Sigma and a0 Correlators in the Staggered Fermion Scheme

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Staggered Chiral Perturbation Theory

- Assume perturbative expansion in lattice spacing
- Map lattice QCD to SXPT (Lee Sharpe & Aubin Bernard)
- Use replica trick to get rooted SXPT
- Observe unitarity violations and “nonlocality” similar to partial quenching

SXPT Tree Level Spectrum

$$\mathcal{L} = \frac{f^2}{8}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\mu U\right) - \frac{\mu f^2}{4}\mathrm{Tr}[\mathcal{M}U^\dagger + U\mathcal{M}^\dagger)] + \frac{m_0^2}{2}\phi_{0I}^2 + a^2\mathcal{V}(U)$$

- Mass matrix with tastes and replicas

$$\mathcal{M}_{frb,f'r'b'} = \delta_{ff'}\delta_{rr'}\delta_{bb'}m_f\,;$$

- Meson spectrum is obtained by diagonalizing the meson mass matrix or by summing diagrams

Tree level meson spectrum

- Meson taste labeling

$$a = \{P, V, A, T, I\}$$

- Masses

$$M^2_{ff'a} = \mu(m_f + m_{f'}) + a^2\Delta_a \quad \text{for } i = 1, 2, \ldots, N.$$

$$M^2_{\eta I} = \frac{2}{3}M^2_{SI} + \frac{1}{3}M^2_{UI}$$

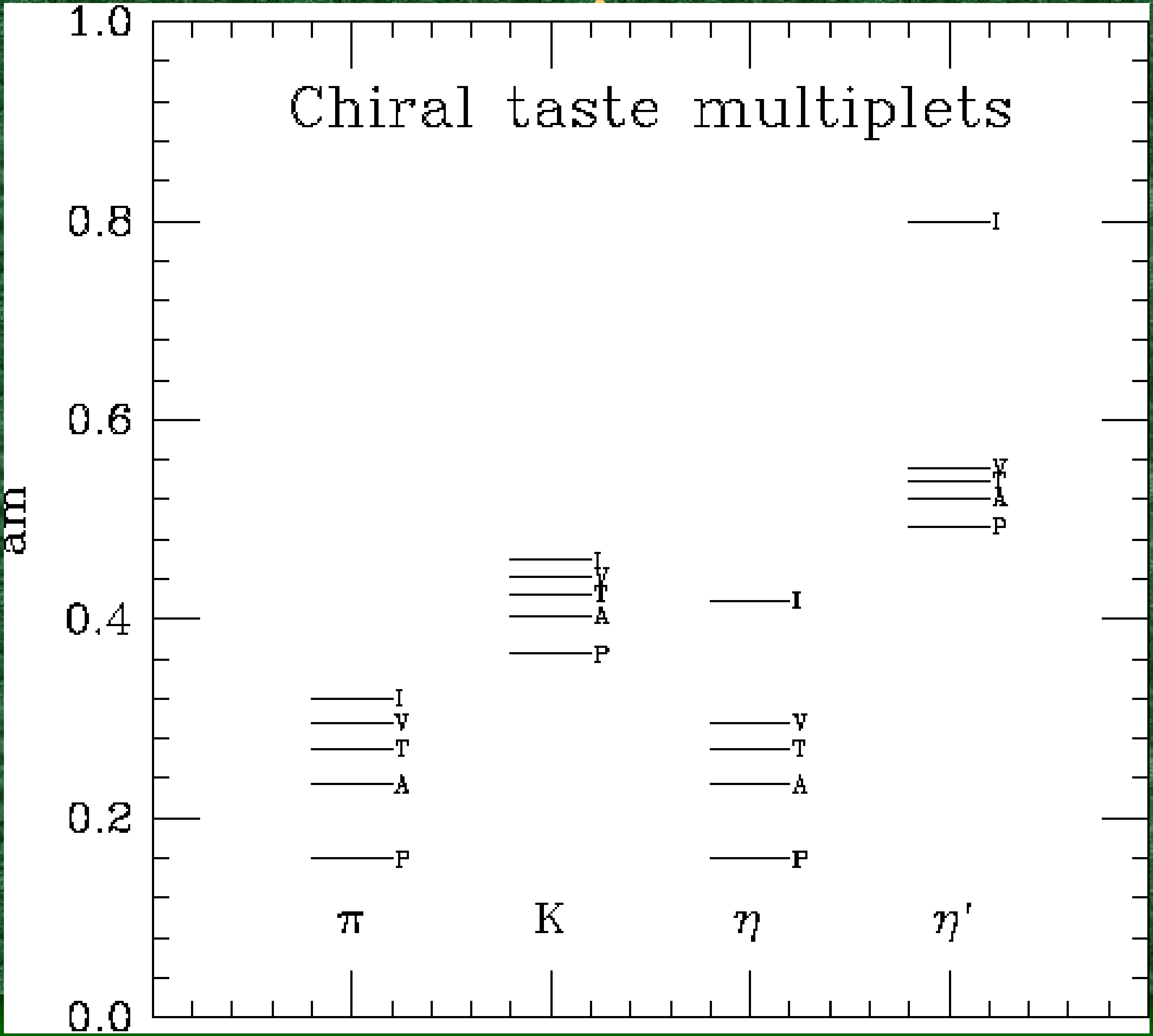
- (Ignore A and V hairpins for now)

Series expansion for taste singlet eta

$$
\begin{aligned}
C_{\eta I}(p) &= u_f^\dagger \left\{ \frac{\delta_{ff'}}{p^2 + M^2_{ffI}} - n_r \frac{1}{p^2 + M^2_{ffI}} \frac{m_0^2}{3} \frac{1}{p^2 + M^2_{f'f'I}} \right. \\
&+ \left. n_r \frac{1}{p^2 + M^2_{ffI}} \frac{m_0^2}{3} \frac{n_r}{p^2 + M^2_{ggI}} \frac{m_0^2}{3} \frac{1}{p^2 + M^2_{f'f'I}} - \dots \right\} u_{f'}
\end{aligned}
$$

$$
C_{\eta I} = \frac{1}{p^2 + M^2_{\eta I}}
$$

Meson Spectrum



Matching Lattice QCD to SXPT

- Standard staggered correlator for a0

$$C_{\rm conn}(\vec{p},t) = \sum_{\vec{x}} (-)^x \cos(\vec{p}\cdot\vec{x}) \left\langle {\rm Tr}[M_u^{-1}(\vec{x},t;0,0) M_u^{-1\dagger}(\vec{x},t;0,0)] \right\rangle$$

- Taste-spin basis

$$q_f^{a\alpha}(2y) = \frac{1}{2}\sum_{\eta} \Gamma_{\eta}^{a\alpha} \chi_{f,2y+\eta}$$

- Scalar density for a0

$$\rho_{udI}(2y) = \frac{1}{4}\bar{q}_d(2y) I \otimes I q_u(2y) = \sum_{\eta} \bar{\chi}_{d,2y+\eta}\chi_{u,2y+\eta}$$

Matching Lattice QCD to SXPT

- Lattice QCD

$$Z(m_{ff'}) = \int dU \exp[-S_g(U)] \det[M(U, m_{ff'})]^{n_r}$$

$$\langle \bar{\rho}_{udI}(2y)\rho_{udI}(0)\rangle = \frac{\partial^2 \log Z}{\partial m_{I,ud}(2y)\partial m_{I,du}(0)}\bigg|_{m_{ff'}(x)=\delta_{ff'}m_f}$$

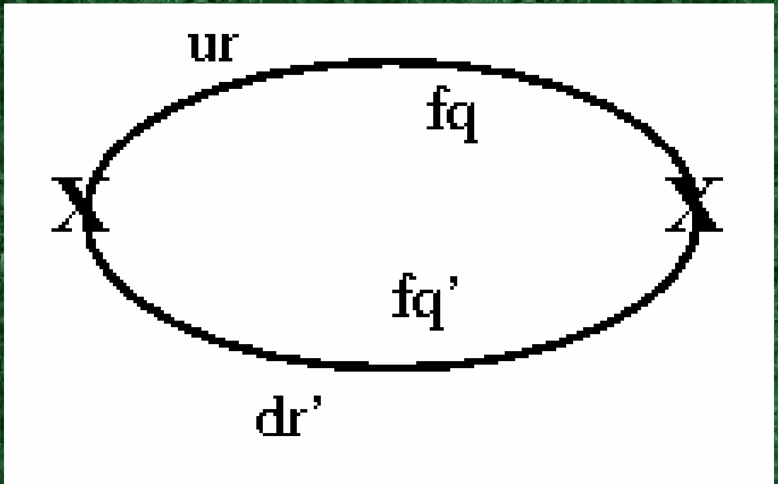
- SXPT

$$Z_{\mathrm{SXPT}}(m_{ff'}) = \int [d\Sigma] \exp[-S(\Sigma, m_{ff'})]$$

$$\langle \bar{\rho}_{udI}(x)\rho_{udI}(0)\rangle = \frac{\partial^2 \log Z_{\mathrm{SXPT}}}{\partial m_{ud}(x)\partial m_{du}(0)}\bigg|_{m_{ff'}(x)=\delta_{ff'}m_f}$$

Two meson "bubble" term

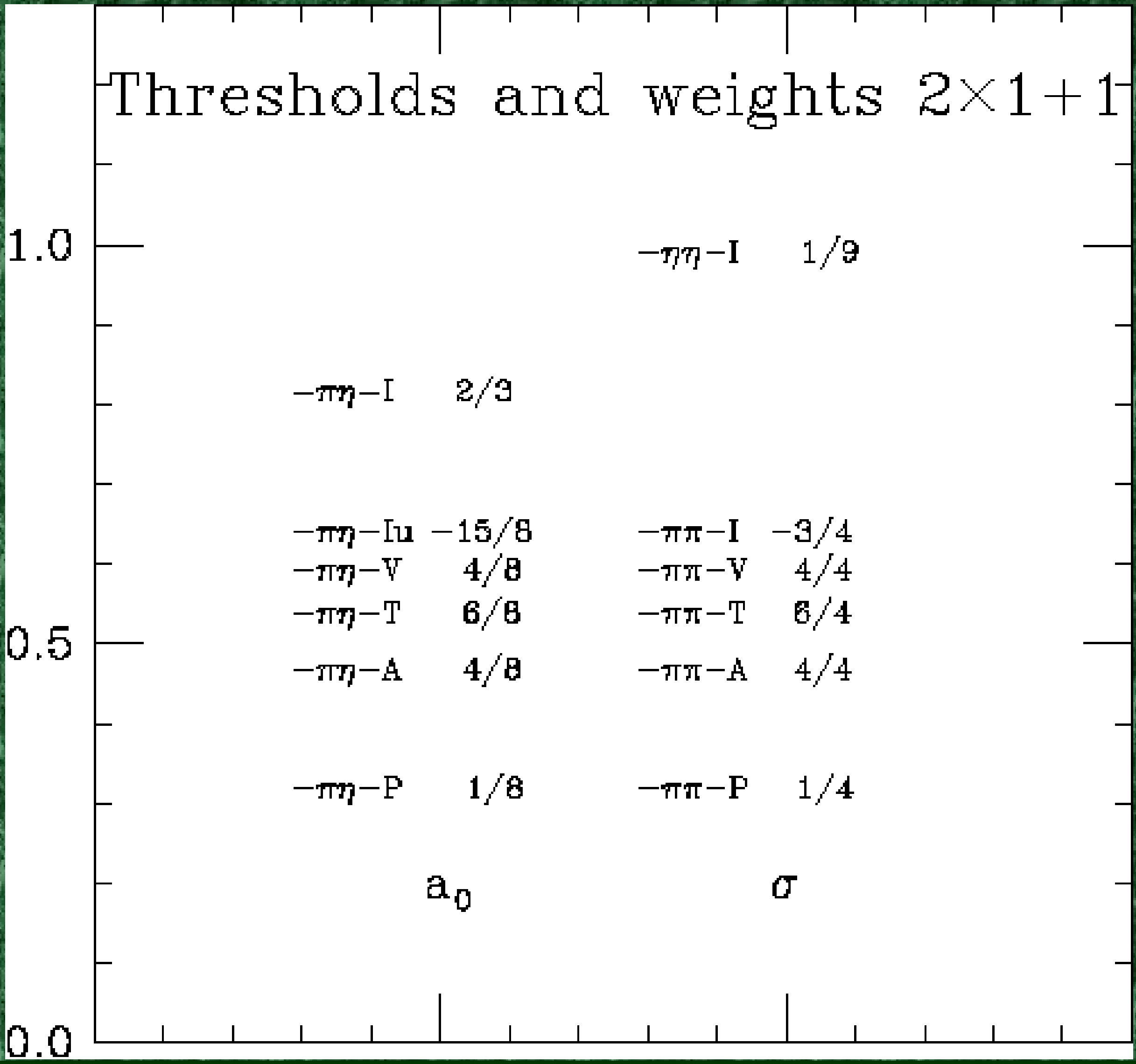
$$\langle \bar{\rho}_{udI}(x)\rho_{udI}(0)\rangle = (2a)^8\mu^2 \sum_{r,q,b,f}\sum_{r',q',b',f'} \langle \phi_{ur,fq}(x)\phi_{fq,dr}(x)\phi_{dr',fq'}(0)\phi_{fq'dr'}(0)\rangle$$



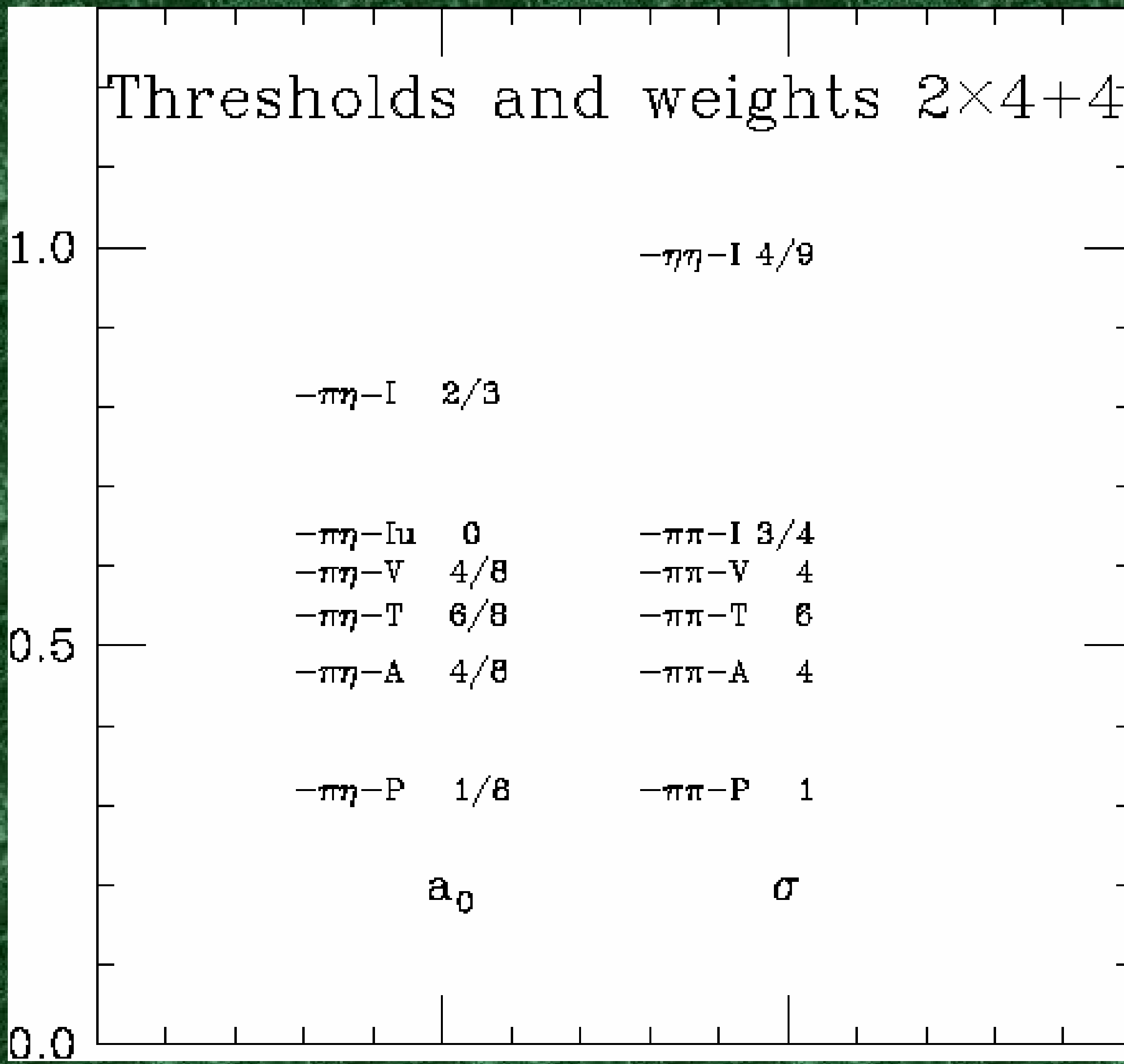
Prelovsek result for a0 “bubble”

$$
\begin{aligned}
C(p,t) &= B_0^2 \sum_k \left\{ \frac{1}{16} \sum_a \left[2 \frac{1}{(k+p)^2 + M_{Ua}^2} \frac{1}{k^2 + M_{Ua}^2} + \frac{1}{(k+p)^2 + M_{Ka}^2} \frac{1}{k^2 + M_{Ka}^2} \right] \right. \\
&- 4 \left[\frac{1}{(k+p)^2 + M_{UI}^2} \frac{1}{3} \frac{k^2 + M_{SI}^2}{(k^2 + M_{UI}^2)(k^2 + M_\eta^2)} \right. \\
&+ \frac{1}{(k+p)^2 + M_{UV}^2} a^2 \delta_V \frac{k^2 + M_{SV}^2}{(k^2 + M_{UV}^2)(k^2 + M_{\eta V}^2)(k^2 + M_{\eta' V}^2)} \\
&+ \left. \left. \frac{1}{(k+p)^2 + M_{UA}^2} a^2 \delta_A \frac{k^2 + M_{SA}^2}{(k^2 + M_{UA}^2)(k^2 + M_{\eta A}^2)(k^2 + M_{\eta' A}^2)} \right] \right\}
\end{aligned}
$$

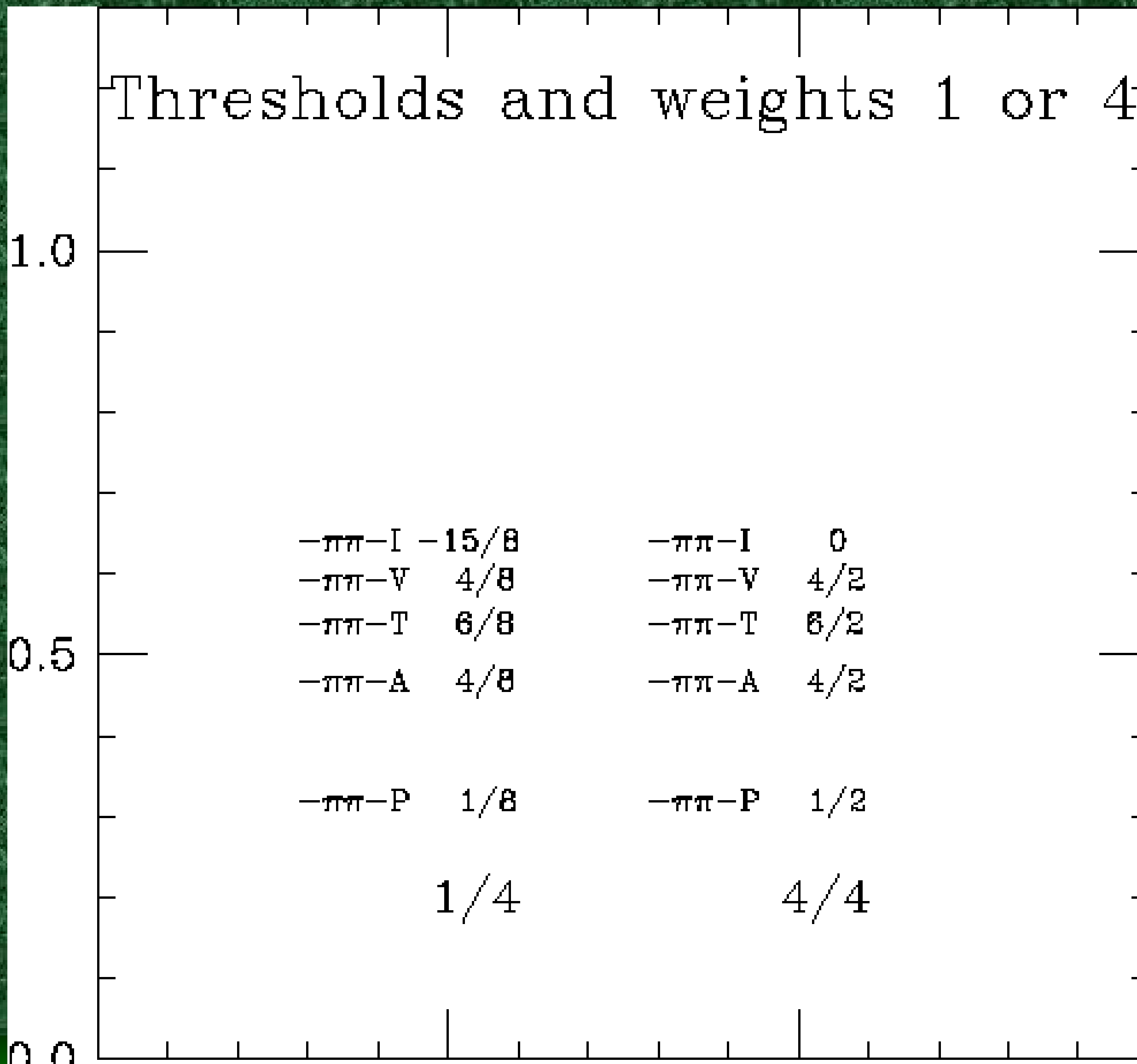
Rooted 2 + 1 Theory



Unrooted 2x4 + 4 Theory



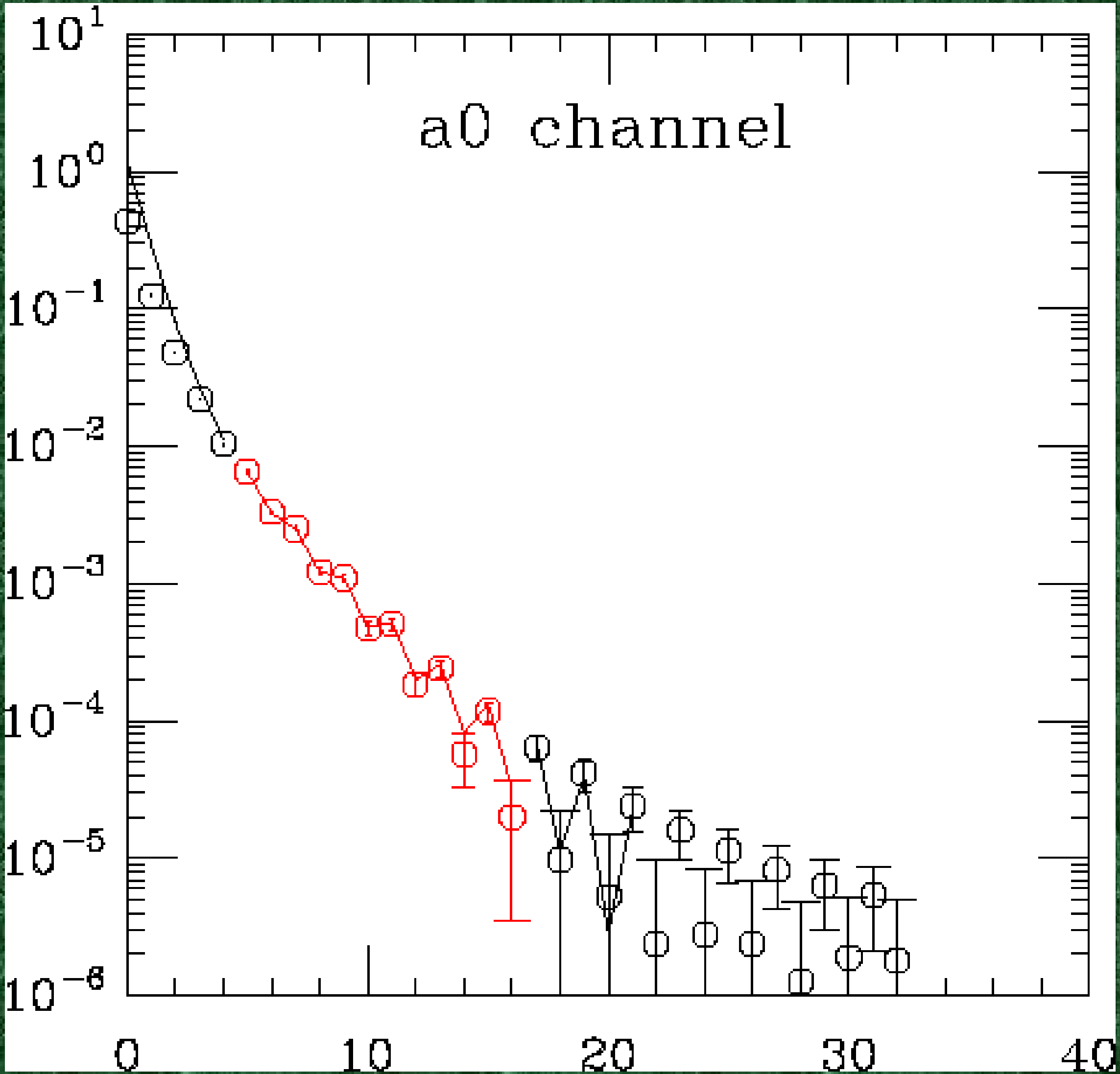
Single flavor



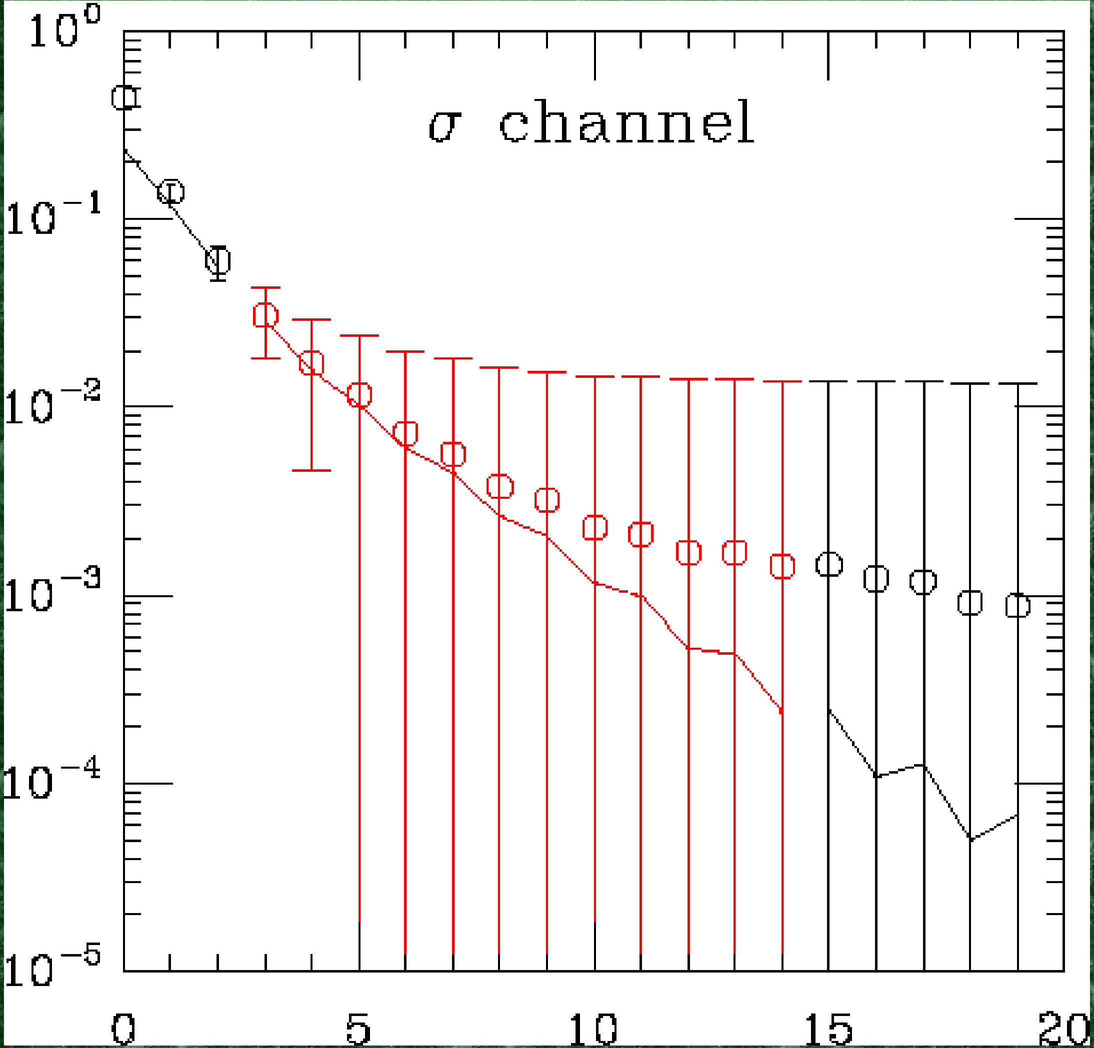
Single flavor from rooting

- No spurious Goldstone modes in the continuum limit.

c f(t) + (-)^t b exp(-Epi t)



c f(t) + (-)^t b exp(-Eeta t)



Fit Result

- Predicted 16 mu^2/L^3 = 0.0033(4) based on bare mu from pion decay constant and pion mass fit.
- Fit from sigma 0.0018(4)
- Fit from a0 0.0018(3)

Conclusion

- Taste symmetry breaking complicates two-particle thresholds
- The parameterization is completely determined.
- Unitarity violations and nonlocality similar to partial quenching.
- No spurious Goldstone modes in the continuum limit

Conclusion

- Better the devil you know than the devil you don't know.

Score Card: Rooted SXPT tests thus far

- Excellent: meson masses
- Excellent: fpi, fK
- Good: Topological susceptibility
- ??: Bubble terms