New techniques and results for excited hadron spectroscopy

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T. Burch et al, hep-lat/0601026
T. Burch et al, in preparation

Why study excited hadrons?

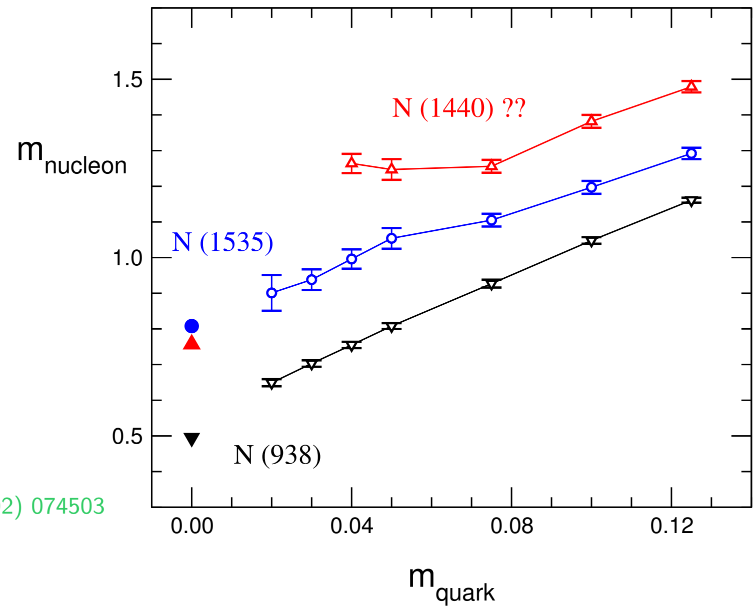
- There is no complete understanding of excited hadrons from a first principles lattice calculation.
- Getting the masses of hadron resonances right is a challenging task and a non-trivial test of QCD.
- The nature of some resonances is disputed - the lattice might be able to help clarifying it.
- Quantum numbers of several resonances are not known
  $\Rightarrow$ chance for predictions.
- Interesting chiral physics to be learned:
  - Important role of chiral symmetry for low-lying excitations.
  - Effective restoration of chiral symmetry for highly excited states.

Excited nucleons - experimental data:

$\frac{1}{2}^+$: N(938) *Nucleon*
N(1440) *Roper*

$\frac{1}{2}^-$: N(1535) *N-star*
N(1650) *N-star'*

A typical lattice result:
(not very old)

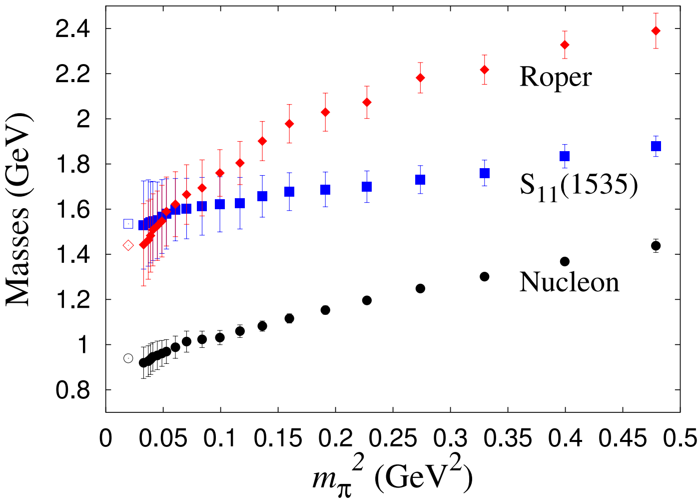


S. Sasaki, T. Blum, S. Ohta, PRD 65 (2002) 074503
Quenched domain wall fermions
$L \sim 1.6 fm\,,\; m_\pi \sim 450 MeV$

Two recent quenched results:

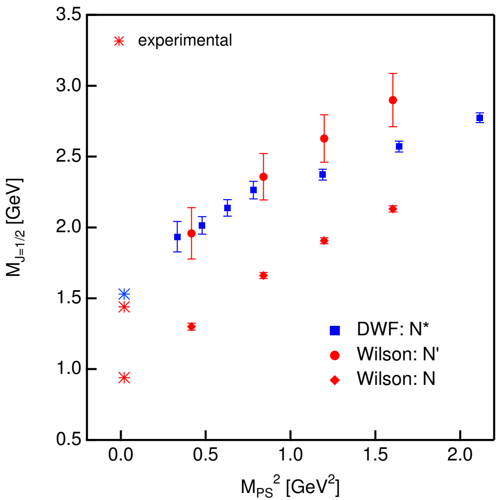
S. Dong *et al*

Phys. Lett. B (2005) 137



S. Sasaki

Prog. Theor. Phys. Suppl. 151 (2003) 143



See also: W. Melnitchouk et al, 2002, 2003; LHP collaboration, 2005, 2006; BGR collaboration, 2004,2005,2006

"Standard" approach to excited states:

- Use e.g. the nucleon interpolator:

$$\chi_1 \;=\; \varepsilon_{abc}\,(\,d_a^T\,C\gamma_5\,u_b\,)\,u_c$$

- Spectral sum of the 2-point function

$$\begin{aligned}\langle\overline{\chi(0)}_1\chi(t)_1\rangle \;&=\; \langle 0\,|\,\chi\,|\,N\rangle\langle N\,|\,\overline{\chi}\,|\,0\rangle\,e^{-\,m\,t} \\ &+\; \langle 0\,|\,\chi\,|\,N'\rangle\langle N'\,|\,\overline{\chi}\,|\,0\rangle\,e^{-\,m'\,t} \;+\; \ldots\end{aligned}$$

- is fitted with

$$A\,e^{-m\,t} \;+\; B\,e^{-m'\,t} \;+\; C\,e^{-m''\,t} + \;\ldots$$

- Extremely challenging fitting problem!!

More information !! $\longrightarrow$ *variational method*

- Use several interpolators χ_i and compute

$$C_{ij}(t) \quad = \quad \langle\, \overline{\chi_i}(0)\, \chi_j(t)\, \rangle$$

- Decomposition in Hilbert space:

$$C_{ij}(t) \quad = \quad \sum_n A_i^n\, A_j^{n\,\star}\, e^{-m_n\, t}$$

- Eigenvalues $\lambda_\alpha(t)$ from the generalized eigenvalue problem

$$C(t)\, \vec{v} \quad = \quad \lambda\, C(t_0)\, \vec{v}\,,$$

behave as

$$\lambda_\alpha\,(t) \quad = \quad e^{-m_\alpha\,(t-t_0)}\, \left[1 \,+\, C\, e^{-\Delta\,(t-t_0)}\right]\,.$$

C. Michael 1985; M. Lüscher & U. Wolff 1990

- Simple stable two parameter fits are possible.

Good hadron interpolators are essential

- The success of the variational approach hinges crucially on the set of basis interpolators. (*garbage in - garbage out*)

- A good set of basis interpolators should:
  - Be sufficiently rich to span the physical states.
  - Contain no irrelevant interpolators.
  - Be easy to be implemented efficiently.

Ingredient I: Different Dirac structures - Baryons

- Nucleon, Σ and Ξ: $$N^{(i)} = \epsilon_{abc}\Gamma_1^{(i)} u_a (u_b^T \Gamma_2^{(i)} d_c - d_b^T \Gamma_2^{(i)} u_c)$$
- Λ-octet: $$\Lambda_8^{(i)} = \epsilon_{abc}\Gamma_1^{(i)} [s_a (u_b^T \Gamma_2^{(i)} d_c - d_b^T \Gamma_2^{(i)} u_c) + u_a (s_b^T \Gamma_2^{(i)} d_c) - d_a (s_b^T \Gamma_2^{(i)} u_c)]$$
- Λ-singlet: $$\Lambda_1 = \epsilon_{abc}\Gamma_1^{(1)} u_a (d_b^T \Gamma_2^{(1)} s_c - s_b^T \Gamma_2^{(1)} d_c) + \text{perms.}$$
- Δ and Ω: $$\Delta = \epsilon_{abc} u_a (u_b^T C\gamma_\mu u_c) \quad \text{(projected to } J = 1/2\text{)}$$

	$\Gamma_1^{(i)}$	$\Gamma_2^{(i)}$
$i=1$	$\mathbb{1}$	$C\gamma_5$
$i=2$	γ_5	C
$i=3$	$i\mathbb{1}$	$C\gamma_4\gamma_5$

Ingredient I: Different Dirac structures - Mesons

General form of interpolator:

$$O = \overline{\psi}^{(f_1)} \, \Gamma \, \psi^{(f_2)}$$

state	J^{PC}	Γ	particles
scalar (SC)	0^{++}	$\mathbb{1}$	a_0
pseudoscalar (PS)	0^{-+}	$\gamma_5\ ,\ \gamma_4\gamma_5$	π, K
vector (V)	1^{--}	$\gamma_i\ ,\ \gamma_4\gamma_i$	ρ, K^*, ϕ
pseudovector (PV)	1^{++}	$\gamma_i\gamma_5$	a_1
tensor (T)	1^{+-}	$\gamma_i\gamma_j$	b_1

Not good enough!

- It is by now well known, that different Dirac structures are not sufficient for capturing the excited states properly Sasaki, Blum, Ohta, 2002; Melnitchouk et al, 2003; Brömmel et al, 2004 ... .

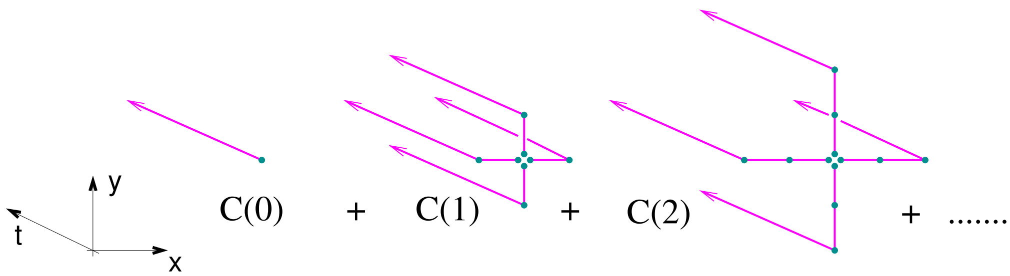
- The spatial wave-function is crucial for a correct description of excited hadrons (nodes in the wave function).

Ingredient II: Jacobi smearing (Güsken et al, 1989; Best et al, 1997)

- Jacobi smearing turns a point source s_0 into a gauge-covariant quark sources with Gaussian shape (similar to a diffusion process):

$$s^{(\alpha,a)} = \sum_{n=0}^{N} H^n \, s_0^{(\alpha,a)}$$

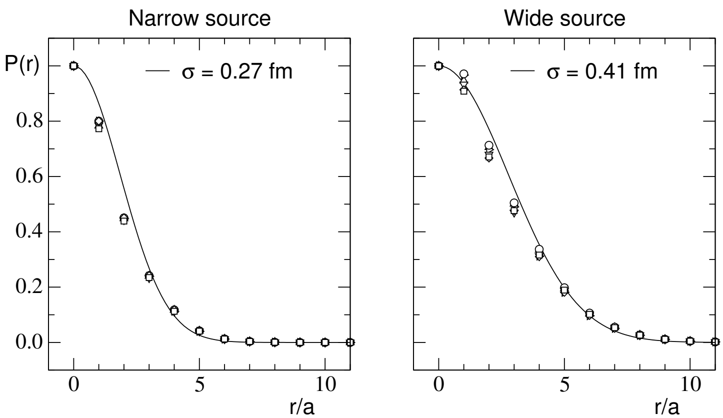
$$H(\vec{x},\vec{y}) = \kappa \sum_{j=1}^{3} \left[U_j(\vec{x},0)\,\delta(\vec{x}+\hat{j},\vec{y}) + U_j(\vec{x}-\hat{j},0)^\dagger\,\delta(\vec{x}-\hat{j},\vec{y}) \right]$$



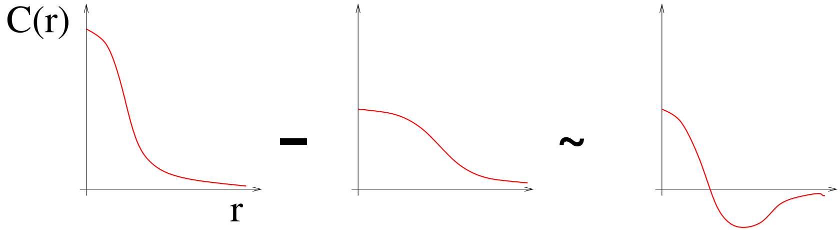
- Cost effective way of improving the overlap with physical wave functions. Two parameters N, κ.

Nodal wave functions (Burch et al, 2004)

- Profiles of the sources:



- Working hypothesis: Sources will combine such that the different widths give rise to a node.



Final form of interpolators

- We combine the different Dirac structures with all possibilities for smearing the quarks.

- Each quark can be either wide, w, or narrow, n:

$$\text{mesons} \quad : \quad (n,n),(n,w),(w,n),(w,w)$$

$$\text{baryons} \quad : \quad (n,n,n),(n,n,w),(n,w,n) \; .... \; (w,w,w)$$

- Thus we have correlation matrices of up to 8×8 for mesons and 24×24 for baryons.

Parameters of the simulation

- Quenched calculation with Lüscher-Weisz gauge action

size	β	confs.	a[fm]	a^{-1}[MeV]
$20^3 \times 32$	8.15	100	0.119	1680
$16^3 \times 32$	7.90	100	0.148	1350

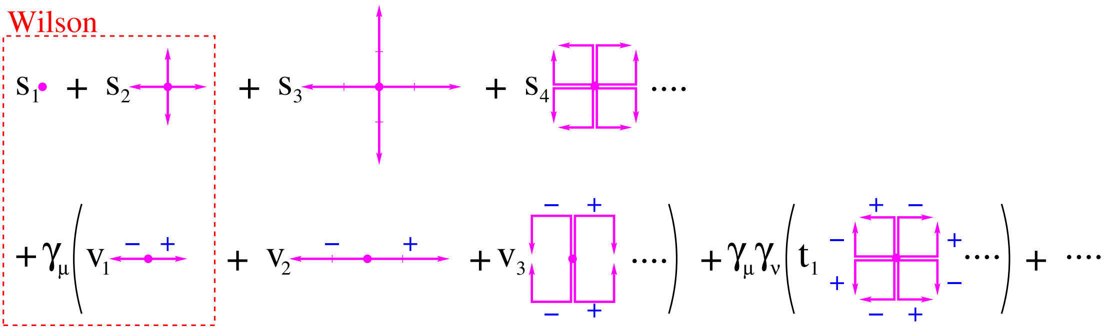
- Spatial extension of $L \sim 2.4$ fm for both lattices.

- We use the Chirally Improved (CI) lattice Dirac operator and use data with pion masses down to 350 MeV.

- We start with the maximally possible correlation matrix, but prune it to optimize the effective mass plateaus. $\Rightarrow$ Removal of operators that couple only weakly to physical states.

Chirally improved fermions (Gattringer, 2001; Gattringer, Hip, Lang, 2001)

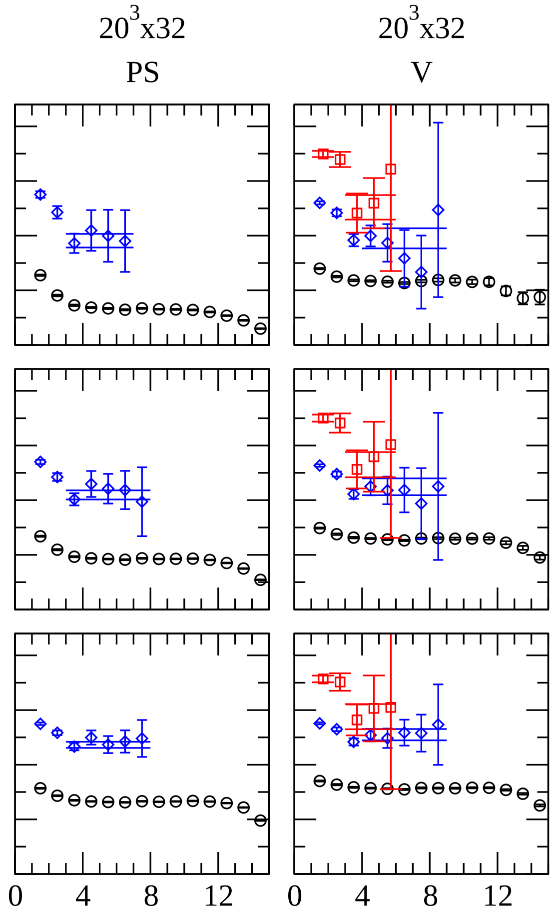
- Expand a general lattice Dirac operator D in terms of paths:



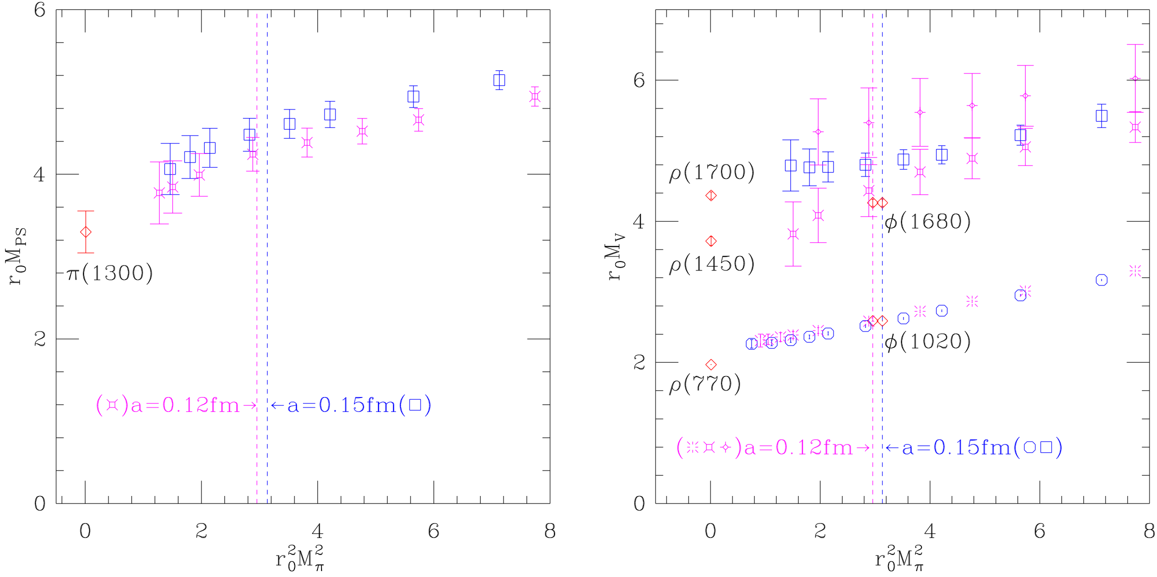
- The Ginsparg-Wilson equation $D\gamma_5 + \gamma_5 D = D\gamma_5 D$ for the Dirac operator turns into a system of coupled quadratic equations for the coefficients s_i, v_i, t_i ....

- Solving this system of equations provides an approximation of a solution of the Ginsparg-Wilson equation (tested down to $m_\pi \sim 280$ MeV).

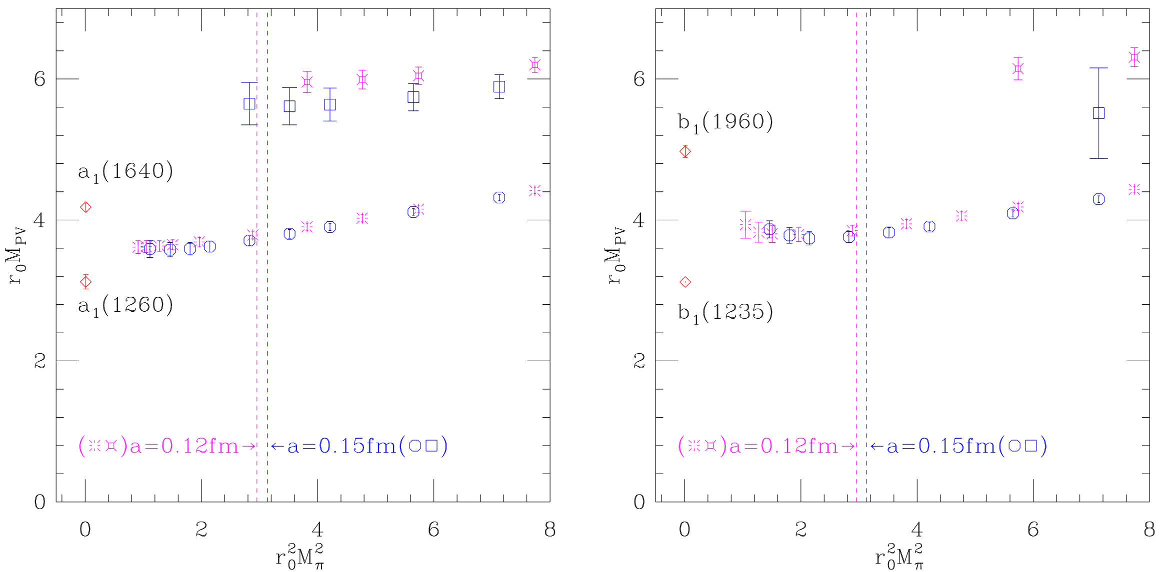
Effective masses:



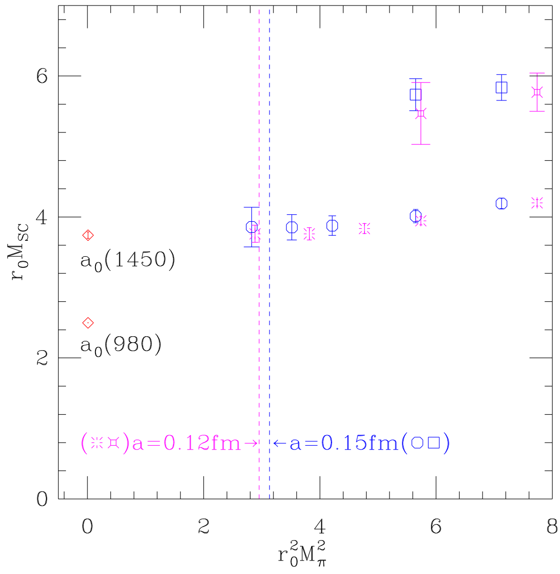
Pseudoscalar and vector mesons (0^{-+} and 1^{--})



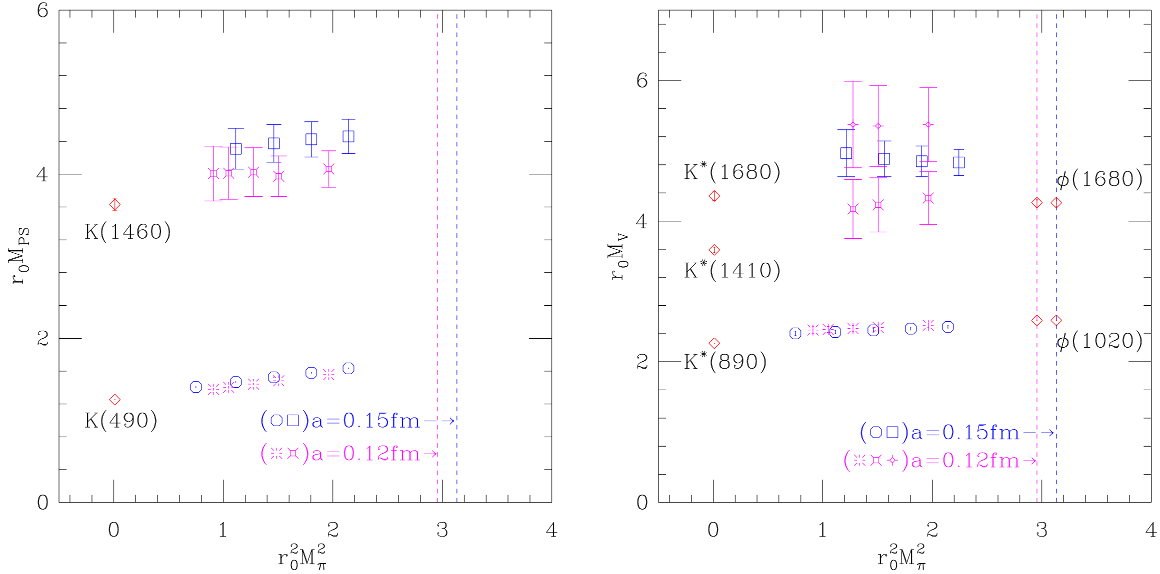
Pseudovector mesons (1^{++} and 1^{+-})



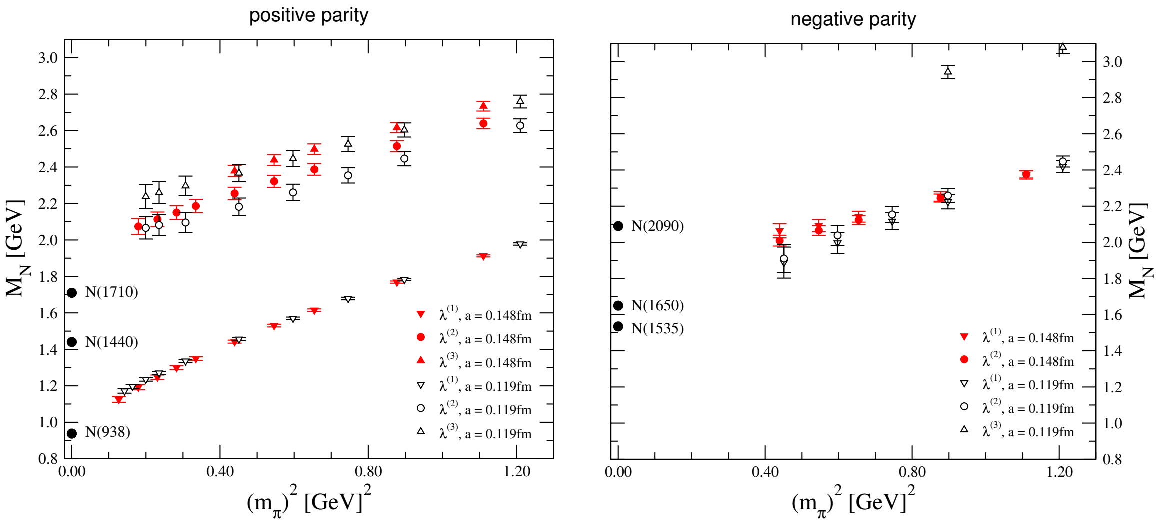
Scalar (0^{++})



Strange pseudoscalar and vector mesons (0^{-+} and 1^{--})



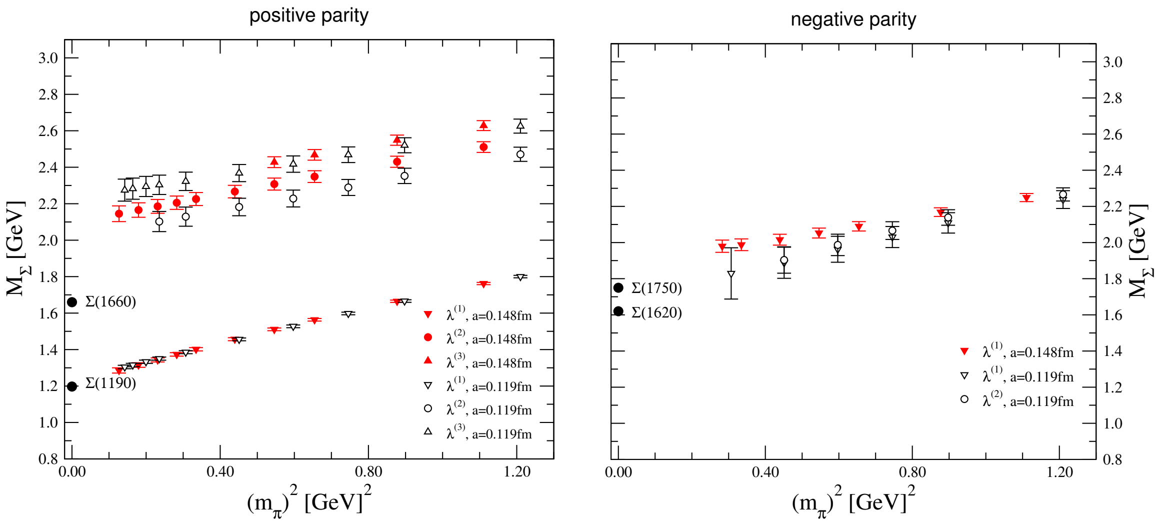
Nucleon system (positive and negative parity)



Positive parity: $w(w,w)^{(1,3)}, w(n,w)^{(1,3)}, n(w,w)^{(1,3)}$

Negative parity: $w(n,n)^{(1,2)}, n(n,n)^{(1,2)}$

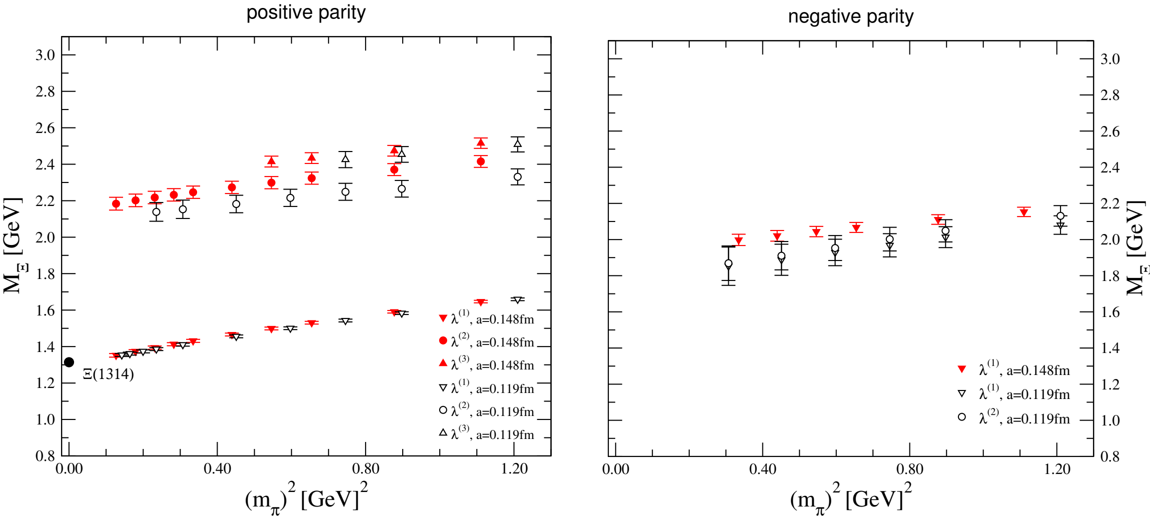
Σ system (positive and negative parity)



Positive parity: $w(w,w)^{(1,3)}, w(n,w)^{(1,3)}, n(w,w)^{(1,3)}$

Negative parity: $w(n,n)^{(1,2)}, n(n,n)^{(1,2)}$

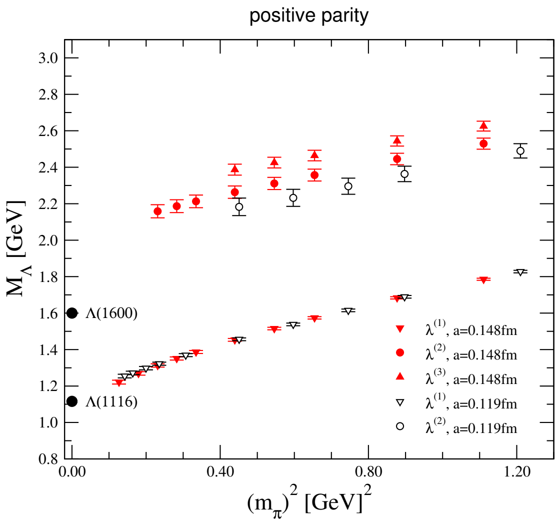
Ξ system (positive and negative parity)

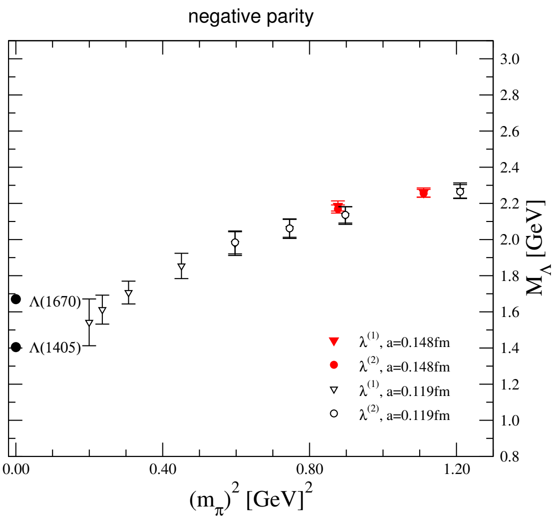


Positive parity: $w(w,w)^{(1,3)}, w(n,w)^{(1,3)}, n(w,w)^{(1,3)}$

Negative parity: $w(n,n)^{(1,2)}, n(n,n)^{(1,2)}$

Λ system (octet operator, positive and negative parity)

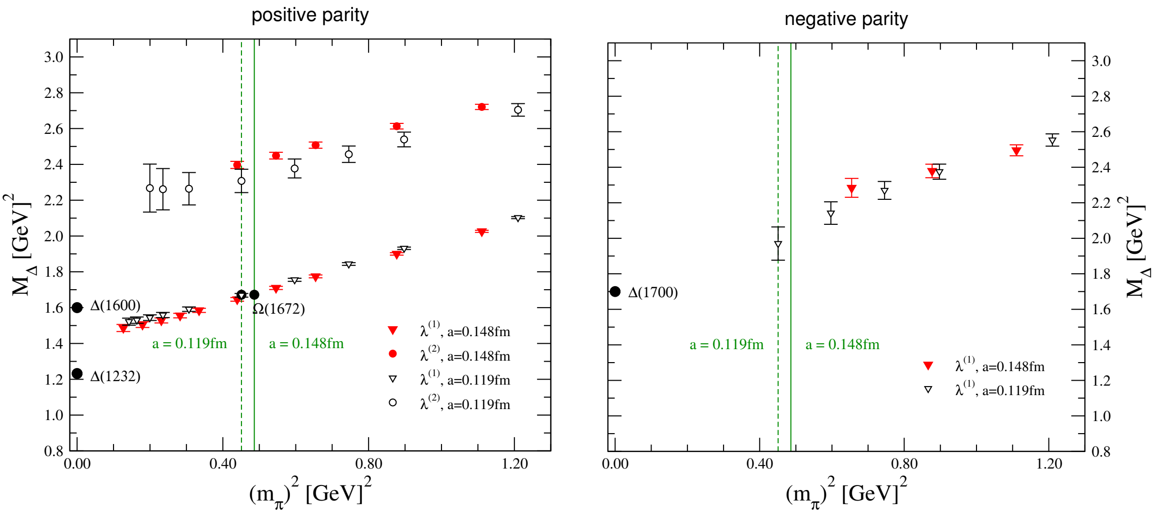




Positive parity: $w(w,w)^{(1,3)}, w(n,w)^{(1,3)}, n(w,w)^{(1,3)}$

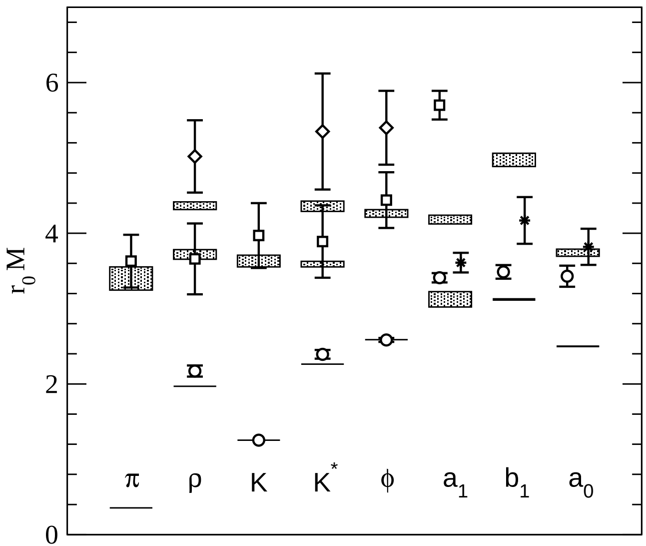
Negative parity: $w(n,n)^{(1,2)}, n(n,n)^{(1,2)}$

Δ (Ω) system (positive and negative parity)



Both parities: $n(n,n), n(n,w), w(n,n), n(w,w), w(n,w), w(w,w)$

Chiral extrapolation - mesons



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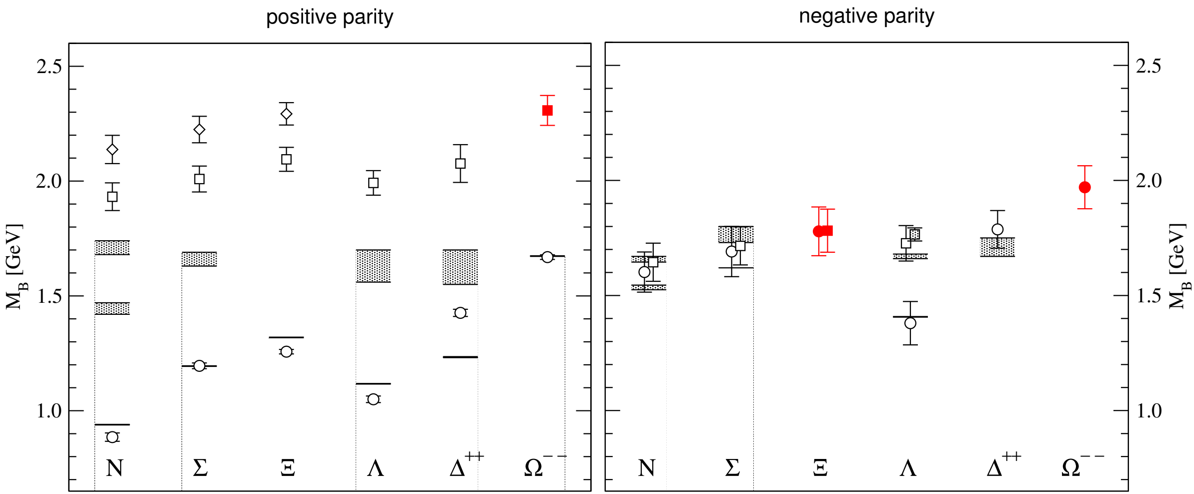
Chiral extrapolation - baryons



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Predictions (being bold)

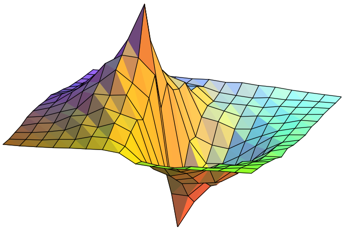
- Ω^{--} first excited state, positive parity: 2300 (70) MeV
- Ω^{--} ground state, negative parity: 1970 (90) MeV
- Ξ ground state, negative parity: 1780 (90) MeV
- Ξ first excited state, negative parity: 1780 (110) MeV

PDB: Candidate resonances exist, spin and parity unknown.

Summary

- Motivation: Check claims on resonances with advanced techniques.
- Methodological improvement:
  - Variational method with large set of basis operators.
  - Non-trivial spatial wave function included.
  - Clean treatment of ghost contributions.
- Larger cutoff effects for excited states.
- Pseudoscalar and vector mesons: good results. Scalars: lowest state seems not to couple to quark bilinears.
- Positive parity excited nucleons systematically too high. Negative parity is quite good $\Rightarrow$ Predictions.

What to improve



- Systematic analysis of cutoff and finite volume effects.
- Larger statistics.
- Inclusion of p-wave sources (covariant derivative on Jacobi source).
  $\Rightarrow$ Many new interpolators: $\overline{u}\gamma_5\gamma_i D_i d\ ,\ D_i\overline{u}\gamma_5 D_i d\ \ \ldots\ \ .$

  Tests under way.
- Non-fermionic interpolators.
- Dynamical fermions.

  Tests under way.