The σ resonance

Gilberto Colangelo

UNIVERSITÄT BERN

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Outline

Introduction

$\pi\pi$ scattering

Model-independent determination of the pole position

Numerics

Work done in collaboration with I. Caprini and H. Leutwyler, `hep-ph/0512364` to appear on PRL

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Thanks to Heiri Leutwyler for some of the figures

$f_0(600)$ or σ

$I^G(J^{PC}) = 0^+(0^{++})$

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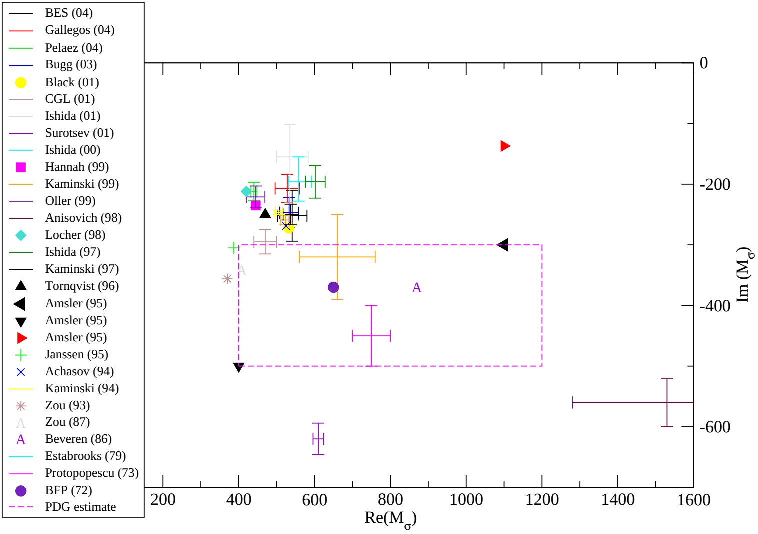
$f_0(600)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\ \mathrm{Im}(\sqrt{s_{\text{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
(400–1200)–*i*(300–500) OUR ESTIMATE			
• • • We do not use the following data for averages, fits, limits, etc. • • •			
$(541 \pm 39)-i(252 \pm 42)$	[1] ABLIKIM 04A	BES2	$J/\psi \to \omega\pi^+\pi^-$
$(528 \pm 32)-i(207 \pm 23)$	[2] GALLEGOS 04	RVUE	Compilation
$(440 \pm 8)-i(212 \pm 15)$	[3] PELAEZ 04A	RVUE	$\pi\pi \to \pi\pi$
$(533 \pm 25)-i(247 \pm 25)$	[4] BUGG 03	RVUE	
$532 - i272$	BLACK 01	RVUE	$\pi^0\pi^0 \to \pi^0\pi^0$
$(470 \pm 30)-i(295 \pm 20)$	[5] COLANGELO 01	RVUE	$\pi\pi \to \pi\pi$
$(535^{+48}_{-36})-i(155^{+76}_{-53})$	[6] ISHIDA 01		$\Upsilon(3S) \to \Upsilon\pi\pi$
$610 \pm 14 - i620 \pm 26$	[7] SUROVTSEV 01	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(558^{+34}_{-27})-i(196^{+32}_{-41})$	ISHIDA 00B		$p\overline{p} \to \pi^0\pi^0\pi^0$
$445 - i235$	HANNAH 99	RVUE	π scalar form factor
$(523 \pm 12)-i(259 \pm 7)$	KAMINSKI 99	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\sigma\sigma$
$442 - i\,227$	OLLER 99	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$469 - i203$	OLLER 99B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$445 - i221$	OLLER 99C	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$
$(1530^{+\ 90}_{-250})-i(560 \pm 40)$	ANISOVICH 98B	RVUE	Compilation
$420 - i\,212$	LOCHER 98	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(602 \pm 26)-i(196 \pm 27)$	[8] ISHIDA 97		$\pi\pi \to \pi\pi$
$(537 \pm 20)-i(250 \pm 17)$	[9] KAMINSKI 97B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, 4π
$470 - i250$	[10,11] TORNQVIST 96	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $K\pi$, $\eta\pi$
$\sim (1100 - i300)$	AMSLER 95B	CBAR	$\overline{p}p \to 3\pi^0$
$400 - i500$	[11,12] AMSLER 95D	CBAR	$\overline{p}p \to 3\pi^0$
$1100 - i137$	[11,13] AMSLER 95D	CBAR	$\overline{p}p \to 3\pi^0$
$387 - i305$	[11,14] JANSSEN 95	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$525 - i269$	[15] ACHASOV 94	RVUE	$\pi\pi \to \pi\pi$
$(506 \pm 10)-i(247 \pm 3)$	KAMINSKI 94	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$370 - i356$	[16] ZOU 94B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$408 - i342$	[11,16] ZOU 93	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$870 - i370$	[11,17] AU 87	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$470 - i208$	[18] BEVEREN 86	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$, ...
$(750 \pm 50)-i(450 \pm 50)$	[19] ESTABROOKS 79	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(660 \pm 100)-i(320 \pm 70)$	PROTOPOP... 73	HBC	$\pi\pi \to \pi\pi$, $K\overline{K}$
$650 - i370$	[20] BASDEVANT 72	RVUE	$\pi\pi \to \pi\pi$

The σ in the PDG



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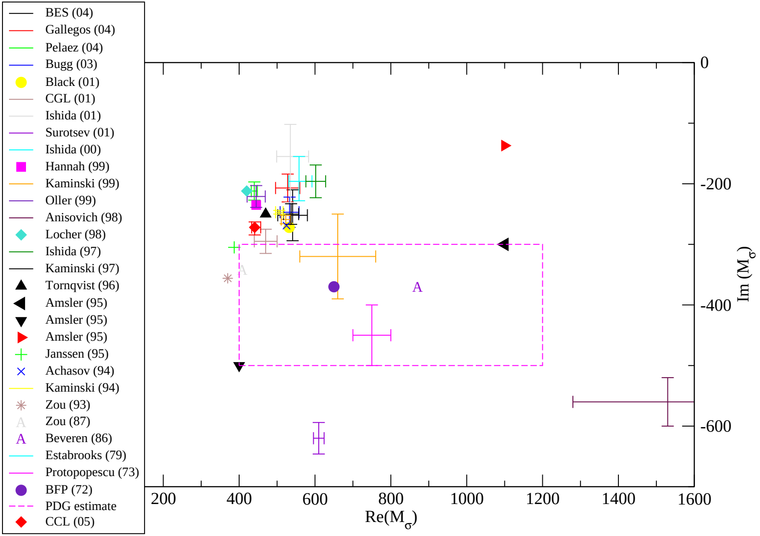
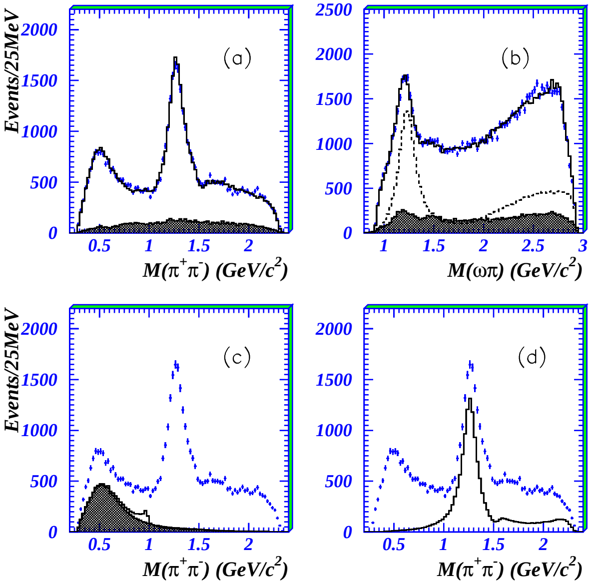


The σ in the data – BES (04), $J/\psi \to \omega\pi^+\pi^-$



How is the σ pole determined?

Fit to the data with a parametrization, e.g.

$$\begin{aligned} f &= \frac{G_\sigma}{M^2 - s - iM\Gamma_{\rm tot}(s)} \\ \Gamma_{\rm tot}(s) &= g_1 \frac{\rho_{\pi\pi}(s)}{\rho_{\pi\pi}(M^2)} + g_2 \frac{\rho_{4\pi}(s)}{\rho_{4\pi}(M^2)} \end{aligned}$$

where $g_{1,2}$ can also be functions of s

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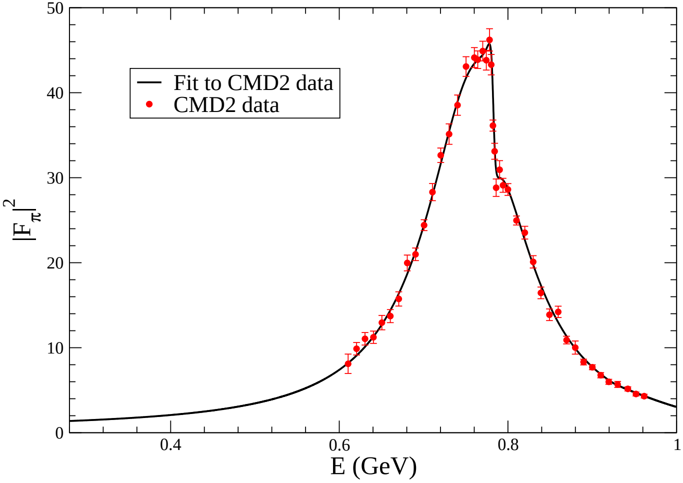
The outcome is parametrization-dependent
Moreover, an obvious shortcoming of many of the parametrizations used to fit data is the neglect of the left-hand cut

Compare to the ρ in $e^+e^- \to \pi^+\pi^-$



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- Understanding the spectrum of QCD – stable particles as well as resonances – is a crucial test for the theory of strong interactions
- Identifying a resonance means determining the position of second-sheet poles in scattering amplitudes
- Finding out where the poles lie is important and sometimes a true challenge

$\pi\pi$ scattering, Roy equations

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- Pioneering work in solving numerically these equations has been performed in the seventies Basdevant, Froggatt, Petersen 1974

Roy equations and chiral symmetry

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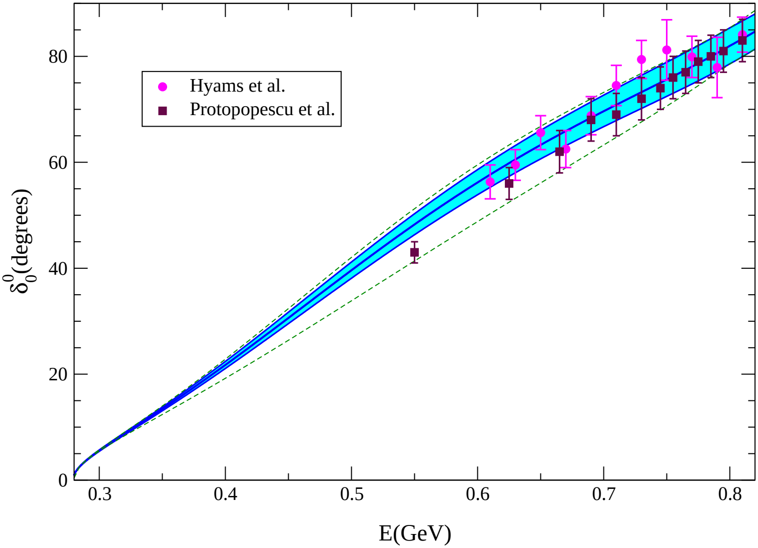
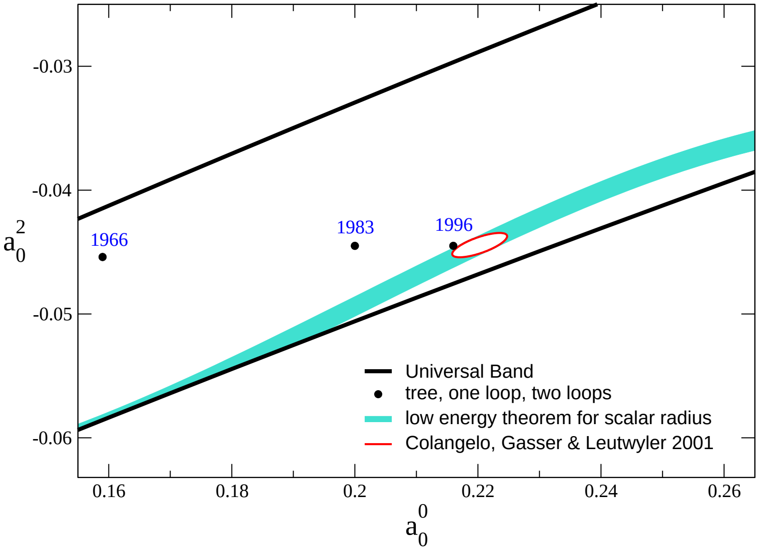
- Chiral symmetry provides precise information about the scattering lengths

  Weinberg (66), Gasser & Leutwyler (84), Bijnens, GC, Ecker, Gasser, Sainio (96)
- Matching the dispersive and chiral representation near $s = 0$ one obtains the $\pi\pi$ scattering amplitude at low energy to a high degree of precision GC, Gasser and Leutwyler (01)

Roy equations and chiral symmetry



Predictions for the scattering lengths



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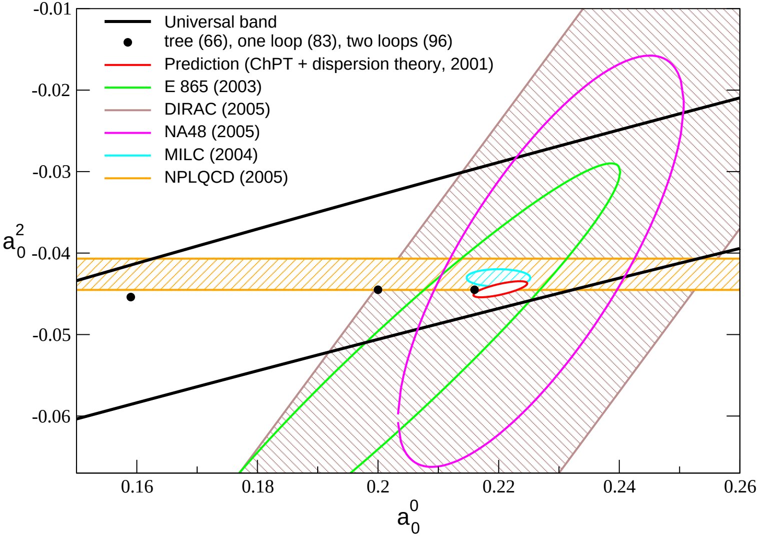

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- Good news! As far as this talk is concerned, all this can be ignored (see below)

A model-dependent σ-pole determination

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$$\sqrt{s} \simeq 840\text{MeV} \qquad]$$

- This determination explicitly relies on parametrizations. The systematic effect due to the choice of a parametrization can only be estimated

Roy representation of t_0^0

Double-subtracted, crossing symmetric dispersion relation for t_0^0

$$\begin{aligned} t_0^0(s) &= a + (s - 4M_\pi^2)\, b + \int_{4M_\pi^2}^{\Lambda^2} ds' \left\{ K_0(s,s')\,\mathrm{Im}\, t_0^0(s') \right. \\ &\quad \left. + K_1(s,s')\,\mathrm{Im}\, t_1^1(s') + K_2(s,s')\,\mathrm{Im}\, t_0^2(s') \right\} + d_0^0(s) \end{aligned}$$

$$a = a_0^0\,, \quad b = (2\,a_0^0 - 5\,a_0^2)/(12M_\pi^2)$$

$$K_0(s,s') = \frac{1}{\pi(s'-s)} + \frac{2\ln((s+s'-4M_\pi^2)/s')}{3\pi(s-4M_\pi^2)} - \frac{5s'+2s-16M_\pi^2}{3\pi s'(s'-4M_\pi^2)}$$

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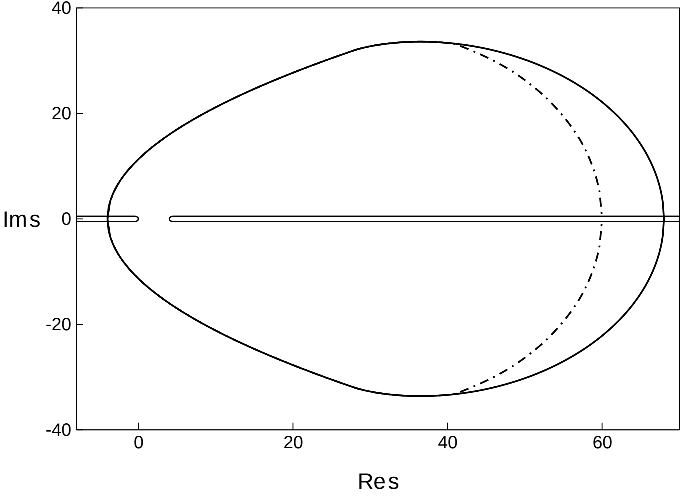

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This representation allows one to evaluate t_0^0 in the complex plane – in its domain of validity on the first sheet.

Poles, however, are to be found on the second sheet

Roy representation of S_0^0

$$S_0^0(s) = 1 - 2\sqrt{\frac{4M_\pi^2}{s} - 1}\, t_0^0(s)\,, \qquad 0 \le s \le 4M_\pi^2$$

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The second sheet is reached by analytic continuation crossing the real axis from above: (for ϵ infinitesimally small)

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By analytic continuation, it is then true everywhere that

$$S_0^{0\,II}(s) = \left[S_0^{0\,I}(s)\right]^{-1}$$

Poles on the second sheet correspond to zeros on the first sheet!

Summary: method to determine the pole position

- Roy equations provide an explicit representation of t_0^0 on the first sheet, in terms of the imaginary parts of the partial waves on the real axis and two subtraction constants:

$$t_0^0(s) = a + (s - 4M_\pi^2)\, b + \int_{4M_\pi^2}^{\Lambda^2} ds' K_0(s, s')\, \mathrm{Im}\, t_0^0(s') + \ldots$$

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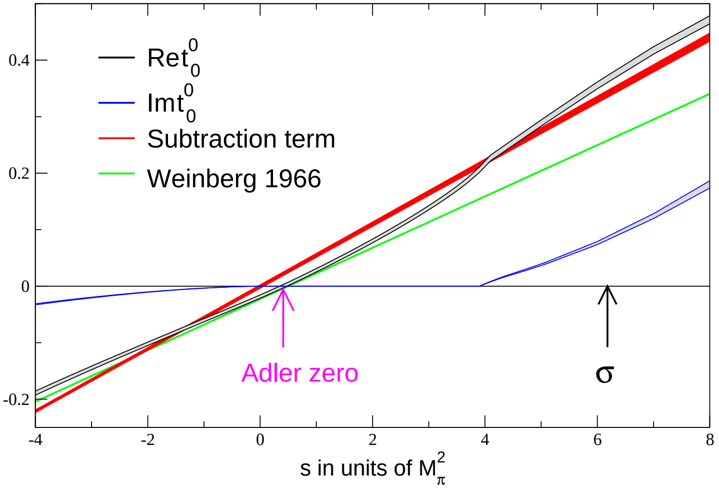
- Using as input the imaginary parts of the partial waves and the two S-wave scattering lengths one can determine the position of the poles of the S-matrix on the second sheet

Importance of the scattering lengths



Zeros of S_0^0 (and S_1^1)

Using as input the imaginary parts determined by solving the Roy equations up to 1.15 GeV [GC, Leutwyler, in preparation] and the central values of the two scattering lengths we find two pairs of zeros

$$m_\sigma^2 = (6.2 \pm i\,12.3)\,M_\pi^2 \qquad m_{f_0}^2 = (51.4 \pm i\,1.4)\,M_\pi^2$$

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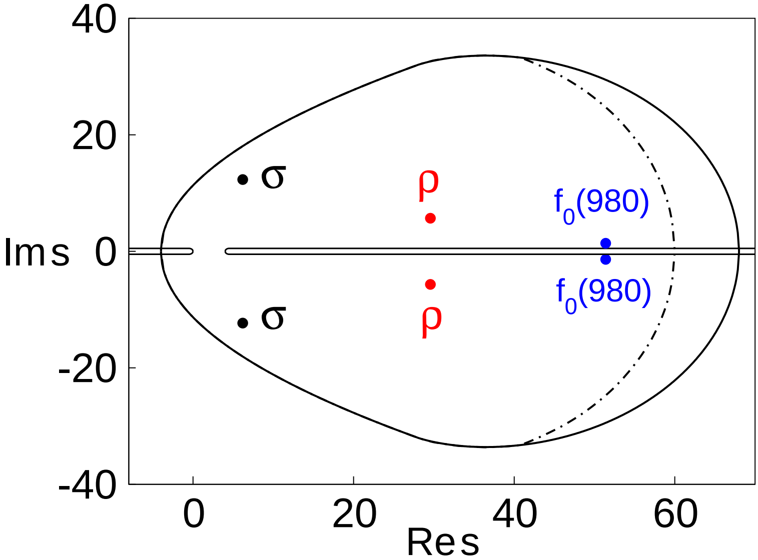

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Error analysis: [at fixed a_0^0, a_0^2 and $\delta_A \equiv \delta_0^0(0.8\text{GeV})$]

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$$\Delta a_0^0 = \frac{a_0^0 - 0.220}{0.005} \qquad \Delta a_0^2 = \frac{a_0^0 + 0.0444}{0.001}$$

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Error analysis:

$$\begin{aligned} m_\sigma &= 441 \pm 4 - i(272 \pm 6)\text{ MeV} + (-2.4 + i3.8)\Delta a_0^0 \\ &\quad + (0.8 - i4.0)\Delta a_0^2 + (5.3 + i3.3)\Delta\delta_A \end{aligned}$$

$$\Delta a_0^0 = \frac{a_0^0 - 0.220}{0.005} \qquad \Delta a_0^2 = \frac{a_0^0 + 0.0444}{0.001} \qquad \Delta\delta_A = \frac{\delta_A - 82.3}{3.4}$$

Zeros of S_0^0 (and S_1^1)

Using as input the imaginary parts determined by solving the Roy equations up to 1.15 GeV [GC, Leutwyler, in preparation] and the central values of the two scattering lengths we find two pairs of zeros

$$m_\sigma^2 = (6.2 \pm i\,12.3)\,M_\pi^2 \qquad m_{f_0}^2 = (51.4 \pm i\,1.4)\,M_\pi^2$$

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$$m_\sigma = 441 \pm 7 - i272 \pm 9$$

Different inputs

- The extension of the Roy equation analysis from 0.8 to 1.15 GeV has no impact on m_σ. Using CGL (01) we get

$$\begin{aligned} m_\sigma^{\rm CGL}(\text{model indep.}) &= 439.4 - i274.5 \text{ MeV} \\ m_\sigma^{\rm CGL}(\text{param.-dep.}) &= 470 \pm 30 - i295 \pm 20 \text{ MeV} \end{aligned}$$

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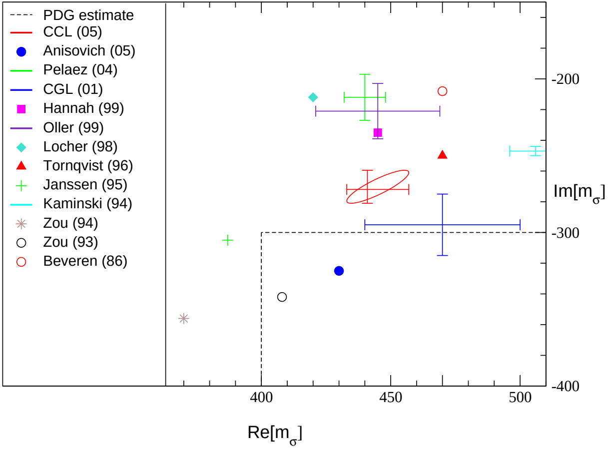
- Using a phenomenological representation of the $\pi\pi$ scattering amplitude [Pelaéz and Ynduráin (05)] we obtain

$$m_\sigma^{\rm PY} = 445 - i241 \text{ MeV}$$

Our formula which describes the dependence on the main three input parameters reproduces this result:

$$a_0^0(PY) = 0.23\,, \quad a_0^2(PY) = -0.048\,, \quad \delta_A(PY) = 90.9^\circ$$
$$\Rightarrow m_\sigma = 447 - i242 \text{ MeV}$$

Comparison to PDG and experimental information



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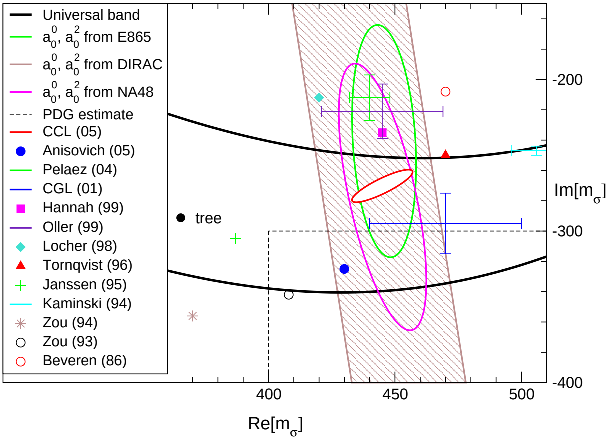

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$$M_\sigma = 441\ {}^{+16}_{-8}\ \text{MeV}\,, \quad \Gamma_\sigma = 544\ {}^{+18}_{-25}\ \text{MeV}$$

- Crucial ingredients of this calculation are:
  - the scattering lengths
  - the left-hand cut

  which often play little or no role in phenomenological determinations of the σ parameters

Quark mass dependence of the σ?