Two Color QCD in the Chiral Limit

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Outline

- **Motivation**
- **Chiral Two color Lattice QCD (2CLQCD)**
  - **Model**
  - **Dimer-Baryonloop representation**
  - **Symmetries and Breaking**
- **Chiral Lagrangian and Predictions**
  - **$\mu = 0$ physics at zero and finite T**
  - **Non-zero μ physics at zero and finite T**
- **T-μ Phase Diagram**
- **A puzzle at finite quark mass**
- **Conclusions**

Motivation

- Chiral Limit in QCD-like theories is difficult to study
  - Algorithms slow down
  - Matching of low energy physics with chiral perturbation theory (CHPT), although widely accepted, remains untested in many interesting cases.
  - T- μ phase diagrams in many cases unclear.

- New opportunities at strong coupling (staggered fermions)
  - confinement and chiral symmetry breaking natural
  - models with all kinds of chiral symmetries can be constructed
  - cluster algorithms can be formulated in the chiral limit
  - diquark correlation functions easy to measure
  - large lattices with relative ease

Two Color Lattice QCD (2CLQCD)

Dagotto, Moreo & Wolff (1987), Klatke & Mutter (1990)

- **Action (infinite gauge coupling)**

$$S = -\sum_{x,\alpha} r_\alpha \eta_{x,\alpha} \left[e^{\mu a_t \delta_{t,\alpha}} \overline{\chi}_x U_{x,\alpha} \chi_{x+\hat{\alpha}} - e^{-\mu a_t \delta_{t,\alpha}} \overline{\chi}_{x+\hat{\alpha}} U^{\dagger}{}_{x,\alpha} \chi_x \right]$$

$$U \in SU(2);\; r_t = \frac{1}{a_t};\; r_{1,2,3} = 1;$$

- **Partition function**

$$Z = \int d\chi \, d\overline{\chi} \, dU \;\; \exp(-S)$$

- **Parameters of the theory**

$$T \equiv \frac{1}{a_t^2}, \mu$$

Dimer-Baryonloop Representation

Rossi & Wolff, Nucl. Phys. B248, 105 (1984)

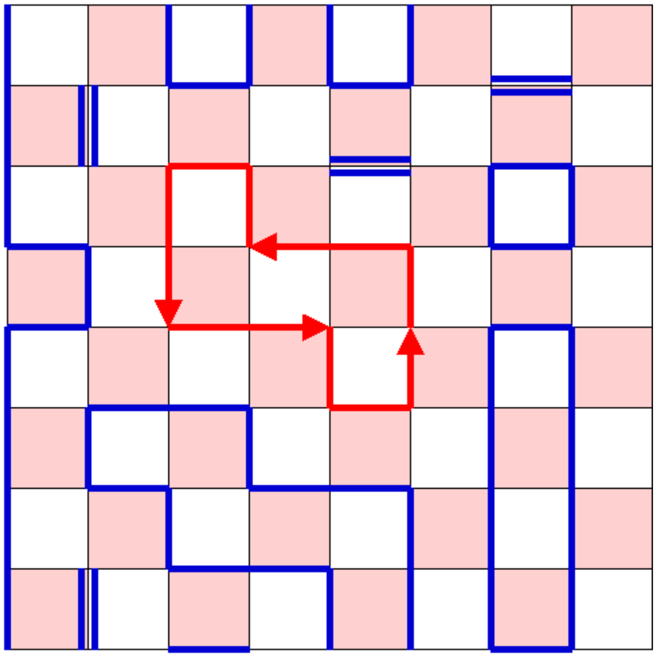
- **Partition function can be rewritten as a statistical mechanics of dimer-baryonloop configurations**

$$Z = \sum_{K} \exp\left\{ \sum_{x} \left(\left(k_{x,t} + | b_{x,t} | \right) \log(T) + 2 \frac{\mu}{\sqrt{T}} b_{x,t} \right) \right\}$$

$$k_{x,\alpha} = 0,1,2; b_{x,\alpha} = -1,0,1; \sum_{\alpha} k_{x,\alpha} + | b_{x,\alpha} | = 2$$

- **A directed-path update algorithm can be constructed.** **Adams & SC (2003)**

- **Many observables are easy to compute.**



Symmetries of 2CLQCD

Hands, Kogut, Lombardo & Morrisson (1999)

- **Define** $\overline{X}_e = \left(\overline{\chi}_e, -\chi_e^T \tau_2\right) \;;\; X_o = \begin{pmatrix} \chi_o \\ -\tau_2 \overline{\chi}_o^T \end{pmatrix}$

- **Action can be rewritten as**

$$S = -\sum_{x,\alpha} r_\alpha \eta_{x,\alpha} \left[e^{\mu a_t \delta_{t,\alpha} \sigma_3} \overline{X}_{x,e} U_{x,\alpha} X_{x+\hat{\alpha},o} - e^{-\mu a_t \delta_{t,\alpha} \sigma_3} \overline{X}_{x,e} U^+_{x-\hat{\alpha},\alpha} X_{x-\hat{\alpha},o} \right]$$

- **Symmetries at $\mu = 0$:**

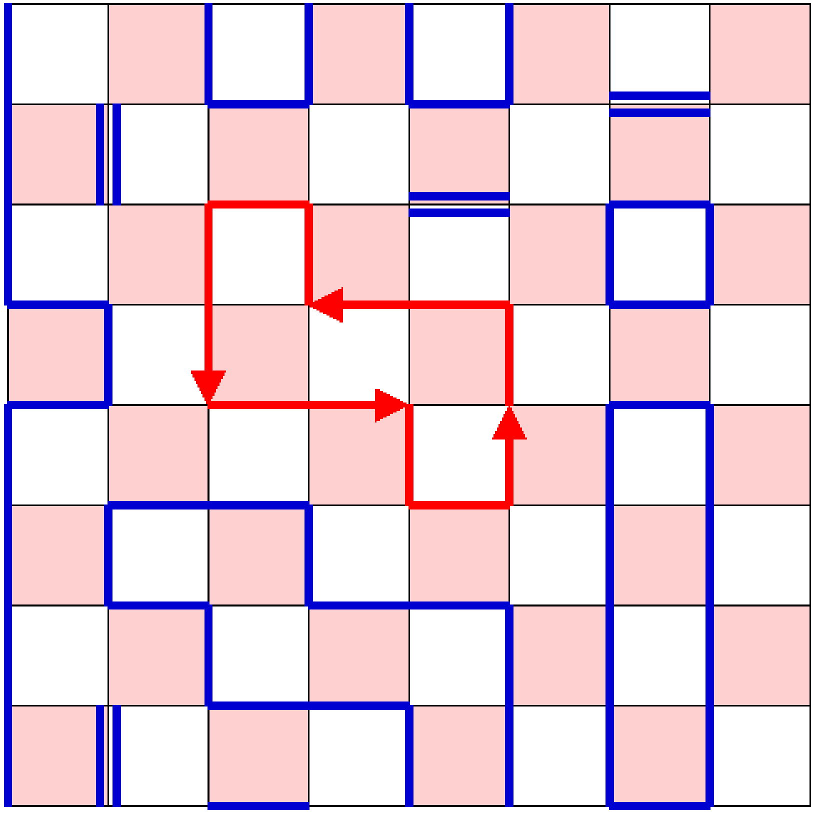
$$X_o \to V X_o \;,\; \overline{X}_e \to \overline{X}_e V^\dagger, \;\; V \in U(2)$$

$$\text{(Baryon) } U_B(1) : X_o \to \exp(i\sigma_3 \phi), \quad \overline{X}_e \to \overline{X}_e \exp(-i\sigma_3 \phi)$$

$$\text{(Chiral) } U_C(1) : X_o \to \exp(i\phi), \quad \overline{X}_e \to \overline{X}_e \exp(-i\phi)$$

$$U(2) \approx SU(2) \otimes U_C(1) \approx O(3) \otimes O(2)$$

Two conserved currents



$$J_B^{\alpha}(x) = b_{x,\alpha}$$

$$J_C^{\alpha}(x) = (-1)^{x_1+x_2+\ldots}(k_{x,\alpha} + |b_{x,\alpha}|)$$

Symmetry Breaking Pattern

- The following three components transform as a complex 3-vector under U(2).

$$\Phi_1 = i\left[\chi_1\chi_2 + \overline{\chi}_1\overline{\chi}_2\right], \Phi_2 = \left[-\chi_1\chi_2 + \overline{\chi}_1\overline{\chi}_2\right], \Phi_3 = \overline{\chi}\chi = \left[\overline{\chi}_1\chi_1 + \overline{\chi}_2\chi_2\right]$$

- Since at $\mu=0$ we expect the chiral condensate to be non-zero, the symmetry breaking pattern should be

$$SU(2)\otimes U(1) \rightarrow U(1)\otimes \mathbb{Z}_2$$

or equivalently

$$O(3)\otimes O(2) \rightarrow O(2)\otimes \mathbb{Z}_2$$

- This is called collinear order in condensed matter physics.
- This symmetry and breaking pattern is encountered in superfluid Helium-3.

Chiral Perturbation Theory

- **One expects three Goldstone bosons**
- **The Chiral Lagrangian (at leading order) is given by**

$$S = \int d^d x \left[\frac{F_B^2}{2} \left(\partial_\mu \vec{s} \cdot \partial_\mu \vec{s} \right) + \frac{F_C^2}{2} \left(\partial_\mu \vec{u} \cdot \partial_\mu \vec{u} \right) \right]$$

$\vec{s}$ is a real unit 3-vector and $\vec{u}$ is a real unit 2-vector.

- **The decay constants can be obtained from**

Hasenfratz & Leutwyler, NPB 343 (1990) 241

$$Y_B = \frac{1}{V} \left\langle \left(\sum_x J_B^i(x) \right)^2 \right\rangle, \quad Y_C = \frac{1}{V} \left\langle \left(\sum_x J_C^i(x) \right)^2 \right\rangle$$

- **Finite size scaling (after some calculation!)**

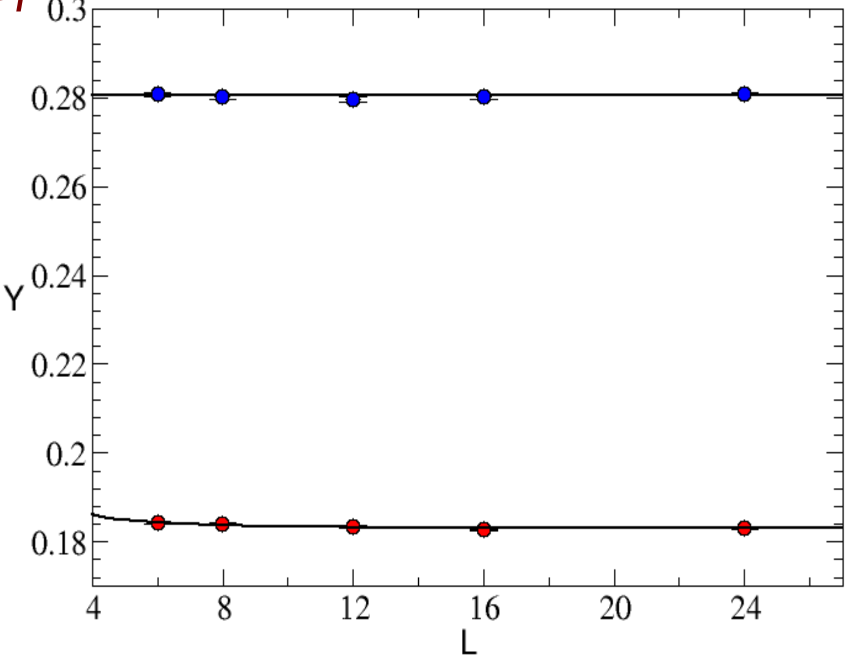
$$Y_B(L) = \frac{2}{3} F_B^2 + \frac{a}{L^2} + \ldots, \quad Y_C(L) = F_C^2 + \frac{a'}{L^2} + \ldots$$

Results: T=1.0, μ=0, LxLxLxL lattice

$$Y_B = 0.2744(3) \times \frac{2}{3} + \frac{0.05(1)}{L^2}, \chi^2/_{dof} = 1.6$$

$$Y_C = 0.2804(2), \chi^2/_{dof} = 1.3$$

$$F_B^2 = 0.2744(3); \quad F_C^2 = 0.2804(2)$$



The two decay constants are almost the same!
Is this an accident
or can we understand it from some symmetry?

A consistency check

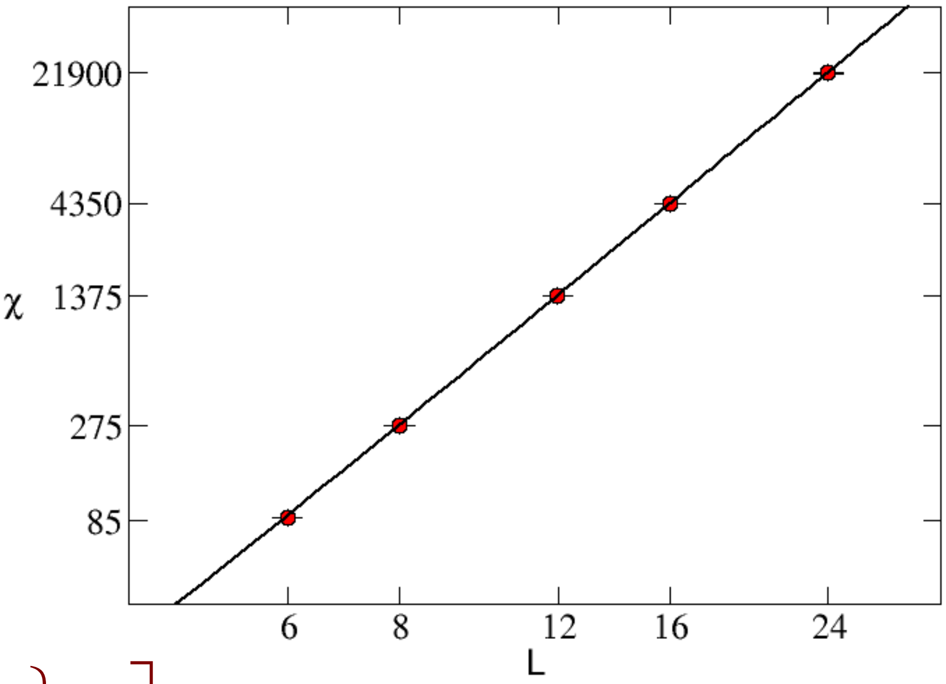
$$\Sigma = \left\langle \overline{\chi}\chi \right\rangle$$

- We can measure

$$\chi_C = \sum_x \left\langle \overline{\chi}\chi(x) \quad \overline{\chi}\chi(0) \right\rangle$$

- Finite size scaling gives

H & L, NPB 343 (1990) 241



$$\chi = \frac{\Sigma^2}{6}\left[L^4 + 0.141\left\{ \frac{2}{F_B^2} + \frac{1}{F_C^2} \right\} L^2 \right]$$

- A one parameter fit using L=8,12,16,24 data gives

$$\Sigma = 0.6283(3), \; \chi^2/dof = 0.8$$

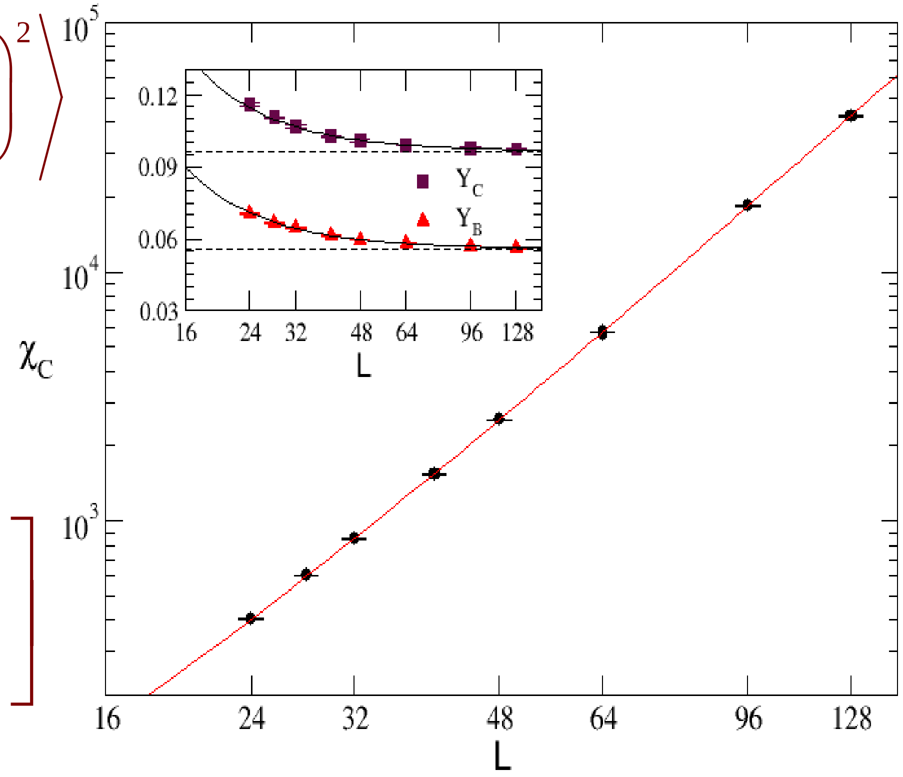
Finite T results: μ = 0, T=2.918 (4xLxLxL lattice)

Need 3d chiral perturbation theory

$$Y_B = \frac{1}{L^3}\left\langle \left(\sum_x J_B^i(x) \right)^2 \right\rangle, Y_C = \frac{1}{L^3}\left\langle \left(\sum_x J_C^i(x) \right)^2 \right\rangle$$

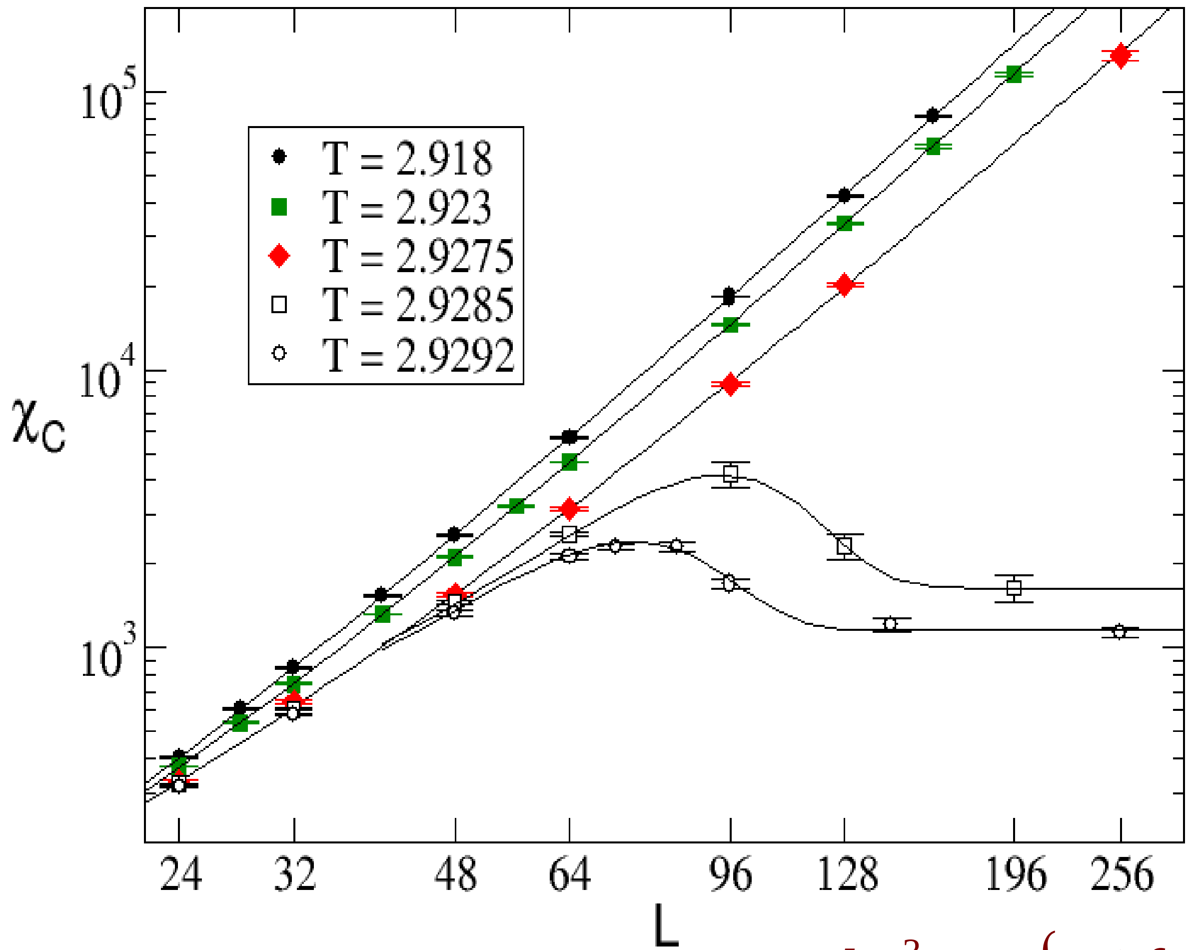
$$Y_B(L) = \frac{2}{3}F_B^2 + \frac{a}{L}, Y_C(L) = F_C^2 + \frac{a'}{L},$$

$$\chi = \frac{\Sigma^2 L_t}{6}\left[L^3 + 0.226\left\{ \frac{2}{F_B^2} + \frac{1}{F_C^2} \right\} L^2 \right]$$



$$F_B^2 = 0.0839(6), \quad F_C^2 = 0.0965(5), \quad \Sigma = 0.1682(4)$$

First Order Phase Transition at $\mu = 0$



For $T > T_c$ we use $\chi_C = \dfrac{a + bL^3 \exp\left\{-\Delta f L^3\right\}}{1 + c \exp\left\{-\Delta f L^3\right\}}$

Physics at non-zero μ

- The symmetry of the action and breaking pattern is

$$U_B(1) \otimes U_C(1) \to \mathbb{Z}_2 \otimes \mathbb{Z}_2$$

- There are two Goldstone bosons governed by the chiral Lagrangian

$$S = \int d^d x \left[\frac{F_B^2}{2} \left(\partial_\mu \vec{s} \cdot \partial_\mu \vec{s} \right) + \frac{F_C^2}{2} \left(\partial_\mu \vec{u} \cdot \partial_\mu \vec{u} \right) \right]$$

where both $\vec{s}$ and $\vec{u}$ are real unit 2-vectors.

- The finite size scaling in 3d is given by

$$Y(L) = F^2 + \frac{a}{L}$$

$$\chi_B = \frac{\Delta^2 L_t}{2} \left[L^3 + 0.226 \left\{ \frac{1}{F_B^2} + \frac{1}{F_C^2} \right\} L^2 \right] \quad \text{where } \Delta = \langle \chi_1 \chi_2 \rangle$$

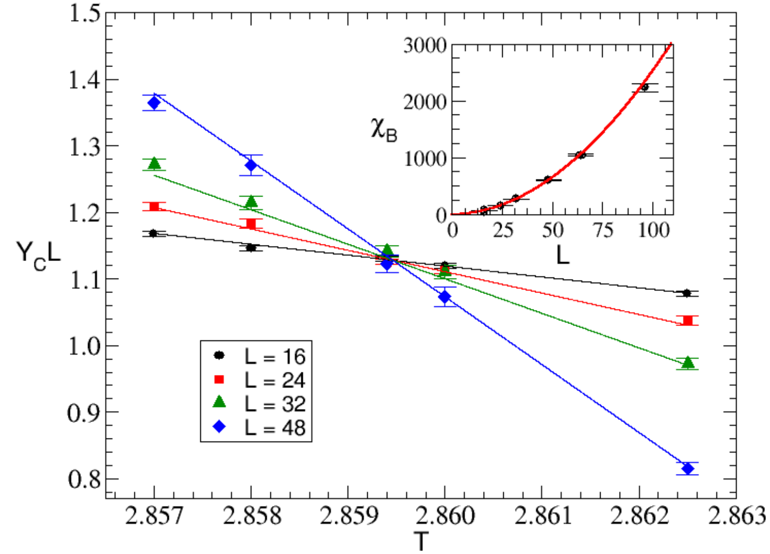
Second order phase transition at μ=0.3

$$YL = f_0 + f_1\left(T - T_c\right)L^{1/\nu} + f_0^{'}L^{-\omega}$$

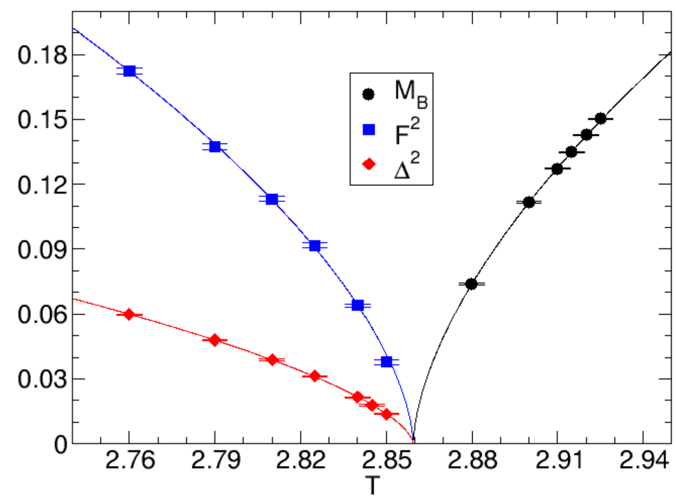
$$F^2 = a\left(T_c - T\right)^{\nu}\left\{1 + a^{'}\left(T_c - T\right)^{\omega\nu}\right\}$$

$$\Delta^2 = d\left(T_c - T\right)^{2\beta}\left\{1 + d^{'}\left(T_c - T\right)^{\omega\nu}\right\}$$

$$M_B = c\left(T - T_c\right)^{\nu}\left\{1 + c^{'}\left(T - T_c\right)^{\omega\nu}\right\}$$



Consistent with two universality classes



New Exponents:

$\nu = 0.610(6), \beta = 0.311(5), \eta = 0.042(2), \omega = 0$

Decoupled XY exponents:

$\nu = 0.6715, \beta = 0.3485, \eta = 0.038, \omega = 0.0218$

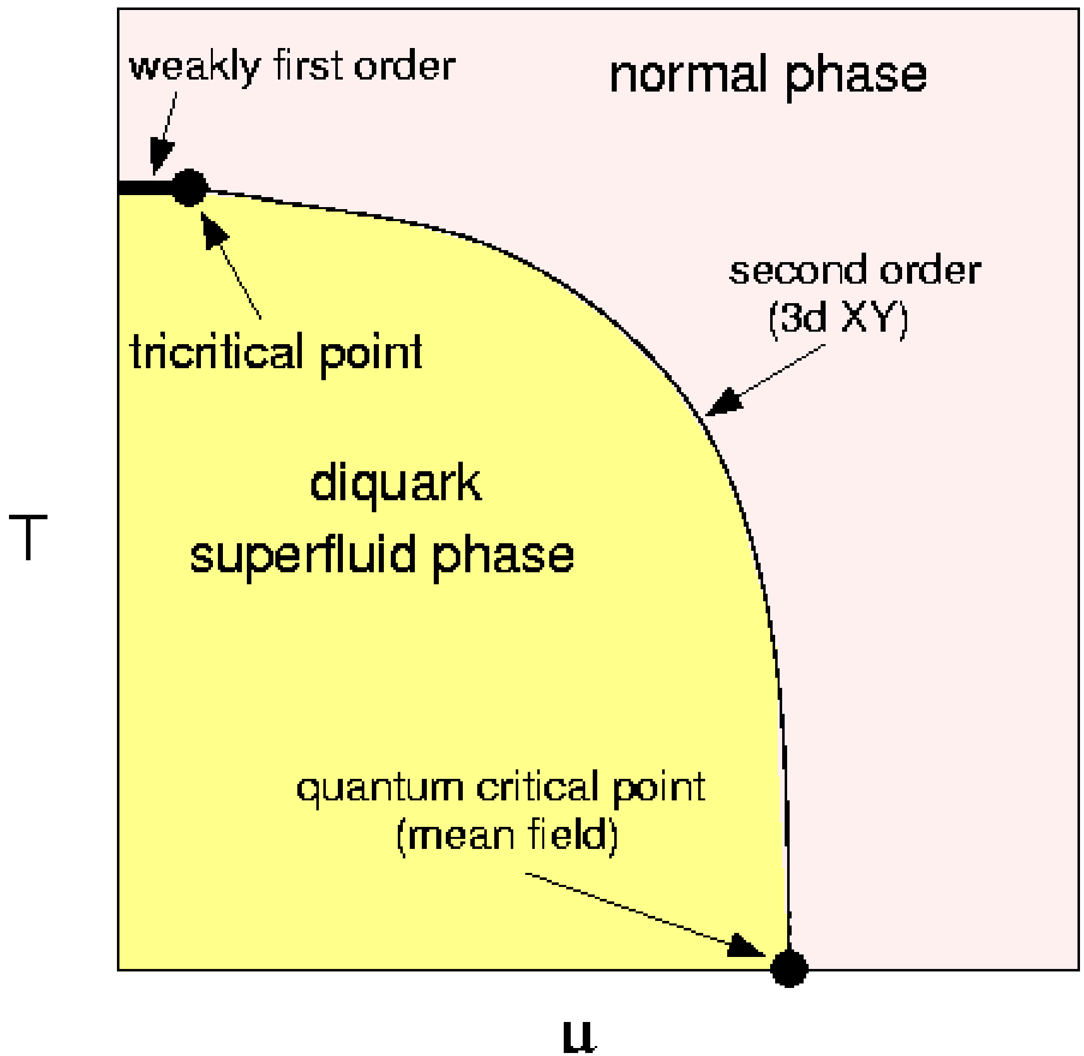
Universality

- Linear sigma model can be used to discuss the transition

$$S = \int d^d x \left\{ \partial_\mu \psi^* \cdot \partial_\mu \psi + r\left(\psi^* \cdot \psi\right) + u\left(\psi^* \cdot \psi\right)^2 + v\,|\psi \cdot \psi|^2 \right\}$$

- Sign of v determines the ordering
  – v < 0 leads to collinear order.
- For complex 2-vectors there is a decoupled fixed point that can dictate the universality
- For complex 3-vectors things are unclear
  – Epsilon exapansion predicts a fluctuation driven first order transition.
  – Recent resummation techniques suggest a second order transition.

Conjectured Phase Diagram



Loose Ends and Puzzles?

- In the presence of quark mass at μ = 0 one obtains

$$S = \int d^d x \left[\frac{F_B^2}{2} \left(\partial_\mu \vec{s} \cdot \partial_\mu \vec{s} \right) + \frac{F_C^2}{2} \left(\partial_\mu \vec{u} \cdot \partial_\mu \vec{u} \right) - m\Sigma s_3 u_2 \right]$$

- Expanding about the minima one gets

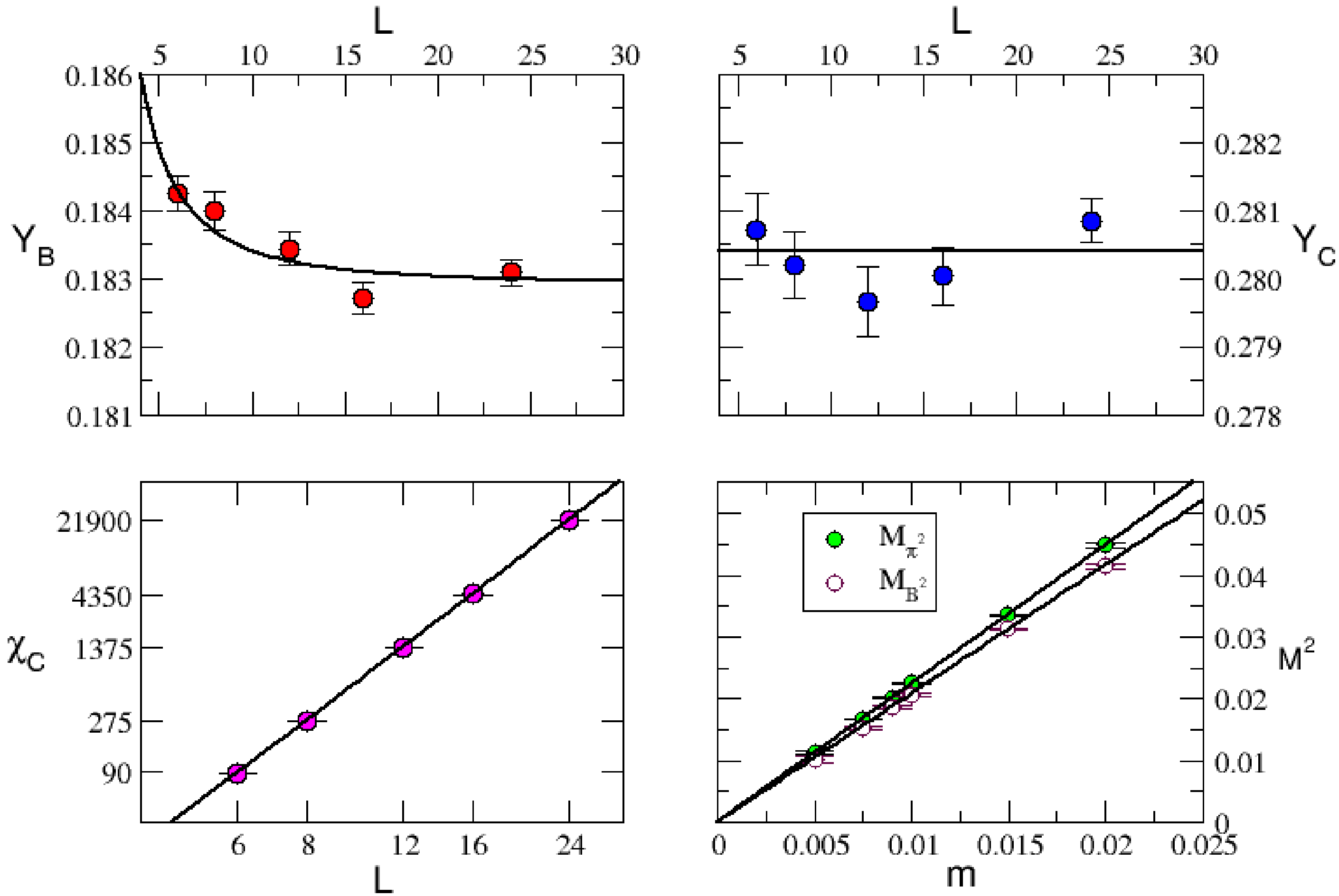
$$s_3 = \sqrt{1 - s_1^2 - s_2^2}, u_2 = \sqrt{1 - u_1^2}$$

$$S = \int d^d x \left[\frac{F_B^2}{2} \left\{ \left(\partial_\mu s_1 \right)^2 + \left(\partial_\mu s_2 \right)^2 + \frac{m\Sigma}{F_B^2} \right\} + \frac{F_C^2}{2} \left\{ \left(\partial_\mu u_1 \right)^2 + \frac{m\Sigma}{F_C^2} \right\} + \ldots \right]$$

- One thus predicts

$$M_B^2 = \frac{m\Sigma}{F_B^2}, M_\pi^2 = \frac{m\Sigma}{F_C^2}$$

Loose Ends and a Puzzle (continued…)



$$F_B^2 = 0.2744(3);\quad F_C^2 = 0.2804(2), \Sigma = 0.6283(3)$$

We find $M_\pi^2 = \dfrac{m\Sigma}{F_C^2}$, $\quad M_B^2 = \dfrac{m\Sigma}{\lambda F_B^2}$, $\lambda \sim 1.1!$

Conclusions

- **Strong coupling QCD provides a unique opportunity to explore the chiral limit of QCD-like theories from first principles**
  - **Two color QCD can be solved very accurately!**
  - **Puzzle 1: why are the two decay constants almost same?**
  - **Puzzle 2: why are pion masses inconsistent with CHPT.**

- **A new approach to chiral Lagrangian on the lattice**
  - **include more number of flavors.**
  - **Baryon chiral perturbation theory also possible.**

- **A new approach to bosonic field theories:**
  - **Similar to D-theory.**
  - **New algorithms for problems not available in the conventional approach.**