Running Coupling and Chiral Phase Boundary in QCD at Finite Temperature

Jens Braun
Heidelberg University

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J. Braun, H. Gies, hep-ph/0512085
J. Braun, H. Gies, hep-ph/0602226

Outline

Motivation

Functional Renormalization Group

running coupling at finite temperature

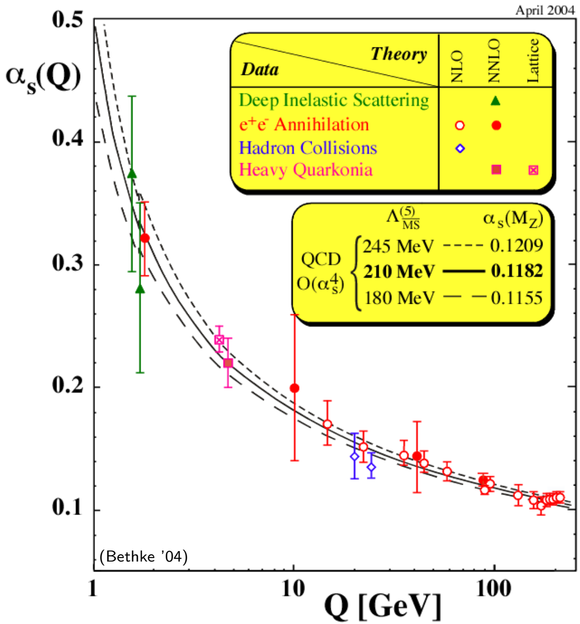
chiral phase boundary of QCD

- critical value of α_s
- results

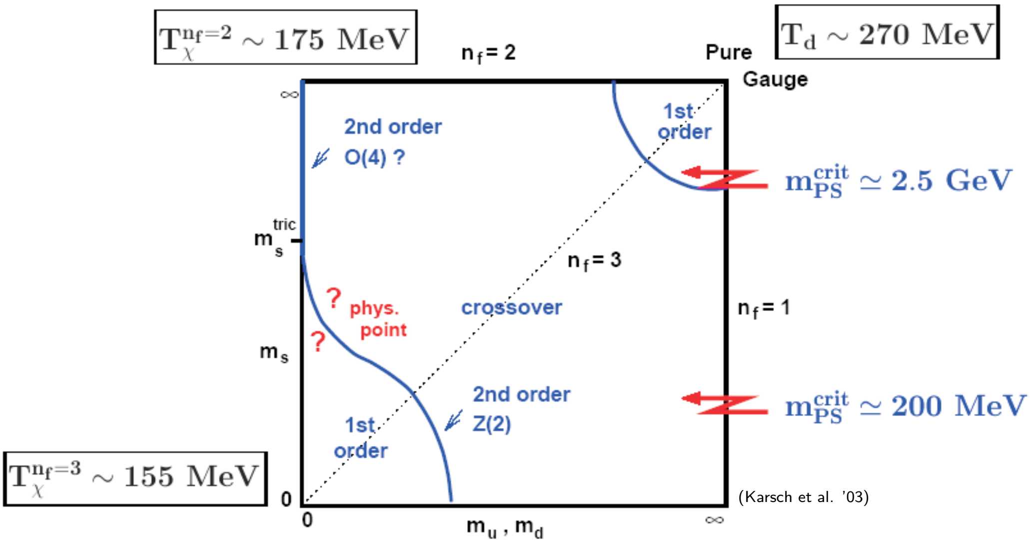
conclusions and outlook

perturbative QCD and experiment

- Asymptotic freedom at high momemtum scales (Politzer '73, Gross and Wilczek '73)
- running coupling at small momentum scales? $\longrightarrow$ pQCD fails *(Landau-pole)*
- confinement, χSB

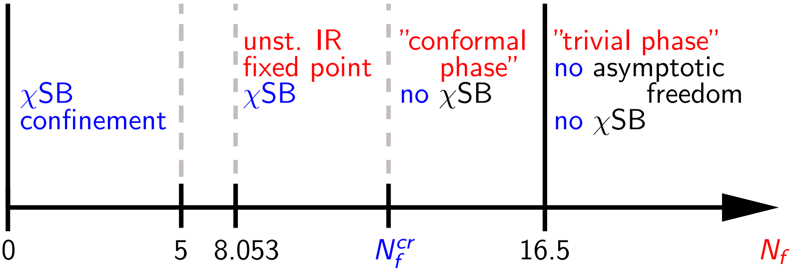


QCD at finite temperature



- 1st order(?) for $N_f = 2$ massless flavors (D'Elia, Di Giacomo, Pica)
- $T_\chi(N_f = 2) - T_\chi(N_f = 3) \approx 20\,\mathrm{MeV}$ (Karsch et al. '03)
- flavor number dependence of T_χ?

many-flavor QCD (a short review)



- two-loop β-function

$$\beta(\alpha) = -\overbrace{\frac{1}{6\pi}(11N_c - 2N_f)}^{=b_1}\alpha^2 - \overbrace{\frac{1}{24\pi^2}\left(34N_c^2 - 10N_cN_f - 3\frac{N_c^2-1}{N_c}N_f\right)}^{=b_2}\alpha^3 - \ldots$$

- $b_1 < 0 \longrightarrow N_f > \frac{11}{2}N_c \overset{N_c=3}{=} 16.5$ (no asymptotic freedom)
- $b_1 > 0$ and $b_2 < 0 \longrightarrow N_f > \frac{34N_c^3}{13N_c^2-3} \overset{N_c=3}{\approx} 8.053$ (non-trivial fixed point)
- $N_f^{cr} \approx \begin{cases} 8 & (\text{Brown et. al. '92}) \\ 12 & (\text{Kogut et. al. '92}) \\ 5 & (\text{Schafer, Shuryak '95}) \\ 12 & (\text{Appelquist et. al. '96}) \\ ? & \text{RG} \end{cases}$

Non-perturbative methods

Lattice QCD

- regularization by a finite lattice spacing ($\rightarrow$ improved actions)
- large current quark masses
  $\longrightarrow$ extrapolation required (chiral perturbation theory, lin. extrapolation)
- finite volume
  $\longrightarrow$ IR-cutoff introduced $\longrightarrow$ extrapolation required (chPT, ...)

quark boundary conditions are important (J. B., B. Klein, H.-J. Pirner)

complementary non-perturbative methods, e. g.

- Dyson-Schwinger Equations
  (truncation required: vertex expansion)
- Functional Renormalization Group
  (truncation required: operator expansion, vertex expansion, ...)

Functional Renormalization Group I

- **Goal**: Flow Equation for Γ_k which interpolates between $S_{\rm bare}$ and $\Gamma_{\rm 1PI}$

$$\Gamma_k \xrightarrow{k\to 0} \Gamma^{\rm 1PI} \qquad\qquad \Gamma_k \xrightarrow{k\to\Lambda} \mathrm{S}^{\rm bare}$$

- **Schwinger Functional**:

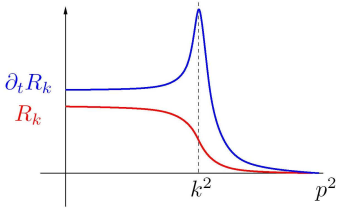
$$Z[J] = \int_\Lambda \mathcal{D}\tilde{\phi}\, \mathrm{e}^{-S[\tilde{\phi}]+\int J\tilde{\phi}} \equiv \mathrm{e}^{W[J]}\,; \qquad \left(\int_\Lambda \equiv \int_{p \lesssim \Lambda} \mathcal{D}\tilde{\phi}(p)\right)$$

- IR-regularization $\longrightarrow$ insertion of an IR-regulator

$$Z_k[J] := \mathrm{e}^{-\Delta S_k[\frac{\delta}{\delta J}]} Z[J] = \int_\Lambda \mathcal{D}\tilde{\phi}\, \mathrm{e}^{-S[\tilde{\phi}]-\Delta S_k[\tilde{\phi}]+\int J\tilde{\phi}} \equiv \mathrm{e}^{W_k[J]}$$

- properties of the IR-regulator $\Delta S_k[\tilde{\phi}] = \frac{1}{2}\int_p \tilde{\phi} R_k(p^2)\tilde{\phi}$

$$\lim_{p^2/k^2 \to 0} R_k(p^2) \sim k^2$$
$$\lim_{k^2/p^2 \to 0} R_k(p^2) = 0$$
$$\lim_{k \to \Lambda} R_k(p^2) \to \infty$$



Functional Renormalization Group II

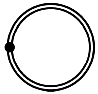
- Legendre-transformation $\rightarrow$ effective average action with $\phi = \frac{\delta W_k[J]}{\delta J}$

$$\Gamma_k[\phi] = -W_k[J] + \int J\phi - \Delta S_k[\phi]$$

- properties of the effective action:
  - minimal at the physical ground-state: $J = \frac{\delta \Gamma_k[\phi]}{\delta \phi} + \phi R_k \rightarrow \langle \phi \rangle_{J=0} \equiv \Phi_k^{\mathrm{gs}}$
  - inverse propagator: $\left(\frac{\delta^2 W_k}{\delta J \delta J}\right)^{-1} = \frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\delta\phi} + R_k$

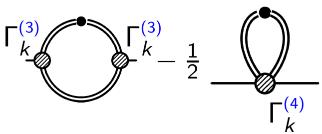
- flow equation for the effective action (Wetterich '93)

$$k\partial_k \Gamma_k \equiv \partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\,\partial_t R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1} = \frac{1}{2}$$



- for example: flow equation for the two-point function

$$\partial_t \Gamma_k^{(2)} = \frac{1}{2}$$



REGULATOR

- fermionic regulator preserves chiral symmetry, i. e. $R_\Psi = R_\Psi(i\partial\!\!\!/)$
- gauge symmetry $\longrightarrow$ modified Ward-Takahashi identities (Reuter & Wetterich '94; Freire et al. '00)

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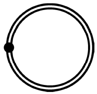

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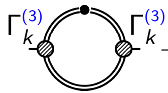
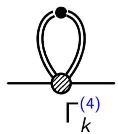


$- \frac{1}{2}$



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running coupling at finite temperature

running coupling - truncation

- ansatz (operator expansion)

$$\Gamma_k = \int_x W_k(\vartheta) + \Gamma^{\rm gf} + \Gamma^{\rm gh} + \bar\psi(i\not{D} + M_{\bar\psi\psi})\psi + \Gamma_k^{\text{q-int}}[\bar\psi,\psi]\,, \qquad \vartheta := \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

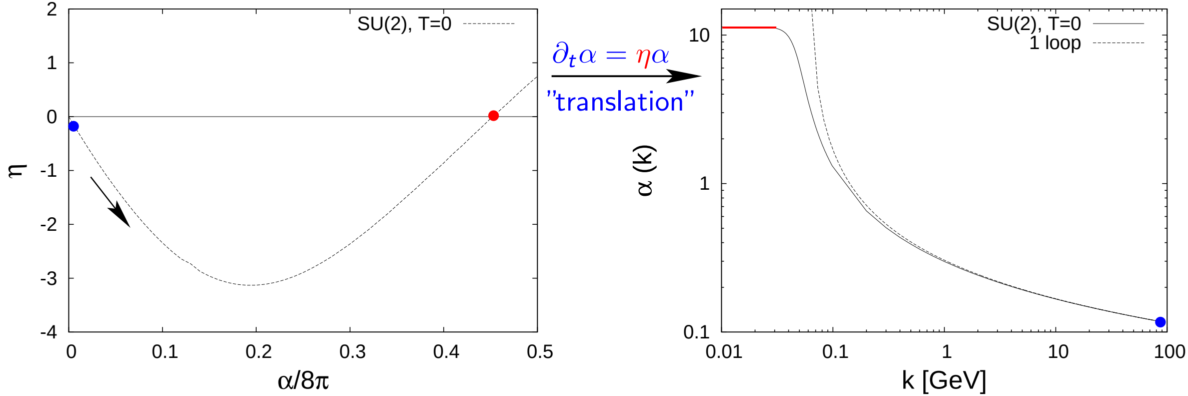
with

$$W_k(\vartheta) = Z_A \vartheta + \frac{w_2}{2!}\vartheta^2 + \frac{w_3}{3!}\vartheta^3 + \frac{w_4}{4!}\vartheta^4 + \dots$$

- running coupling: $g^2 = Z_A^{-1} \bar g^2$ (Abbott '82: background gauge)
- β-function is related to the "anomalous dimension" η via

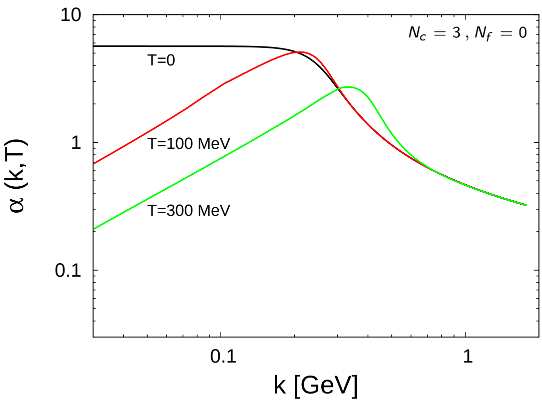
$$\partial_t \alpha \equiv \partial_t \frac{g^2}{4\pi} \equiv \beta(\alpha) = \eta \alpha \qquad \text{with} \qquad \eta = -\partial_t \ln Z_A$$

running coupling - zero temperature results



- take initial condition for $\alpha = \frac{g^2}{4\pi}$ from experiment (Bethke '04)
- IR fixed point ...
  ... for SU(2), $N_f = 0$: $\alpha^* \approx 11.3$ (Gies '02)
  ... for SU(3), $N_f = 0$: $\alpha^* \approx 7.7$ (Gies '02)

running coupling - finite temperature results



- pQCD at high scales $k \gg T$
- $k_{max} \propto T$: "finite-size effect"

$$p_{g,0}^2 = \omega_n^2 = 4n^2\pi^2 T^2 \qquad (n \in \mathbb{Z}) \quad \longrightarrow \quad \omega_0^2 = 0$$

- "3d-running" for $T \gtrsim k$: $g_{4\mathrm{D}}^2 = \frac{k}{T}\tilde{g}_{3\mathrm{D}}^2$

IR-fixed point: $\alpha^* \equiv \alpha_{3\mathrm{D}}^* = \frac{(g_{3\mathrm{D}}^*)^2}{4\pi} \approx 2.7$

running coupling - intermediate conclusions

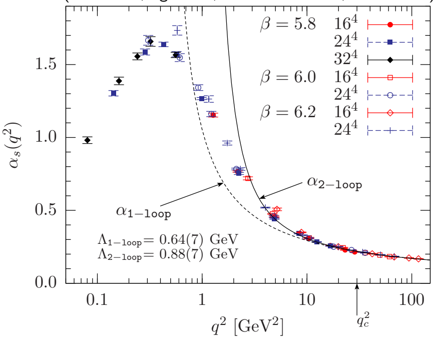
- consistent with a mass gap in QCD
- consistent with Kugo-Ojima confinement scenario
- consistent with results from vertex-expansion in Landau-gauge QCD

  SDE: v. Smekal, Alokofer & Hauck '97; Fischer & Alkofer '02; Maas, Wambach, Alkofer '05

  RG: Pawlowski et al. '04, Fischer & Gies '04

- consistent with lattice calculations (e. g. Sternbeck, Ilgenfritz, Muller-Preussker, Schiller '05)

(Sternbeck, Ilgenfritz, Muller-Preussker, Schiller '05)



Chiral Phase Boundary of QCD

Nambu-Jona-Lasinio (NJL) - model (a short reminder)

- NJL - model:

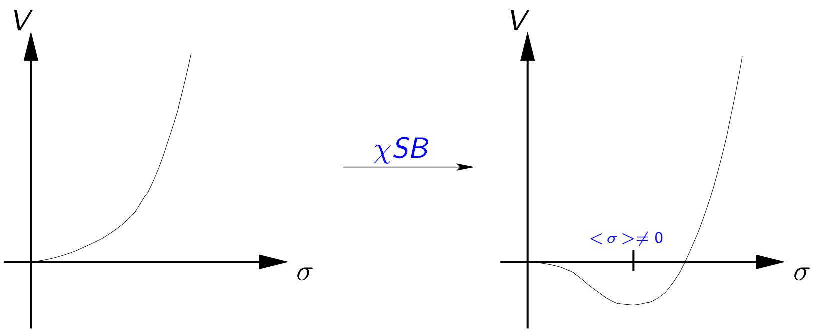
$$S_{\rm NJL} = \int_x \bar\psi(i\partial\!\!\!/)\psi + \lambda\left[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\psi)^2\right]$$

- spontaneous breakdown of chiral symmetry, if $\langle\bar\psi\psi\rangle \neq 0$
- "bosonization" of $S_{\rm NJL}$ yields ($\sigma = -2\lambda\bar\psi\psi$, $\vec\pi = -2\lambda\bar\psi i\gamma_5\psi$)

$$S = \int_x \bar\psi(i\partial\!\!\!/)\psi - \bar\psi(\sigma + i\gamma_5\vec\pi)\psi - \frac{1}{4\lambda}(\sigma^2 + \vec\pi^2)$$

$\Longrightarrow$ λ is inverse proportional to the scalar mass parameter, $m^2 \propto \frac{1}{\lambda}$

- large coupling λ signals onset of chiral symmetry breaking



critical value of α_s - truncation

- definition of the "critical" value of the strong coupling:

  The strong coupling α_s must exceed the "critical" value $\alpha_{\rm cr}$ in order to have χSB

- ansatz ($SU(N_c)$ gauge symmetry + chiral $SU(N_f)_L \times SU(N_f)_R$ symmetry):

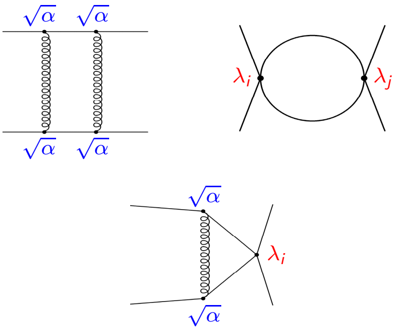
$$\Gamma_k = \Gamma_k^{\rm gauge} + \int_x \bar{\psi} i \partial\!\!\!/ \psi + \frac{1}{2}\Big[Z_- \bar{\lambda}_- ({\rm V}-{\rm A}) + Z_+ \bar{\lambda}_+ ({\rm V}+{\rm A}) + Z_\sigma \bar{\lambda}_\sigma ({\rm S}-{\rm P}) + Z_{\rm VA} \bar{\lambda}_{\rm VA} [2({\rm V}-{\rm A})^{\rm adj} + (1/N_c)({\rm V}-{\rm A})] \Big]$$

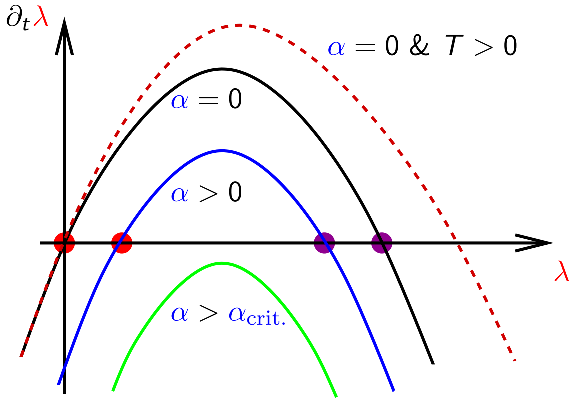
- four-fermion interactions ($\bar{\lambda}_i \to 0$ for $k \to \Lambda$)

$$\begin{aligned}
({\rm V}-{\rm A}) &= (\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5\psi)^2 \\
({\rm V}+{\rm A}) &= (\bar{\psi}\gamma_\mu\psi)^2 - (\bar{\psi}\gamma_\mu\gamma_5\psi)^2 \\
({\rm S}-{\rm P}) &= (\bar{\psi}_i^a\psi_i^b)^2 - (\bar{\psi}_i^a\gamma_5\psi_i^b)^2 \\
({\rm V}-{\rm A})^{\rm adj} &= (\bar{\psi}\gamma_\mu T^z\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5 T^z\psi)^2
\end{aligned}$$

- ansatz for the four-fermion vertices: $\bar{\lambda}_i(p_1, p_2, p_3, p_4) \to \bar{\lambda}_i(p_i = 0)$; $Z_\psi(p) \to Z_\psi(p=0)$

critical value of α_s - flow equations



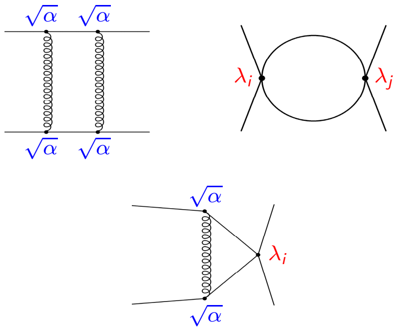


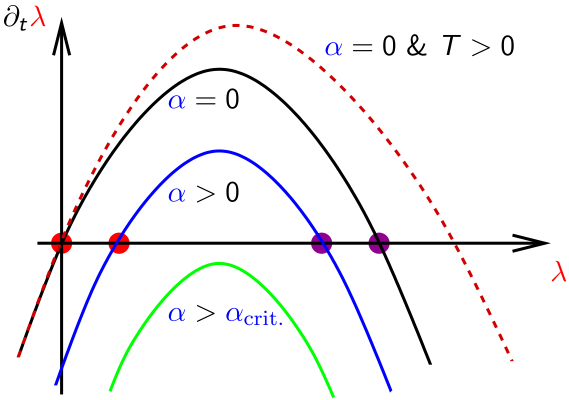
- an example: flow equation for λ_- (zero temperature: H. Gies, J. Jaeckel, C. Wetterich '03)

$$
\begin{aligned}
\partial_t \lambda_- &= 2\lambda_- - \frac{1}{8\pi^2} l_{1,1}^{(\mathrm{FB}),4}\left(\tfrac{T}{k}\right)\left[\frac{3}{N_c} g^2 \lambda_- - 3 g^2 \lambda_{\mathrm{VA}}\right] - \frac{1}{256\pi^2} l_{1,2}^{(\mathrm{FB}),4}\left(\tfrac{T}{k}\right)\left[\frac{12 + 9N_c^2}{N_c^2} g^4\right] \\
&\quad - \frac{1}{4\pi^2} l_1^{(\mathrm{F}),4}\left(\tfrac{T}{k}\right)\left\{2\lambda_{\mathrm{VA}}^2 - N_f N_c(\lambda_-^2 + \lambda_+^2) + \lambda_-^2 - 2(N_c + N_f)\lambda_- \lambda_{\mathrm{VA}} + N_f \lambda_+ \lambda_\sigma\right\}
\end{aligned}
$$

- $\alpha = \frac{g^2}{4\pi} > \alpha_{\rm cr} \longrightarrow$ no fixed points $\longrightarrow \chi SB$
- $l_{\cdots}^{(\ldots),4}(T{=}0) > l_{\cdots}^{(\ldots),4}(\tfrac{T}{k}) \quad \Longrightarrow \quad \alpha_{\rm cr}(\tfrac{T}{k}) > \alpha_{\rm cr}(T{=}0) \overset{N_c=N_f=3}{\approx} 0.85$

critical value of α_s - flow equations





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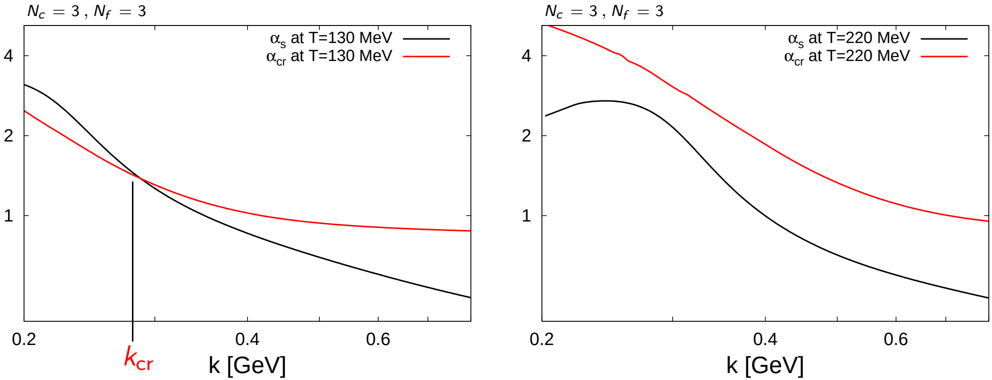
$$- \frac{1}{4\pi^2} l_1^{(\mathrm{F}),4}(\tfrac{T}{k}) \left\{2\lambda_{\mathrm{VA}}^2 - N_f N_c(\lambda_-^2 + \lambda_+^2) + \lambda_-^2 - 2(N_c+N_f)\lambda_-\lambda_{\mathrm{VA}} + N_f \lambda_+ \lambda_\sigma\right\}$$

- $\alpha = \frac{g^2}{4\pi} > \alpha_{\mathrm{cr}} \longrightarrow$ no fixed points $\longrightarrow \chi SB$
- $l_{\dots}^{(\dots),4}(T{=}0) > l_{\dots}^{(\dots),4}(\tfrac{T}{k}) \quad \Longrightarrow \quad \alpha_{\mathrm{cr}}(\tfrac{T}{k}) > \alpha_{\mathrm{cr}}(T{=}0) \overset{N_c=N_f=3}{\approx} 0.85$

chiral phase transition temperature

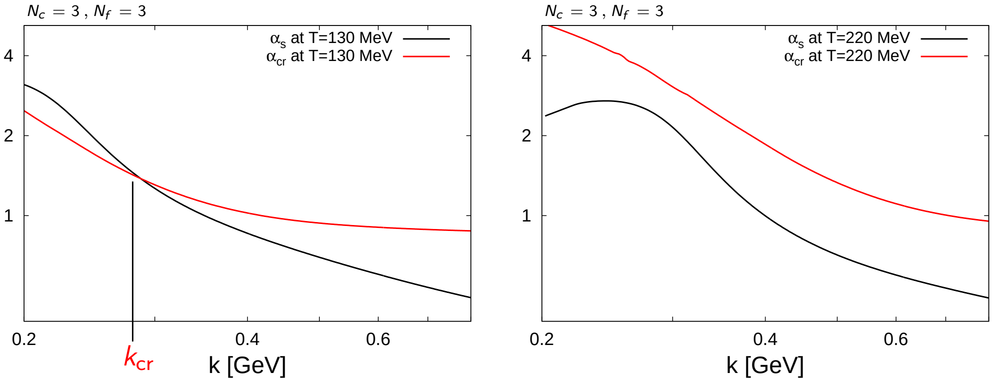


- initial value for α_s: $\alpha_s(m_\tau) \approx 0.322$
- intersection point of α_s and α_{cr} indicates onset of χSB (left panel)
- results for T_{cr} compared to lattice calculations:

N_f	T_{cr}	T_{cr} (lattice) (Karsch et al. 03)
2	186 MeV	175 ± 8 MeV
3	161 MeV	155 ± 8 MeV

- $\Delta(N_f) = T_{cr}(N_f = 2) - T_{cr}(N_f = 3) \approx 25\,\mathrm{MeV}$

chiral phase transition temperature



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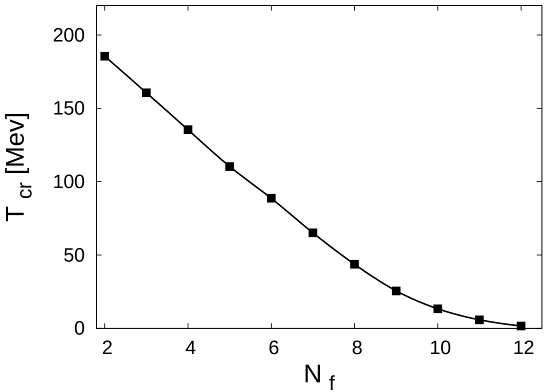
many-flavor QCD

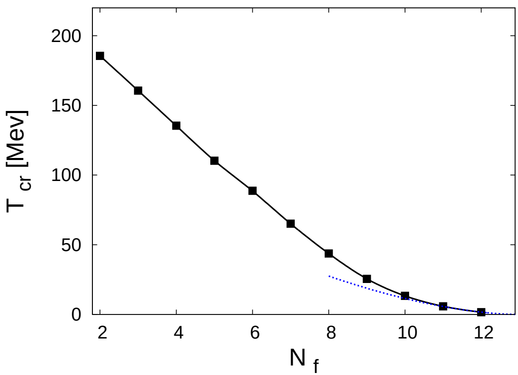


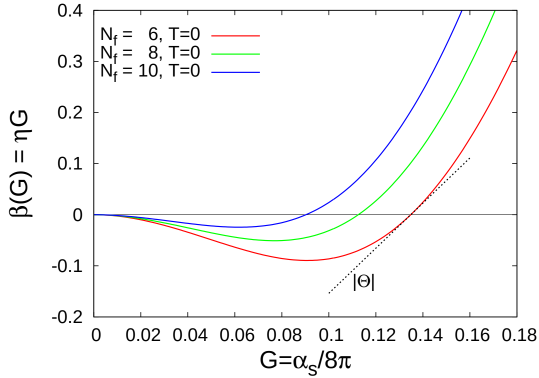
- approximately linear decrease of $T_{\rm cr}$ with increasing number of (massless) quark flavors
- critical number of (massless) quark flavors ($N_f > N_f^{\rm cr}$: no χSB)

$$N_f^{\rm cr} \approx \begin{cases} 8 & (\text{Brown et. al. '92}) \\ 12 & (\text{Kogut et. al. '92}) \\ 5 & (\text{Schafer, Shuryak '95}) \\ 12 & (\text{Appelquist et. al. '96}) \\ 10 & (\text{Gies \& Jaeckel '05}) \\ 12 & (\text{J.B. \& Gies '05}) \end{cases}$$

$$N_f^{\rm cr} < N_f < N_f^{\rm af} = 16.5 \longrightarrow \text{asymptotic freedom but no } \chi\text{SB}$$

scaling of $T_{\rm cr}$ at large N_f





- "expansion" around $T = 0$ and $N_f \approx N_f^{\rm cr}$ yields (J.B. & H. Gies '06)

$$T_{\rm cr} \approx k_0 |N_f - N_f^{\rm cr}|^{\frac{1}{|\Theta|}}$$

- shape of phase boundary is related to "critical" exponent Θ: $|\Theta| \approx 0.6$ for $N_f = N_f^{cr}$

$\Longrightarrow$ relation between two universal quantities

- testable prediction $\rightarrow$ can be checked by other approaches, e.g. lattice QCD

Conclusions and outlook

CONCLUSIONS

- fixed point of the running coupling at finite temperature
  $\longrightarrow$ consistent with DSE- and lattice-results
- $\Delta T_\chi = T_\chi(N_f) - T_\chi(N_f + 1) \approx 25\,\mathrm{MeV}$
  $\longrightarrow$ good agreement with lattice-results
- critical number of quark flavours for $SU(3)$: $N_f^{\mathrm{cr}} \approx 12$
- shape of the phase boundary near N_f^{cr} is determined by the underlying IR fixed point scenario (testable prediction!)

OUTLOOK

- extension to finite quark chemical potential
- "penetrate" phase boundary: apply rebosonization technique
- extend study of confinement-/deconfinement-phase transition with Polyakov-loop (J.B., H. Gies, H.-J. Pirner ´05)