Chiral extrapolation in the Interacting Instanton Liquid Model

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Lattice QCD and the chiral extr. problem

Axial Charge:

The quark mass dependence is strongly channel-dependent

Understanding such a dependence is crucial for a comparison of LQCD calculations vs experiment.

DIS SF Moments:
In the small pion mass regime, the pion-mass dependence can be analyzed in Chpt.

Encodes the QCD dependence

- Convergence of finite-order calculation
- Uncertainties on effective parameters
- Very weak non-linear dependence on quark mass is seen in lattice simulations: do present lattice data lie inside the radius of convergence of Chpt?
We present the first step towards a study of the transition between the chiral and the lattice regime. At present this can be performed only in a model-dependent way.

A good choice for the model approach should:

1. Provide a realistic description of QCD in the chiral regime
2. Provide a realistic description of QCD in the “lattice” regime
3. Provide an insight which can guide lattice QCD through the exploration of the transition regime
More specific questions being addressed here:

- What is the extent of the chiral regime? Does it connect with the lattice regime?

- At which value of the quark mass does the quenched approximation become accurate. Why?
Instantons and chiral dynamics

Problem: how can we filter the content of gauge configuration responsible for near-zero modes?

Solution (Gattringer): use fermionic representation of $F_{\mu\nu}$:

$$-\frac{1}{4} \text{Tr}_D \left[ \gamma_\mu \gamma_\nu D^2(x) \right] = F_{\mu\nu}(x),$$

Interacting Instanton Liquid Model (IILM):

$$Z_{QCD} \approx Z_{ILM} = \sum_{N_+,N_-} \frac{1}{N_+! N_-!} \int \prod_i \left[ N_i \right] d\Omega_i d(\rho_i) e^{-s_{\text{int}}} \prod_i \det(iD + im_f).$$
Part I:
Interacting Instanton Liquid Model in the chiral regime

Main question:
Can the IILM reproduce Chpt predictions in the chiral regime?
Exploring The Chiral Regime in the IILM

We need to find quantities for which we can directly compare a chpt prediction with an ILM simulations.

Spectrum of the Dirac Operator

\[ \rho(\lambda) = -\frac{1}{\pi} \frac{B}{f_0^2} + \frac{B^2 (N_f^2 - 4)}{32\pi^2 N_f f_0^2} |\lambda| + \mathcal{O}(\lambda) \]

(Smilga and Stern)

\[ (m_q \rightarrow 0) \]

Chpt prediction: For \( N_f=2 \) \& \( m_q=0 \), the density of eigenvalues of the Dirac Spectrum should become flat near the origin:

What about finite-mass corrections?
Finite mass corrections and chiral divergences

To see if also mass corrections are compatible with Chpt we study

\[ K^{abc} = \int d^4x d^4y d^4z \langle 0 | S^a(x) S^b(y) S^c(z) | 0 \rangle, \]

\[ K^{abc} = -d^{abc} m \int_0^\infty \rho(\lambda) \frac{(m^2 - 3\lambda^2)}{(m^2 + \lambda^2)^3} d\lambda. \]

(spectral decomposition)  
(LO Ch-pt)

At \( N_f=2 \) the polynomial chiral divergence \( 1/m^3 \) can only develop through mass corrections to the spectral function.

We can use the calculated spectral density at different quark Mass and see if we get the correct behavior.
Clean $\frac{1}{m_\pi^2}$ behavior...

In the small-quark mass regime mass corrections are compatible with Chpt predictions
What about QCD?

Having established that the model is compatible with Chpt let’s use it to address questions which can be of interest for real QCD calculations.

- What is the regime in which we expect chiral dynamics to dominate?

- What is the regime in which the quenched approximation breaks down?

- Are such two regimes related by some microscopic mechanism?
Instantons and the Chiral Dynamics regime:

**Def:** Chiral dynamics $\leftrightarrow$ Dynamics associated with small eigenvalues of the Dirac Op.

**Speculation:** Chpt is hardly to be trusted in a regime in which chiral dynamics does not dominate.

In QCD the definition of chiral dynamics is not sharp. Not so in the ILM, where the propagator splits in two parts:

\[
S(x, y) = S_{zm}(x, y) + S_{nzm}(x, y)
\]

\[
S_{zm}(x, y) = \sum_{I, J} \frac{\psi^{0}{}_{I}(x)\psi^{0+}{}_{I}(x)}{T_{I,J} - im}
\]

\[
T_{IJ} = \int d^{4}z \psi^{0+}{}_{I}(z) \partial \psi^{0}{}_{J}(z)
\]

• For small quark masses correlators are dominated by $S_{zm}$

• For $im \gg \langle T_{IJ} \rangle$ chiral dynamics becomes subleading
For $m_q \gg 100$ MeV:

- Chiral dynamics in the ILM becomes suppressed

- The properties of the spectrum of DO stop depending on the quark mass.

This scale explains the region of validity of the quenched approximation.
Part II:
Interacting Instanton Liquid Model in the lattice regime

Main question:
Can the IILM reproduce lattice predictions in the lattice regime?
Effective mass plot analysis
### Pion and nucleon mass

<table>
<thead>
<tr>
<th>Pion [MeV]</th>
<th>Nucleon [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>120.300 ± 0.004</td>
<td>1.11 ± 0.01</td>
</tr>
<tr>
<td>200.380 ± 0.004</td>
<td>1.16 ± 0.01</td>
</tr>
<tr>
<td>400.460 ± 0.004</td>
<td>1.21 ± 0.008</td>
</tr>
<tr>
<td>600.530 ± 0.004</td>
<td>1.28 ± 0.008</td>
</tr>
<tr>
<td>800.600 ± 0.004</td>
<td>1.35 ± 0.008</td>
</tr>
</tbody>
</table>
Fixing the units from the ultra-violet cut-off
Chiral extrapolation of the nucleon mass

\[ M_N = M_0 - 4c_1 + \frac{3}{32}\pi^2 f^2 \mu_g A^2 M_0 - c_2^2 + 4c_3^3 \ln m_\pi \lambda_i m_4^4 \pi^5 + \frac{3g A^2}{256\pi^2 f^2 \pi} M_0^2 m_5^5 \pi^5 + O\left(\frac{m}{\pi}\right) \]

Input parameters

Fitted parameters
Chiral regime connects with the lattice regime!
Nucleon sigma term

what microscopic dynamics generates the modification of the nucleon sigma term which is encoded in the chiral fits?

We now have an efficient calculational tool to address these type of questions...
Conclusions

• IILM can be used to study the transition into the chiral regime:
  – Chiral regime connects with lattice regime!

• It can provide information which help explain facts of life of LQCD:
  – Unification of the chiral dynamics scale and the scale where quenched approximation breaks down
  - Possibility for phenomenological investigation (what microscopic dynamics generates the modification of the nucleon sigma term which is encoded in the chiral fits?)