Searching for Diquarks on the Lattice

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with C. Alexandrou and B. Lucini

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hep-lat/0509113 and in progress
”Exploration of Hadron Structure...”

Are diquarks for real?
First encounter with diquarks

Baryonic static potential $V_{qqq}$:

- $qq$ (antisymmetrized in color) behaves like $\bar{q}$
- one-gluon exchange: $V_{qq} = \frac{1}{2} V_{q\bar{q}}$ attractive
Diquark advocacy

Diquarks

- are Cooper pairs of color superconductivity
- can explain the $\Delta I = 1/2$ rule in weak non-leptonic decays
- can explain some features of excited baryon spectrum
- can explain stability of exotics
  - $(\Theta^+ \rightarrow$ pentaquark, $X$ and $Y \rightarrow$ tetraquarks)
- can explain hadronic parity violation
Diquarks are a combination of quarks in the color antitriplet

\[ 3 \otimes 3 = \bar{3} \oplus 6 \]

In particular the **parity even flavor antisymmetric spinless** combination is the most attractive channel in this sector.
Diquark operators: complete classification

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>Color</th>
<th>Flavor</th>
<th>Operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>0$^+$</td>
<td>3</td>
<td>3</td>
<td>$\bar{q}_C \gamma_5 q$, $\bar{q}_C \gamma_0 \gamma_5 q$</td>
</tr>
<tr>
<td>1$^+$</td>
<td>3</td>
<td>6</td>
<td>$\bar{q}_C \vec{\gamma} q$, $\bar{q}_C \sigma_0 i q$</td>
</tr>
<tr>
<td>0$^-$</td>
<td>3</td>
<td>6</td>
<td>$\bar{q}_C q$, $\bar{q}_C \gamma_0 q$</td>
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<td>1$^-$</td>
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</tr>
</tbody>
</table>

From R. Jaffe, hep-ph/0409065

Spin-color effective interaction $\rightarrow$ wealth of predictions:

- parity-odd states heavier (suppressed in non-relat. limit)
- $M(0^+) < M(1^+)$: $0^+$ is “good” diquark, while $1^+$ is “bad”
- in the real world $\Delta M \equiv M(1^+) - M(0^+) \simeq 200$ MeV
- $\Delta M$ from spin-spin interaction $\propto \frac{1}{m_1 m_2}$ for heavy quarks
Problem: diquarks are colored

\[ \text{combine (diquark+static quark) into color singlet: static-light-light baryon} \]

i.e. diquark in the background color field of static quark

\[ \text{robustness of results vs distance from static quark} \]
Wilson fermions, quenched: $3 \times \beta$, $3 \times \kappa$ (heavy, medium, light) 
unquenched (thanks to Th. Lippert & co.)

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$N_f$</th>
<th>size</th>
<th>#conf</th>
<th>$a(r_0)$ (fm)</th>
<th>$\kappa$</th>
<th>$m_\pi$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.8</td>
<td>0</td>
<td>$16^332$</td>
<td>2-500</td>
<td>0.136</td>
<td>0.156-0.159</td>
<td>690-910</td>
</tr>
<tr>
<td>6.0</td>
<td>0</td>
<td>$16^332$</td>
<td>2-500</td>
<td>0.093</td>
<td>0.153-0.155</td>
<td>620-900</td>
</tr>
<tr>
<td>6.2</td>
<td>0</td>
<td>$20^340$</td>
<td>200</td>
<td>0.068</td>
<td>0.151-0.1523</td>
<td>570-870</td>
</tr>
<tr>
<td>5.6</td>
<td>2</td>
<td>$24^340$</td>
<td>100</td>
<td>$\sim 0.1(m_N)$</td>
<td>0.1575</td>
<td>530</td>
</tr>
</tbody>
</table>

Technical difficulty:
static quark propagator very noisy $\Rightarrow$ HYP-smearing

- I. Masses
- II. Wavefunctions
I. Masses: the [unbearable] lightness of smearing

\[ \beta = 5.8 \text{ medium quarks} \]
Effective masses: 3 groups

Static quark $\rightarrow$ mass UV divergent
Look at mass differences
The good, the bad and the ugly

\[ m(\text{scalar}) < m(\text{vector}) \ll m(\text{others}) \quad \text{as predicted} \]
Diquarks

The Calculations

Conclusions

\( m(\text{vector}) - m(\text{scalar}) \) versus pion mass

Ansatz

\[ \Delta m = \frac{c_1}{c_2 + m_\pi^4} \rightarrow \text{extrapolation} \sim 150 \text{ MeV} \text{ (200 MeV expected)} \]

Larger in full QCD (Orginos) ?

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II. Wavefunction: density-density correlator

\[ \rho(x) \rho(y) \]

Fix distance from static quark \( \rightarrow \) fixed background field

Look at angular distribution
II. Wavefunction: density-density correlator

\[ C_{\text{tot}}(r) = \int \langle N | \rho^u \rho^d | N \rangle \sin \theta \, d\theta \, d\phi \equiv \int d(\cos \theta) C(\theta, r), \quad \rho^q = : \bar{q} \gamma_0 q : \]

No correlation \iff flat in \cos \theta
Spatial correlations?

$C(r/\alpha=7.35,\theta)$ vs $\cos(\theta)$

Attraction in scalar channel > vector channel as predicted
Smaller mass $\leftrightarrow$ stronger correlation?

- Excited states contamination?
- Concentrate on light channels (scalar and vector)
Excited states contamination?

Vary separations between source – measurement – sink:

$\text{beta}=6.0$ light quarks

Better groundstate

→ more correlation in scalar diquark, less in vector diquark
Diquark size?

Very large size for vector; $\mathcal{O}(1)$fm for scalar box size $\sim 1.5$ fm $\rightarrow$ wrap-around effects
Size vs distance from static quark

Very stable (slowly increasing?)
Lighter quarks seem to give larger size – systematics when size $\gtrsim L_s/2$?
Size summary

From hep-lat/0509113
Size summary

- Scalar: size $\lesssim 1$ fm robust vs background field
- Vector: size $\gtrsim 2$ fm
- Light quarks? Full QCD?
Conclusions

- "Good" diquark more than phenomenological construct: diquarks are for real!
- Surprise: all measurements consistent with predictions.
- Scalar diquark $\sim$ robust versus background field
  $\rightarrow$ unchanged in color superconductivity?
- Tighter binding, smaller size for lighter quarks?
  $\rightarrow$ fit diquark inside nucleon?
- Outlook:
  static $\bar{q}$ + diquark + diquark $\rightarrow$ diquark-diquark interactions
- Density-density correlators: powerful gauge-invariant tool
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