2}$

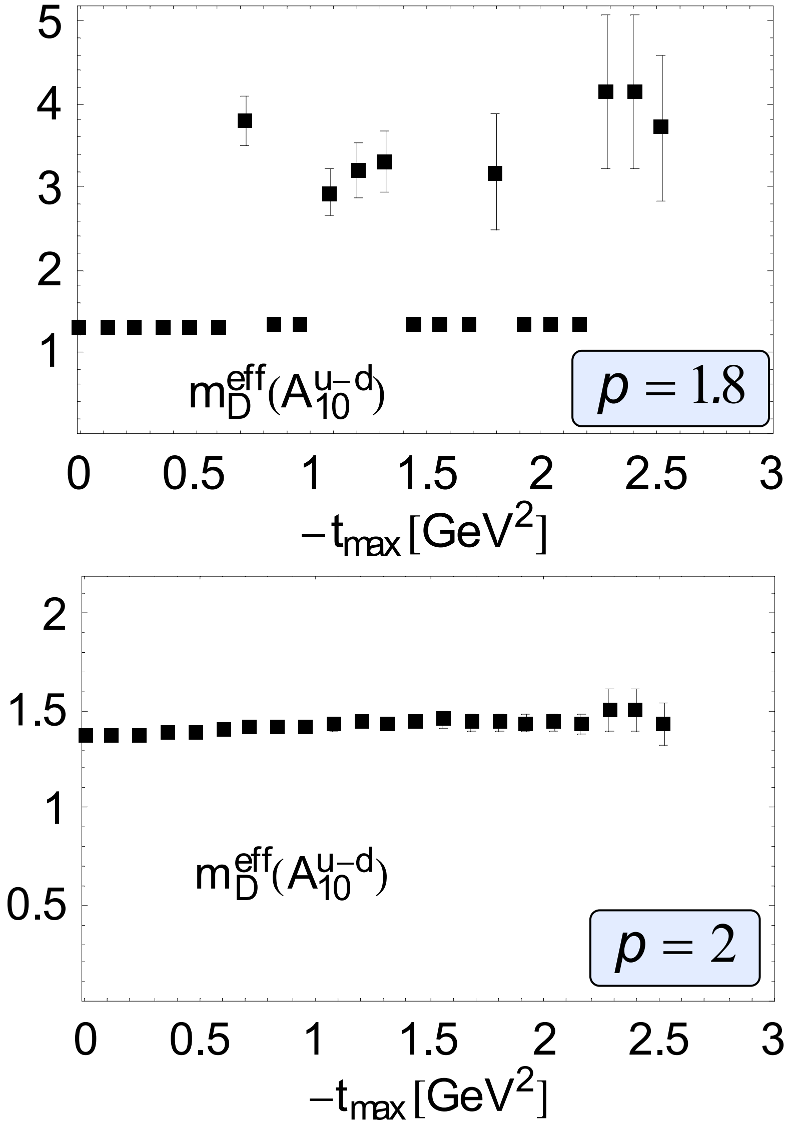
$\tilde{H}_T : \quad p > 2$

Testing the dipole parametrization

$$A_{n0}(t) = \frac{A_{n0}(t)}{\left(1 - t / m_{n,p}^2\right)^p} \rightarrow m_p^2 = m_p^2(t, A_{n0}(t))$$



global fit



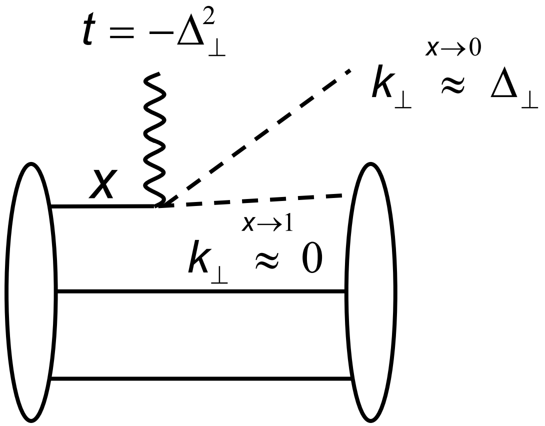
Ph. Hägler, INT, 2006

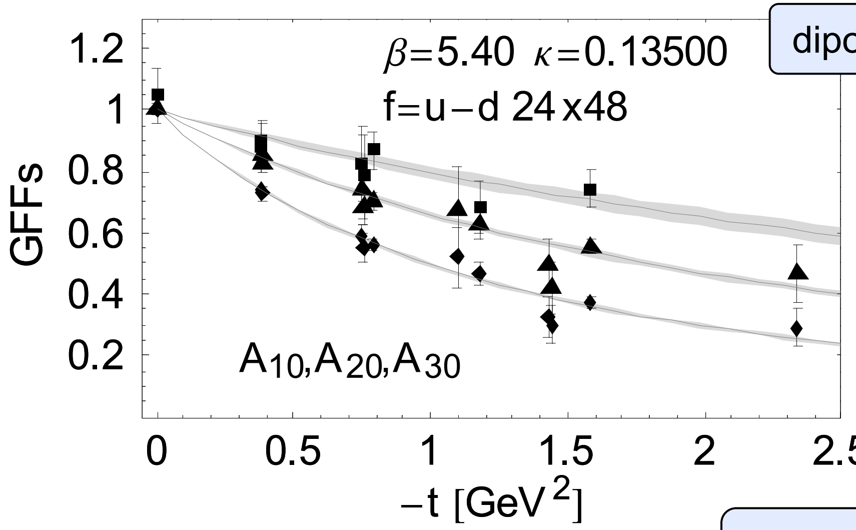
„3d“-nucleon structure (unpolarized) – momentum space

higher moments *n* correspond to larger momentum fraction *x*

$n \to \infty \quad \Leftrightarrow \quad x \approx 1$

$$\left\langle P^+, R_\perp = 0 \middle| \hat{\rho}(x, b_\perp) \middle| P^+, R_\perp = 0 \right\rangle = H(x, b^2) = q(x, b^2)$$



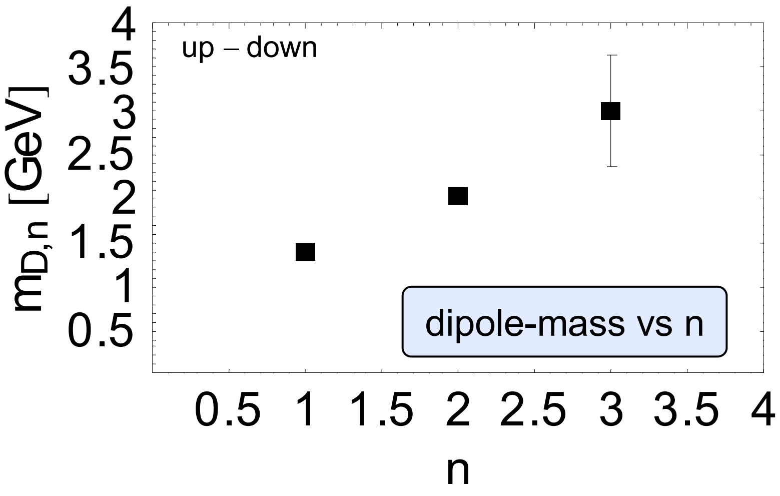


dipole

strong coupling larger <x>, n=3

weak coupling smaller <x>, n=1

first shown in LHPC/SESAM PRL 2004



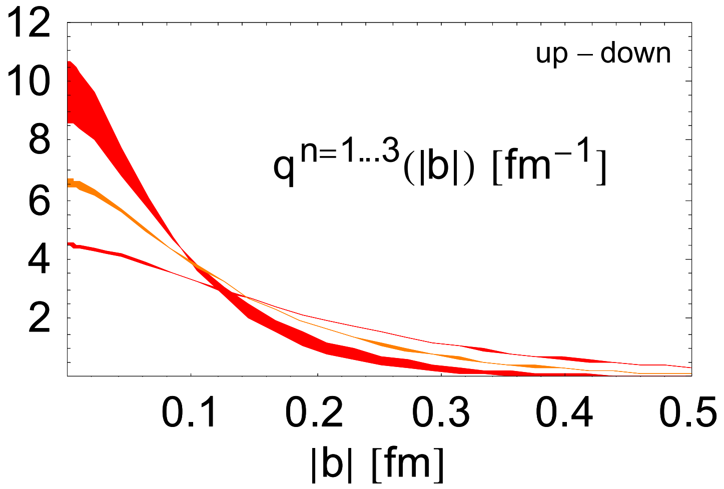
dipole-mass vs n

$\int dx\, x^{n-1}$

„3d“-nucleon structure (unpolarized) – coordinate space

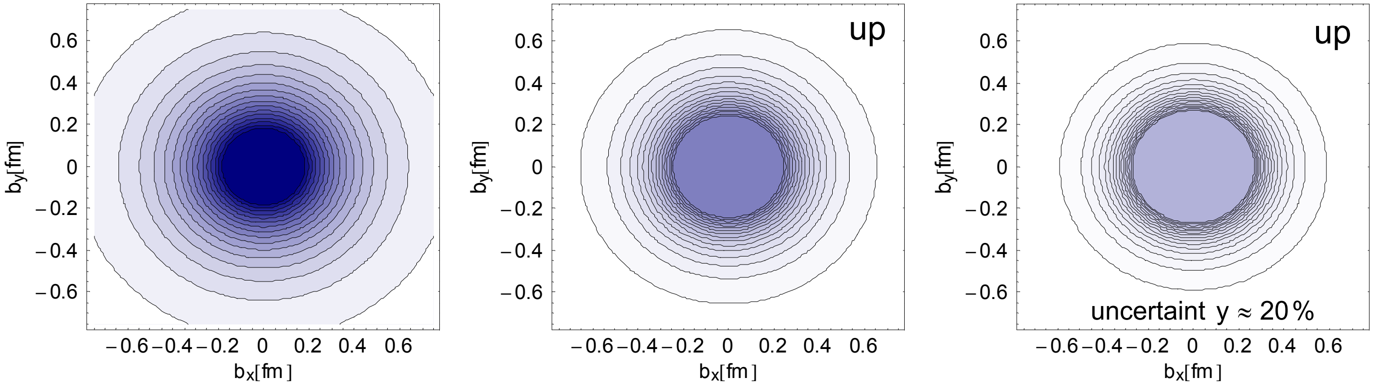
$b_\perp \hat{=}$ distance of active quark to the CM $R_\perp = \frac{\sum_i x_i r_{\perp,i}}{\sum_i x_i}$

as $x \to 1$, the distribution peaks around $R_\perp$ and $\langle b_\perp^2 \rangle^{1/2} \to 0$



charge radius vs <x>

$< x >= .265(6)$ $< x >= 0.37(2)$ $< x >= 0.40(5)$



(Transverse) spin structure of the nucleon

based on M.Diehl and Ph.H., EPJC 2005

goal: construct general spin density of quarks in the nucleon

nucleon states $|P,\Lambda,S_\perp\rangle$

$\Lambda \neq 0, S_\perp = 0 \quad \Leftrightarrow |P,\Lambda=\pm\rangle$ longitudinal

$\Lambda = 0, S_\perp = (1,0) \Leftrightarrow \sqrt{2}^{-1}\left(|P,+\rangle + |P,-\rangle\right)$ "transversity"

Goldstein/Moravcsik, 1976

quark projection operators

longitudinal $\hat{\rho}_L(x) \equiv \int \frac{d\eta}{2\pi} e^{i\eta x}\bar{q}(-\eta n/2)\gamma^+\left[1+\lambda\gamma_5\right]q(\eta n/2)$

transverse $\hat{\rho}_T(x) \equiv \int \frac{d\eta}{2\pi} e^{i\eta x}\bar{q}(-\eta n/2)\gamma^+\left[1+(s_\perp\cdot\gamma_\perp)\gamma_5\right]q(\eta n/2)$

$$= \int \frac{d\eta}{2\pi} e^{i\eta x}\bar{q}(-\eta n/2)\left[\gamma^+ + i s_{\perp j}\sigma^{+j}\gamma_5\right]q(\eta n/2)$$

$\langle P^+, P'_\perp, \Lambda, S_\perp | \hat{\rho}_L(x) | P^+, P_\perp, \Lambda, S_\perp \rangle$

$\langle P^+, P'_\perp, \Lambda, S_\perp | \hat{\rho}_T(x) | P^+, P_\perp, \Lambda, S_\perp \rangle$

parameterize in terms of the GPDs $H, \widetilde{H}, H_T \ldots$

almost there…

Fourier-transform to impact parameter space

$$\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ib_\perp \Delta_\perp}$$

$$\langle P^+, R_\perp = 0, \Lambda, S_\perp | \hat{\rho}_L(x, b_\perp) | P^+, R_\perp = 0, \Lambda, S_\perp \rangle$$
$$\langle P^+, R_\perp = 0, \Lambda, S_\perp | \hat{\rho}_T(x, b_\perp) | P^+, R_\perp = 0, \Lambda, S_\perp \rangle$$

densities of quarks with $x, b_\perp, \lambda, s_\perp$ in a nucleon with $P^+, R_\perp = 0, \Lambda, S_\perp$

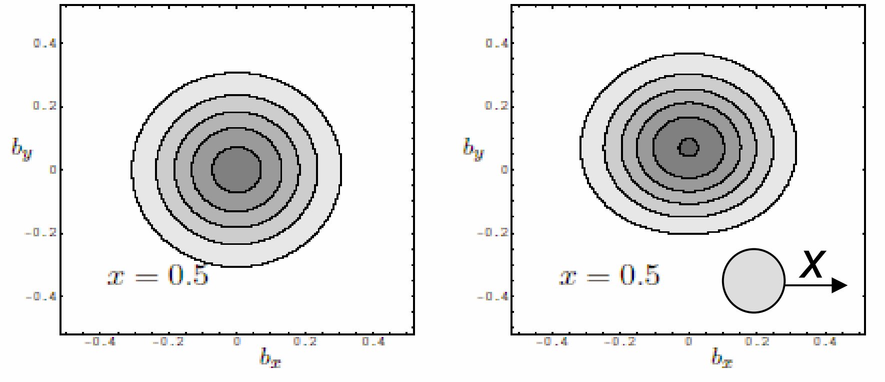
longitudinal quarks

$$\langle P^+, R_\perp = 0, \Lambda, S_\perp | \hat{\rho}_L(x, b_\perp) | P^+, R_\perp = 0, \Lambda, S_\perp \rangle = \frac{1}{2}\left[H(x, b^2) - \frac{S^i \epsilon^{ij} b^j}{M} \frac{\partial}{\partial b^2} E(x, b^2) + \lambda\Lambda \widetilde{H}(x, b^2) \right]$$

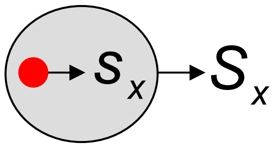
forward case: integrate over impact parameter

$$\langle P^+, P_\perp, \Lambda, S_\perp | \hat{\rho}_L(x) | P^+, P_\perp, \Lambda, S_\perp \rangle = \frac{1}{2}[q(x) + \lambda\Lambda\Delta q(x)]$$

in particular $H(x,b^2) - \frac{S^i \epsilon^{ij} b^j}{M} \frac{\partial}{\partial b^2} E(x,b^2)$ has been discussed by M. Burkardt and interpreted as transverse distortion of the nucleon

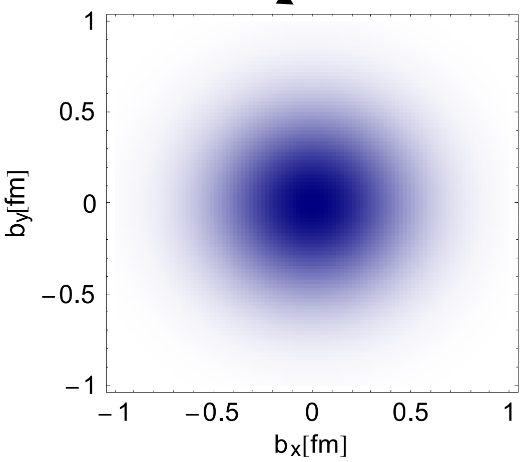


Spin densities in the transverse plane

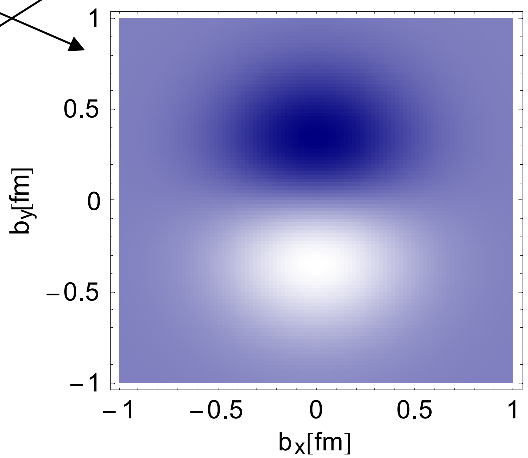


spin density for transversely polarized quarks

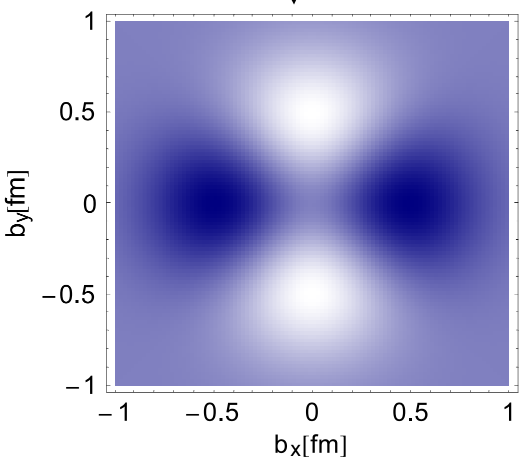
$$\left\langle P^+, R_\perp = 0, \Lambda, S_\perp \left| \hat{\rho}^n(b_\perp, s_\perp) \right| P^+, R_\perp = 0, \Lambda, S_\perp \right\rangle =$$
$$\frac{1}{2}\left[A_{n0} - \frac{S^i \epsilon^{ij} b^j}{M} B_{n0}' - \frac{s^i \epsilon^{ij} b^j}{M} \overline{B}_{Tn0}' + s^i S^i \left(A_{Tn0} - \frac{1}{4M^2} \Delta_b \tilde{A}_{Tn0} \right) + s^i \left(2b^i b^j - b^2 \delta^{ij} \right) S^j \frac{1}{M^2} \tilde{A}_{Tn0}'' \right]$$



monopole



dipole

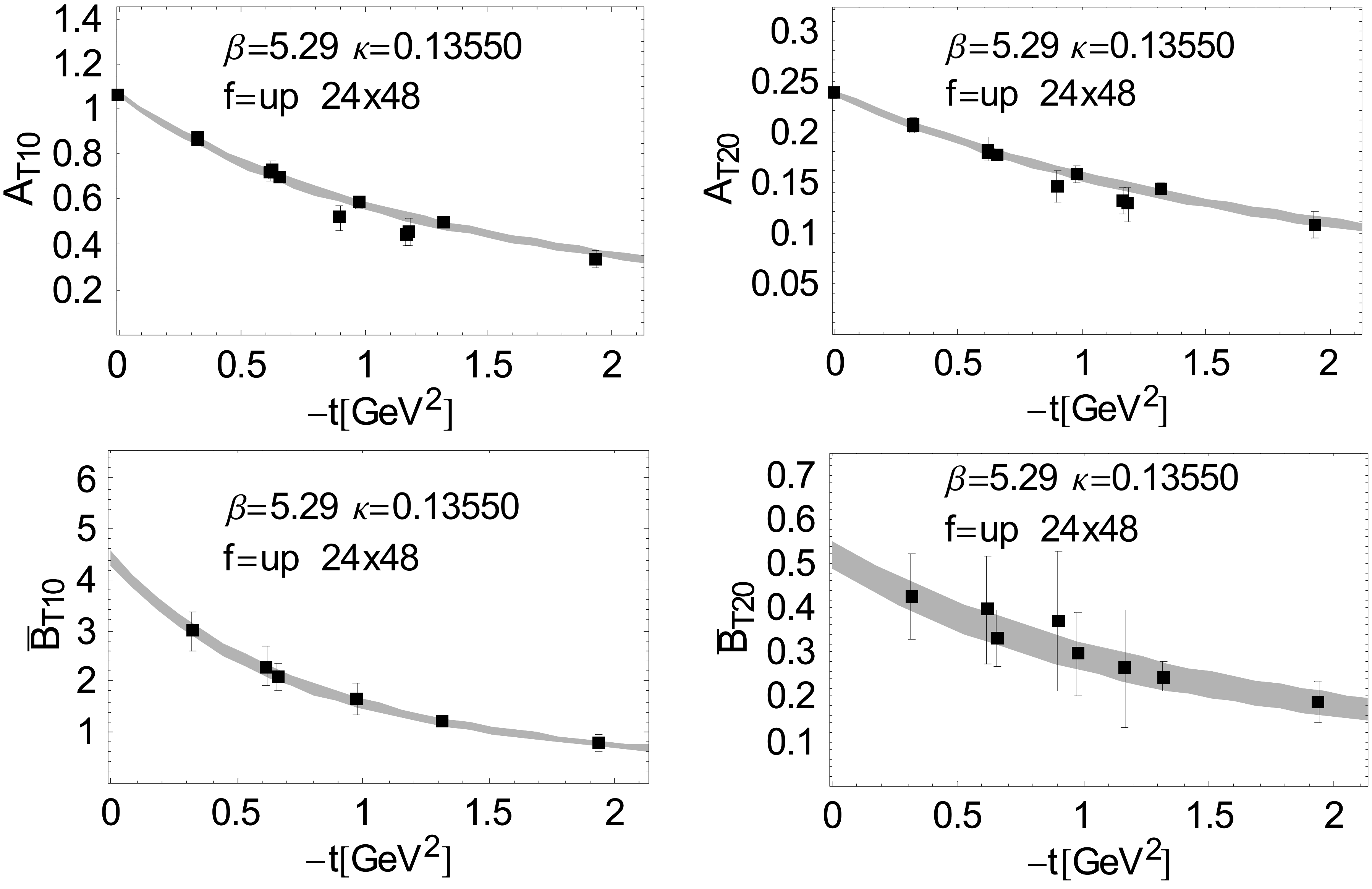


quadrupole

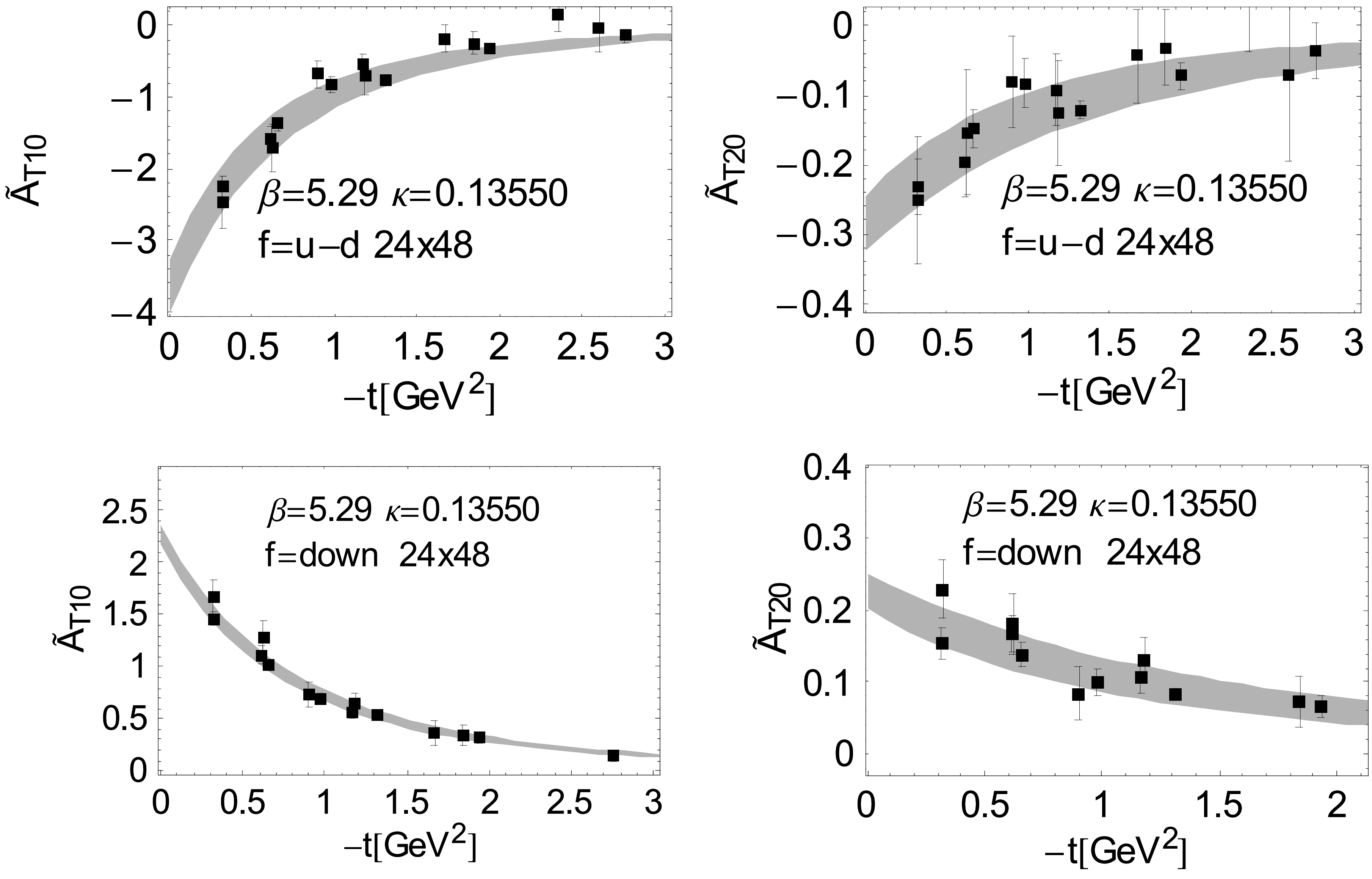
$$\hat{\rho}^n(b_\perp, s_\perp) = \int_{-1}^{1} dx x^{n-1} \int \frac{d\eta}{2\pi} e^{i\eta x} \overline{q}(-\eta n/2 b_\perp)\left[\gamma^+ + i s_{\perp j} \sigma^{+j} \gamma_5 \right] q(\eta n/2, b_\perp)$$

Ph. Hägler, INT, 2006

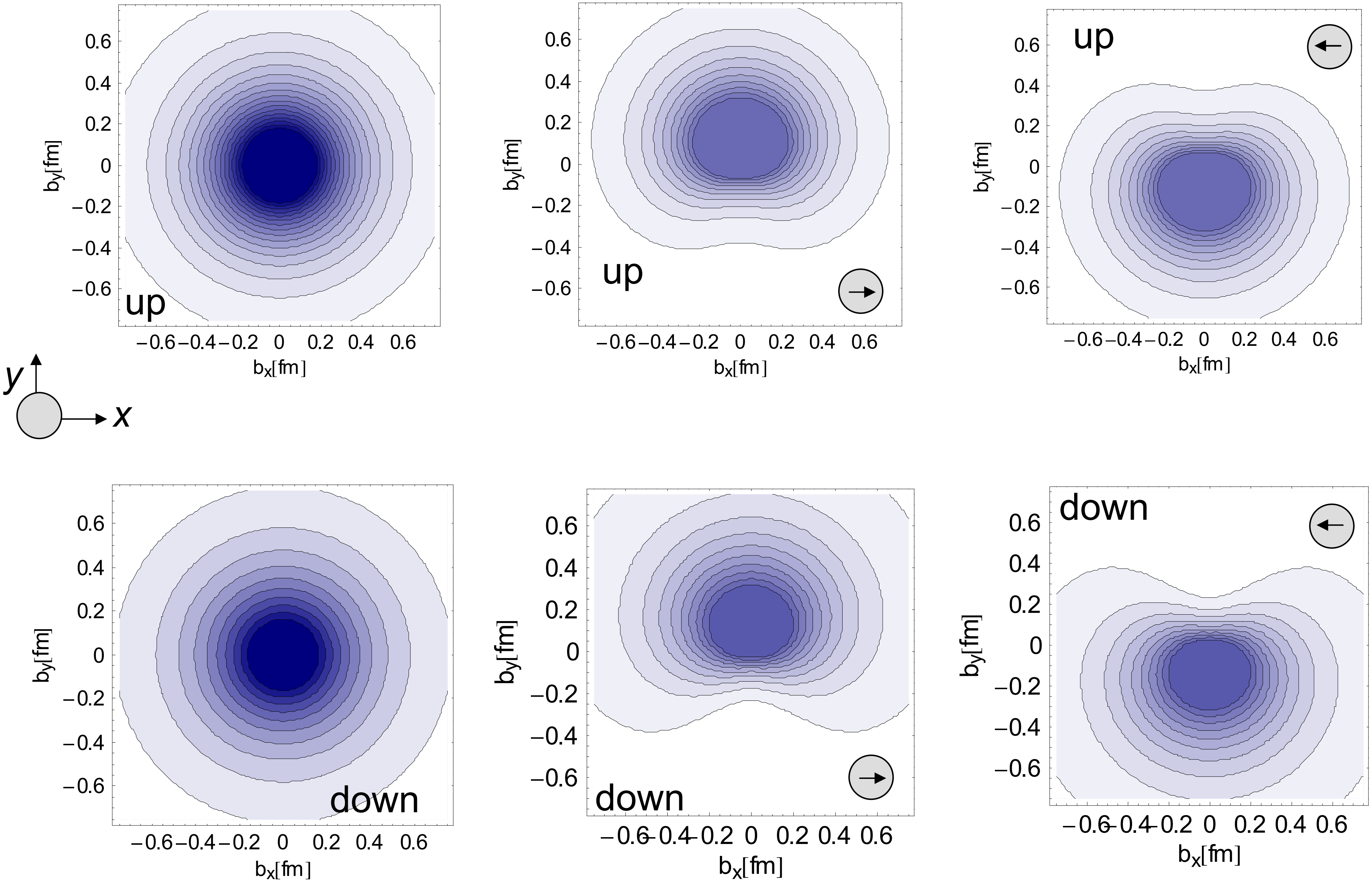
tensor GPDs – some „raw“ data



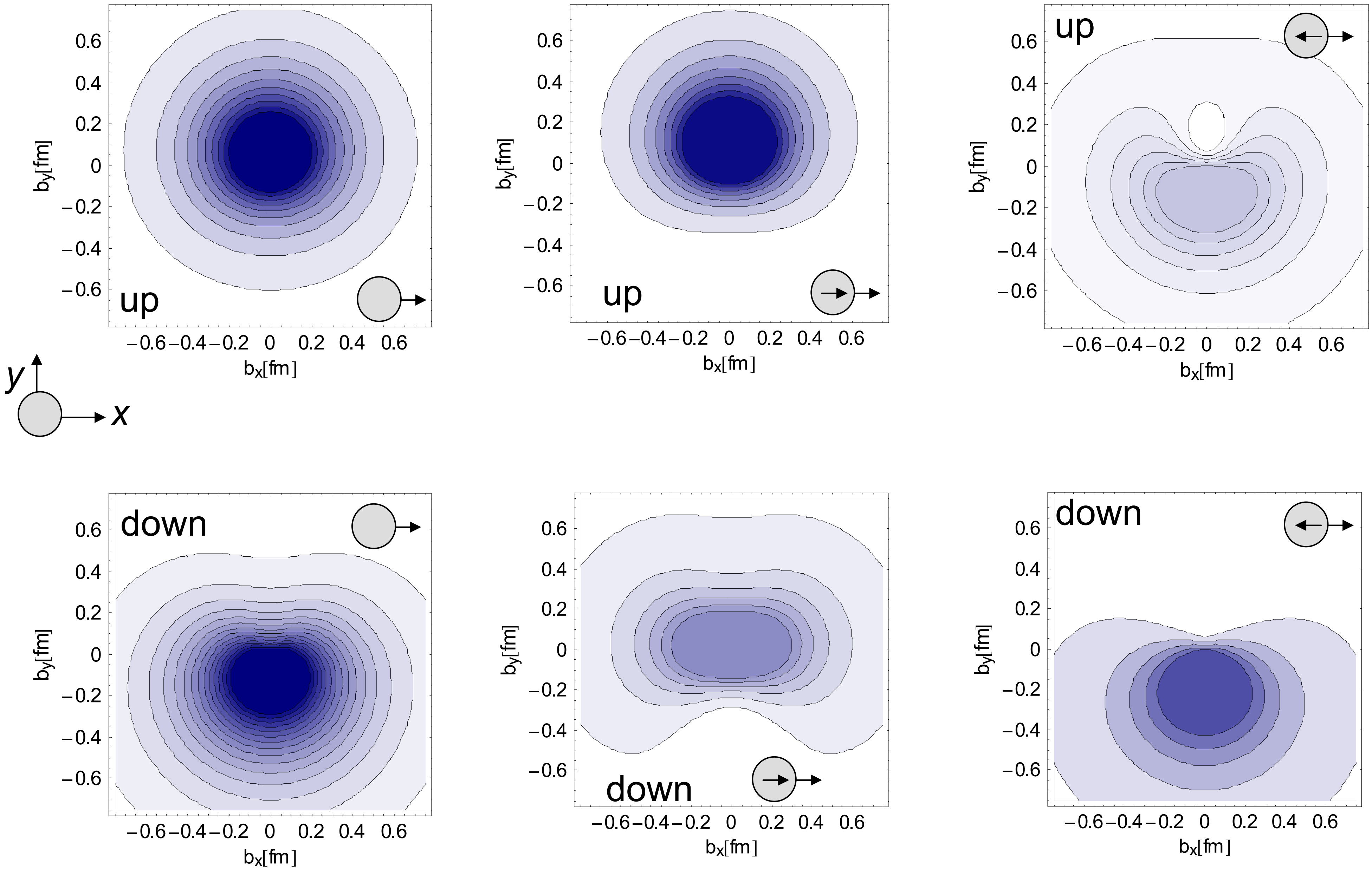
tensor GPDs – continued



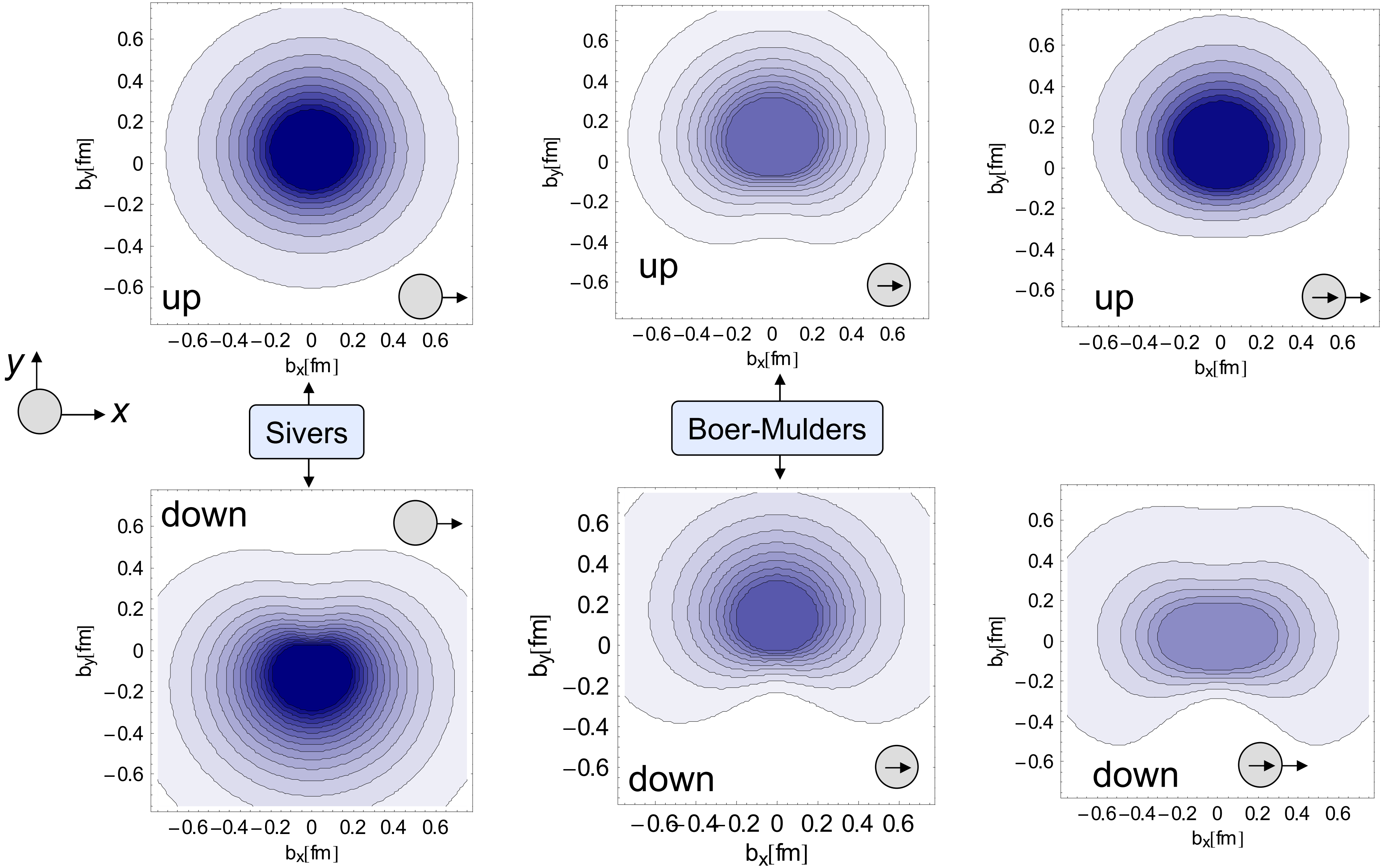
preliminary results for the spin densities I



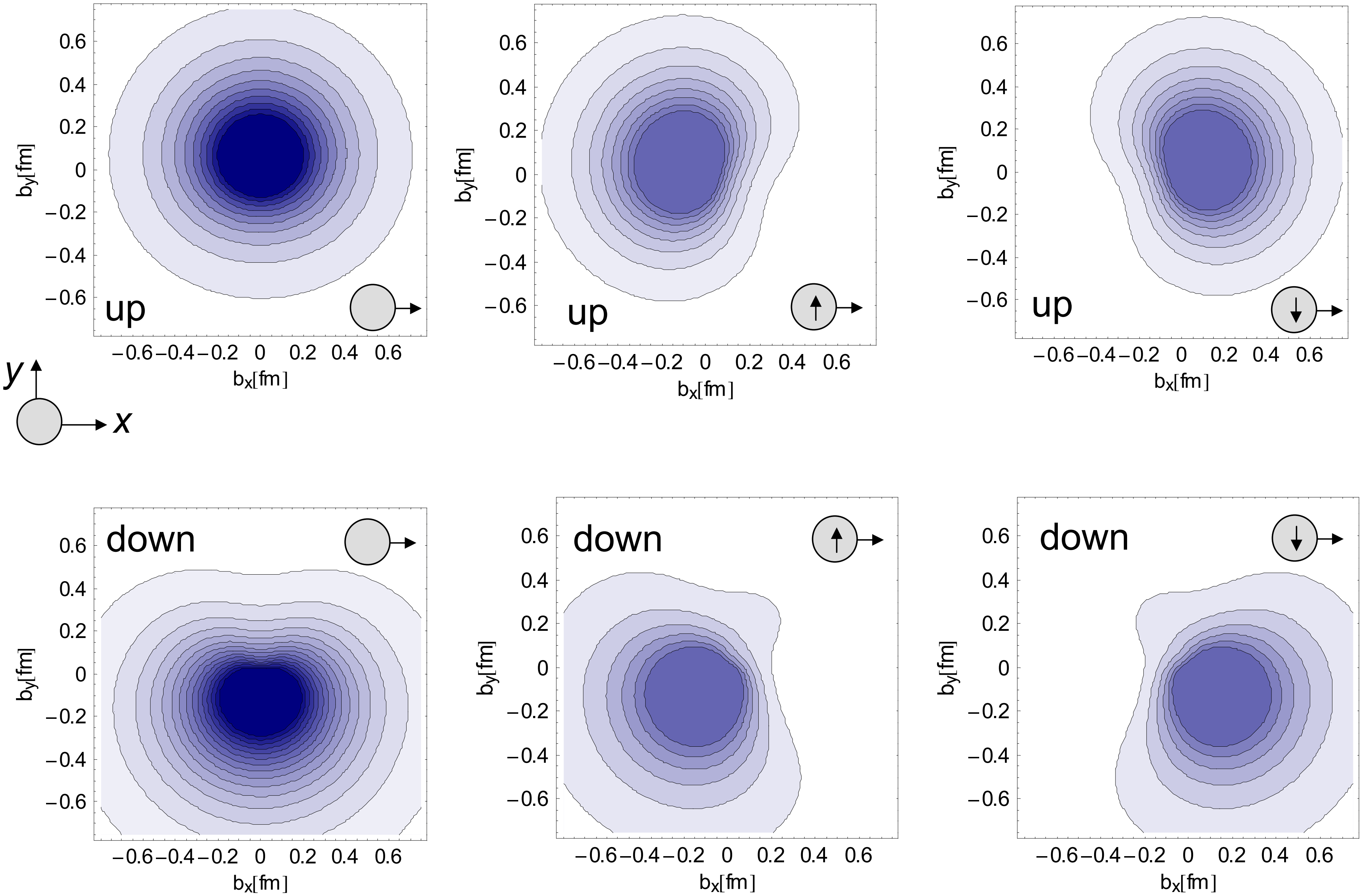
preliminary results for the spin densities II



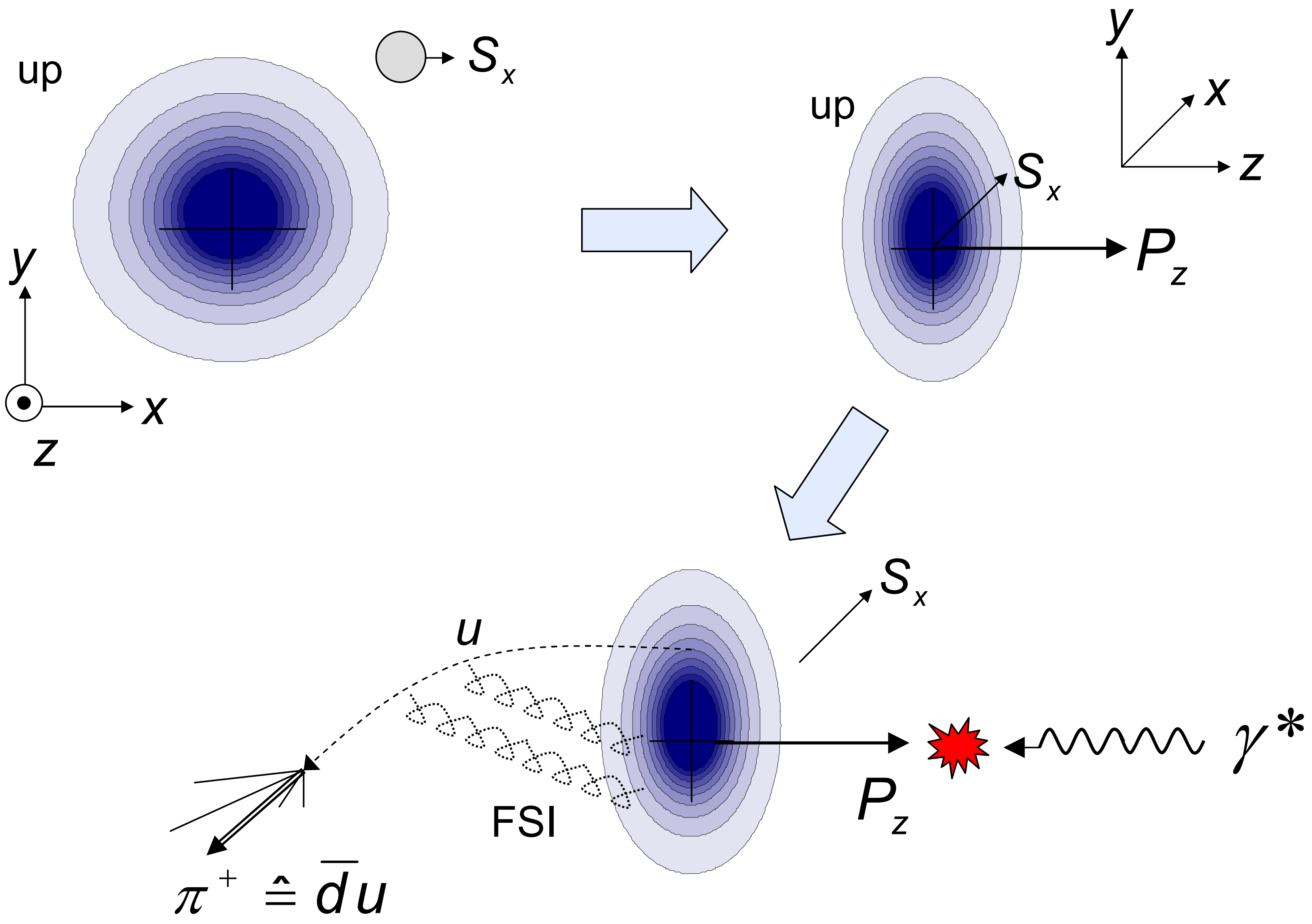
preliminary results for the spin densities III



preliminary results for the spin densities IV



Deformed quark densities and spin asymmetries



Sivers asymmetry/effect

M. Burkardt, 2003

Deformed quark densities and spin asymmetries II

Sivers - function $f_{1T}^{\perp}(x,p_{\perp})$ probes correlation of transverse nucleon spin and intrinsic transverse momentum

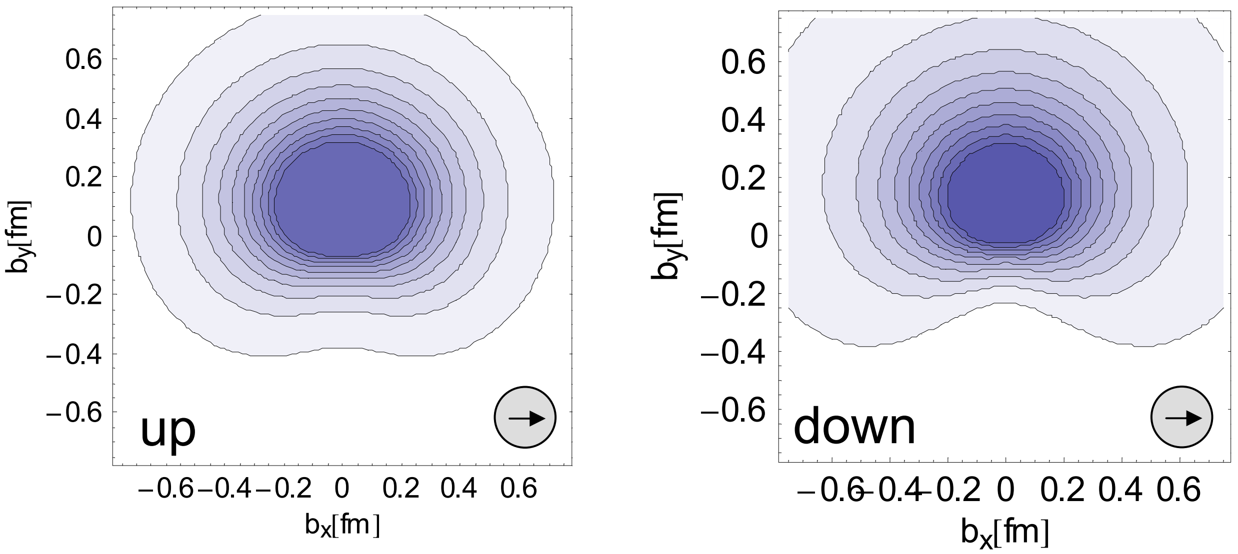
M.Burkardt : $f_{1Tq}^{\perp}(x,p_{\perp}) \sim -\kappa_q$

Boer - Mulders - function $h_1^{\perp}(x,p_{\perp})$ probes correlation of transverse quark spin and intrinsic transverse momentum

M.Burkardt :

$$h_{1q}^{\perp}(x,p_{\perp}) \sim -\int dx\overline{E}_{Tq}(x,0,0) = -\overline{B}_{Tq}(0)$$

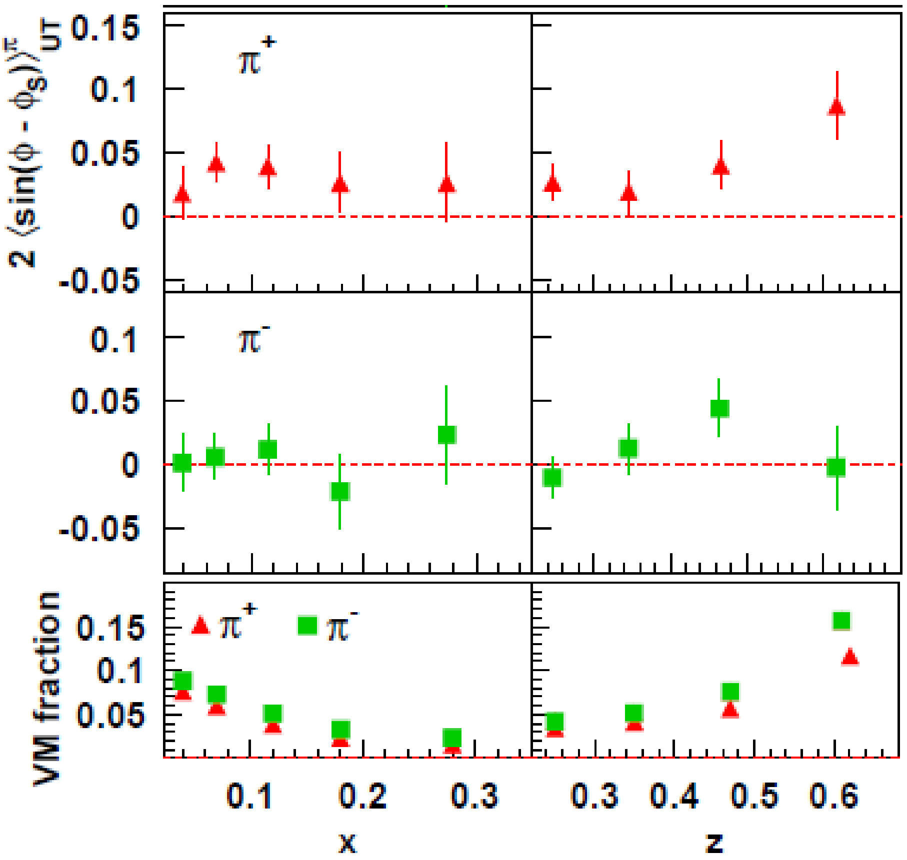
$$\overline{B}_{T(u,d),lattice}(0) \approx 2\ldots3$$



could lead to sizeable Boer-Mulders-effect

Single spin asymmetries in experiment

pion production from positrons on a transversely polarized hydrogen target



Sivers asymmetry

HERMES collaboration
PRL 2005

Ph. Hägler, INT, 2006

Improved positivity bounds including tensor GPDs

M.Diehl/Ph.H., EPJC 2005

full spin matrix, $\xi = 0$

$$\begin{array}{c|cccc}
\Lambda', \lambda' \downarrow \;\; \Lambda, \lambda \rightarrow & ++ & -+ & +- & -- \\
\hline
++ & H+\tilde{H} & -ie^{-i\varphi}\frac{b}{M}E & ie^{i\varphi}\frac{b}{M}\left(E_T'+2\tilde{H}_T'\right) & 2\left(H_T-\frac{1}{4M^2}\Delta_b\tilde{H}_T\right) \\
-+ & ie^{i\varphi}\frac{b}{M}E & H-\tilde{H} & 2ie^{2i\varphi}\frac{b^2}{M^2}\tilde{H}_T'' & ie^{i\varphi}\frac{b}{M}\left(E_T'+2\tilde{H}_T'\right) \\
+- & -ie^{-i\varphi}\frac{b}{M}\left(E_T'+2\tilde{H}_T'\right) & 2ie^{-2i\varphi}\frac{b^2}{M^2}\tilde{H}_T'' & H-\tilde{H} & -ie^{-i\varphi}\frac{b}{M}E \\
-- & 2\left(H_T-\frac{1}{4M^2}\Delta_b\tilde{H}_T\right) & -ie^{-i\varphi}\frac{b}{M}\left(E_T'+2\tilde{H}_T'\right) & ie^{i\varphi}\frac{b}{M}E & H+\tilde{H}
\end{array}$$

positivity of eigenvalues

Soffer - bound $\left[H+\tilde{H} \geq 2\left|H_T\right|\right]_{t=0} \Leftrightarrow q+\Delta q \geq 2\left|\delta q\right|$

bounds by Pobylitsa and Burkardt

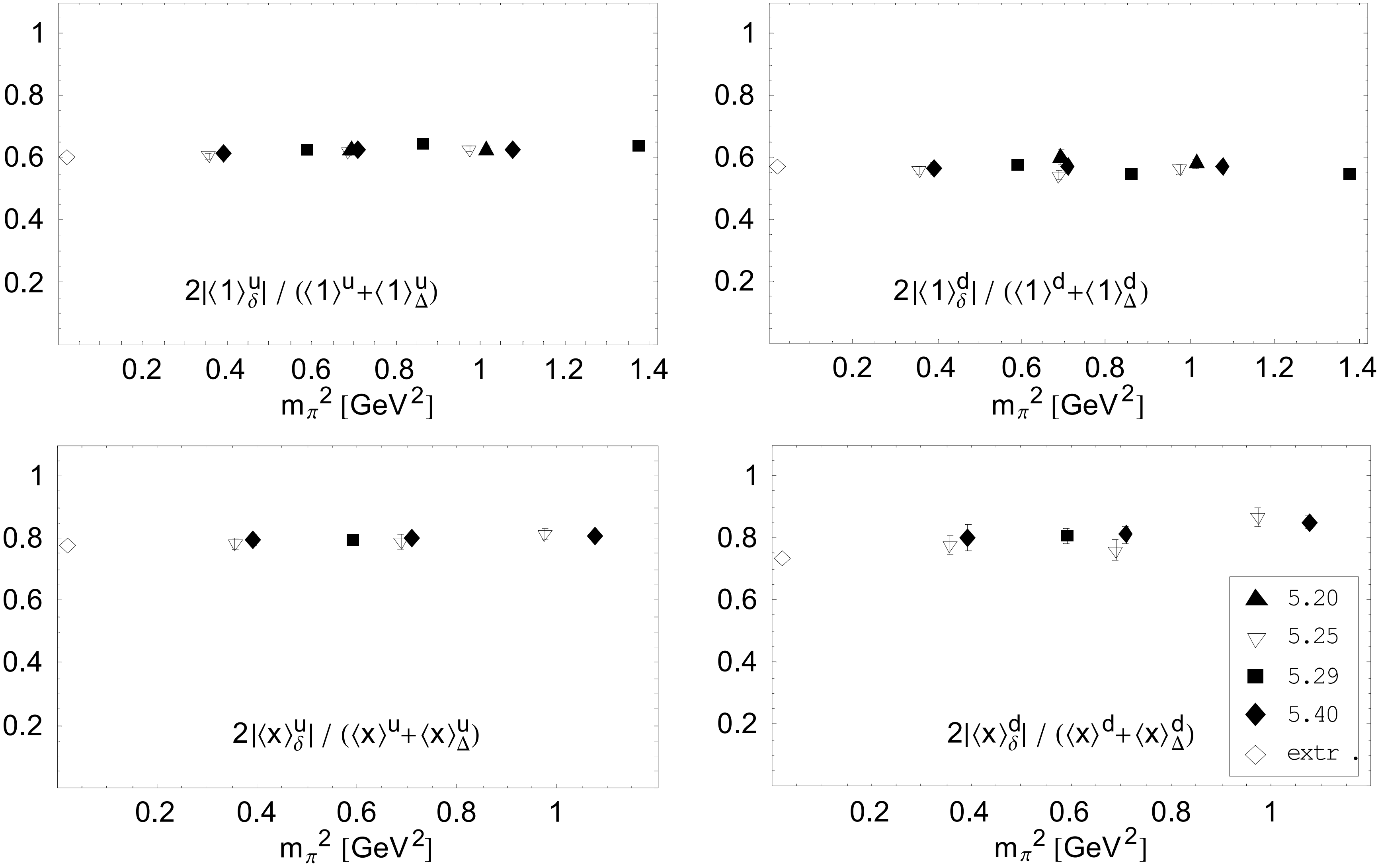
improved bounds including tensor GPDs

$$H \geq \left|H_T+\frac{1}{M^2}\tilde{H}_T'-\frac{1}{2M^2}\Delta_b\tilde{H}_T\right|$$

$$\left(H \pm H_T \pm \frac{1}{M^2}\tilde{H}_T' \mp \frac{1}{2M^2}\Delta_b\tilde{H}_T\right)^2-\left(\tilde{H} \pm H_T \mp \frac{1}{M^2}\tilde{H}_T'\right)^2 \geq \frac{b^2}{M^2}\left(E \pm E_T' \pm 2\tilde{H}_T'\right)^2$$

Lowest two moments of the Soffer - bound $q + \Delta q \geq 2|\delta q|$ in lattice QCD

based on QCDSF/UKQCD PLB 2005



Summary/Outlook

preliminary lattice results on transverse spin densities show complex interplay of spin and coordinate degrees of freedom

spin densities are strongly distorted
for transversely polarized quarks and nucleons

pointing towards sizeable single spin asymmetries

possible implications for sign/size of Sivers and Boer-Mulders functions (M. Burkardt)

(improve) chiral extrapolation – see e.g. hep-ph/0602200 by Ando, Chen, Kao

plan to study the second moment of the transverse spin-densities

investigate new positivity bounds on GPD