A Variational Approach to Searching for Diquarks in the Nucleon Wavefunction

Jonathan Bratt (MIT)

work done under the supervision of John Negele

outline

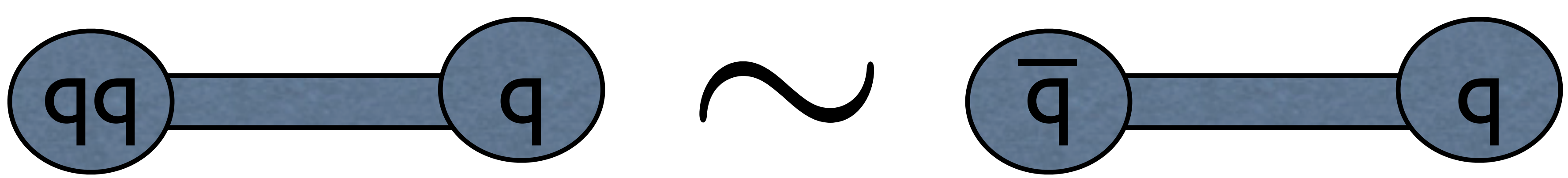
- physical motivation
- method
- results
- conclusions

Physical Picture

- diquarks (see R. Jaffe's *Exotica*, hep-ph/0409065)
- "good" (scalar) vs. "bad" (vector) diquarks

$$(\mathrm{u}\ C\gamma_5\ \mathrm{d}) \qquad\qquad (\mathrm{u}\ C\gamma_\mu\ \mathrm{d})$$

- Jaffe-Wilczek "bone" model



- c.f. Regge trajectories (Wilczek, hep-ph/0409168)

The Method

For a trial source

$$| \, \mathrm{J} \, \rangle = \mathrm{J} \, | \, \Omega \, \rangle$$

we calculate the (momentum-projected) two-point function

$$\langle \, \bar{\mathrm{J}}(t) \, | \, \mathrm{J}(0) \, \rangle \quad = \quad \sum_n \left| \, \langle \, \bar{\mathrm{J}}(0) \, | n \rangle \, \right|^2 \, e^{-E_n t}$$

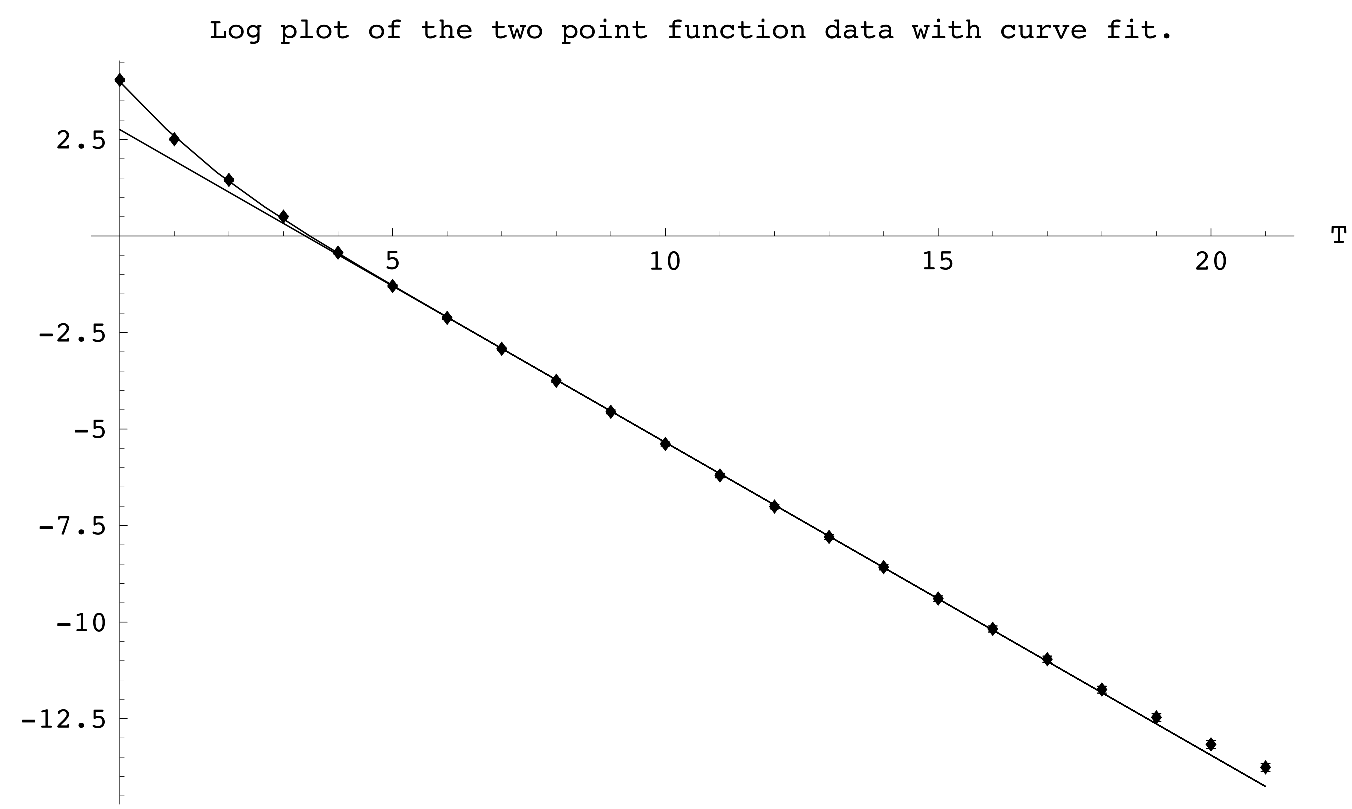
For large *t*, the ground state dominates the sum...

... so we fit the two-point function to a sum of exponentials:

$$C(t) \equiv \langle\, \bar{\mathrm{J}}(t) \,|\, \mathrm{J}(0) \,\rangle \quad \doteq \quad A_0\, e^{-m_0 t} + A_1\, e^{-m_1 t}$$

and estimate the (normalized) ground-state overlap for our trial source:

$$\left|\langle\, \bar{\mathrm{J}}_{\mathrm{N}}(0) \,|\, 0 \,\rangle\right|^2 = \frac{A_0}{C(0)}$$



computational side note:
propagator correction at t = t_{source}

The "naive" Wilson propagator is incorrect at t_{source}! This can be seen by examining the transfer matrix formalism:

$$\langle \hat{O}_1 \hat{O}_2 \rangle = \frac{\int DU D\bar{q} Dq \, e^{-S(U,\bar{q},q)} O_1 O_2}{\int DU D\bar{q} Dq \, e^{-S(U,\bar{q},q)}} = \frac{\langle 0 | N\{\hat{T} \dots \hat{T} \hat{O}_1 \hat{T} \dots \hat{T} \hat{O}_2 \hat{T} \dots \hat{T}\} | 0 \rangle}{\langle 0 | \hat{T} \dots \hat{T} | 0 \rangle}$$

We want to find T (and appropriate coordinate transform) such that we recover the action exponential in the path integral.
Writing out the Wilson action, one finds that it is necessary to define the following normal ordering convention for the fermion field operators:

$$N\{q^{\uparrow}\bar{q}^{\uparrow}\} = q^{\uparrow}\bar{q}^{\uparrow}$$
$$N\{q^{\downarrow}\bar{q}^{\downarrow}\} = \bar{q}^{\downarrow}q^{\downarrow}$$

... where upper (lower) refers to the upper (lower) half spinor in the Dirac basis.
The operators anticommute ***except*** when on the same timeslice, in which case we pick up a correction term from the anticommutator:

$$P_{correct}(x,y) \quad = \quad P_{naive}(x,y) - \frac{(1-\gamma_0)}{2} B^{-1}(\vec{x},\vec{y}) \, \delta_{x_0,y_0}$$

where

$$B(\vec{x},\vec{y}) \quad = \quad \mathbb{I} - \kappa \sum_{j=1}^{3} \left[U_j^{\dagger}(y) \delta_{x-y-\hat{\jmath}} + U_j(x) \delta_{x-y+\hat{\jmath}} \right]$$

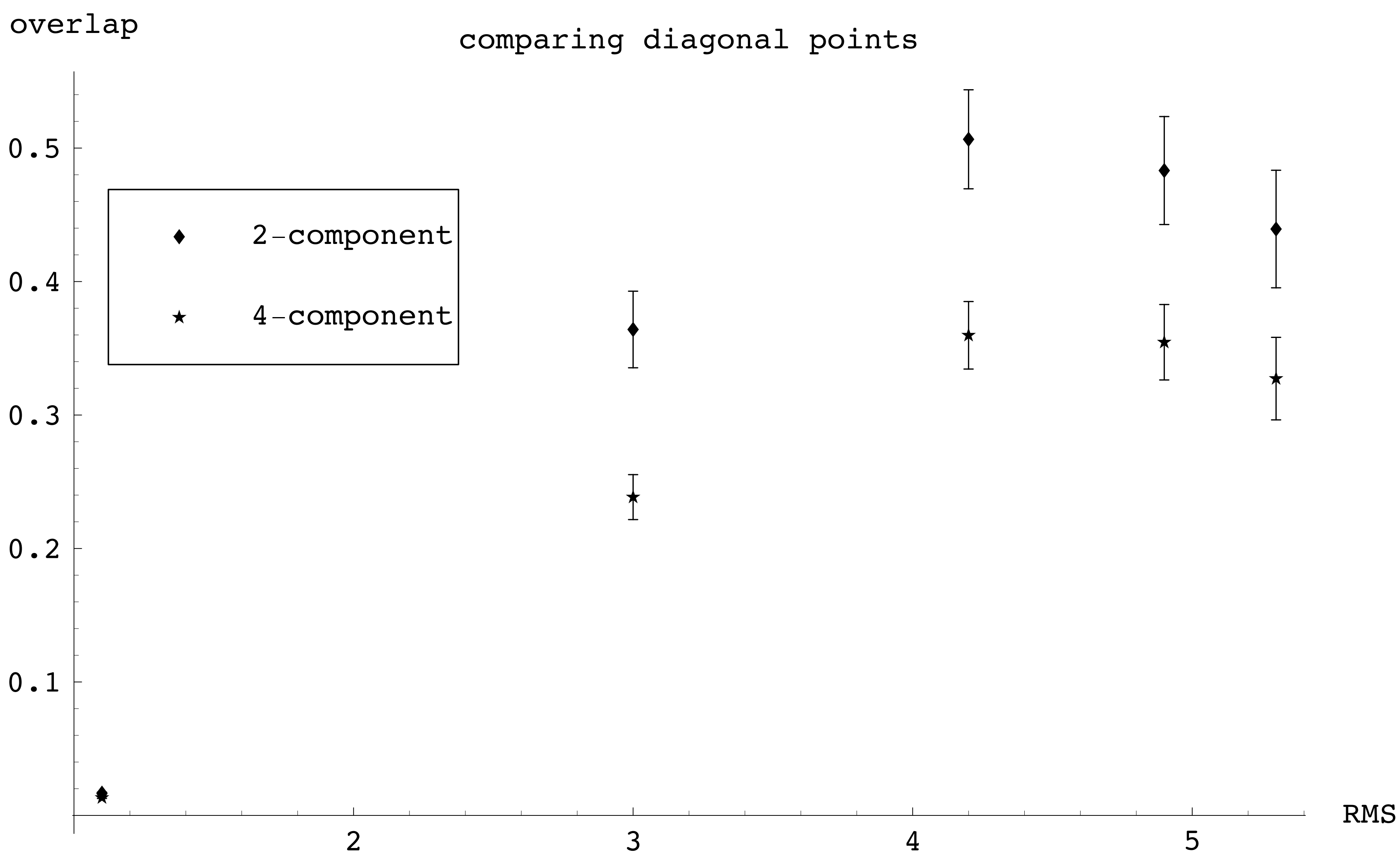
What I did.

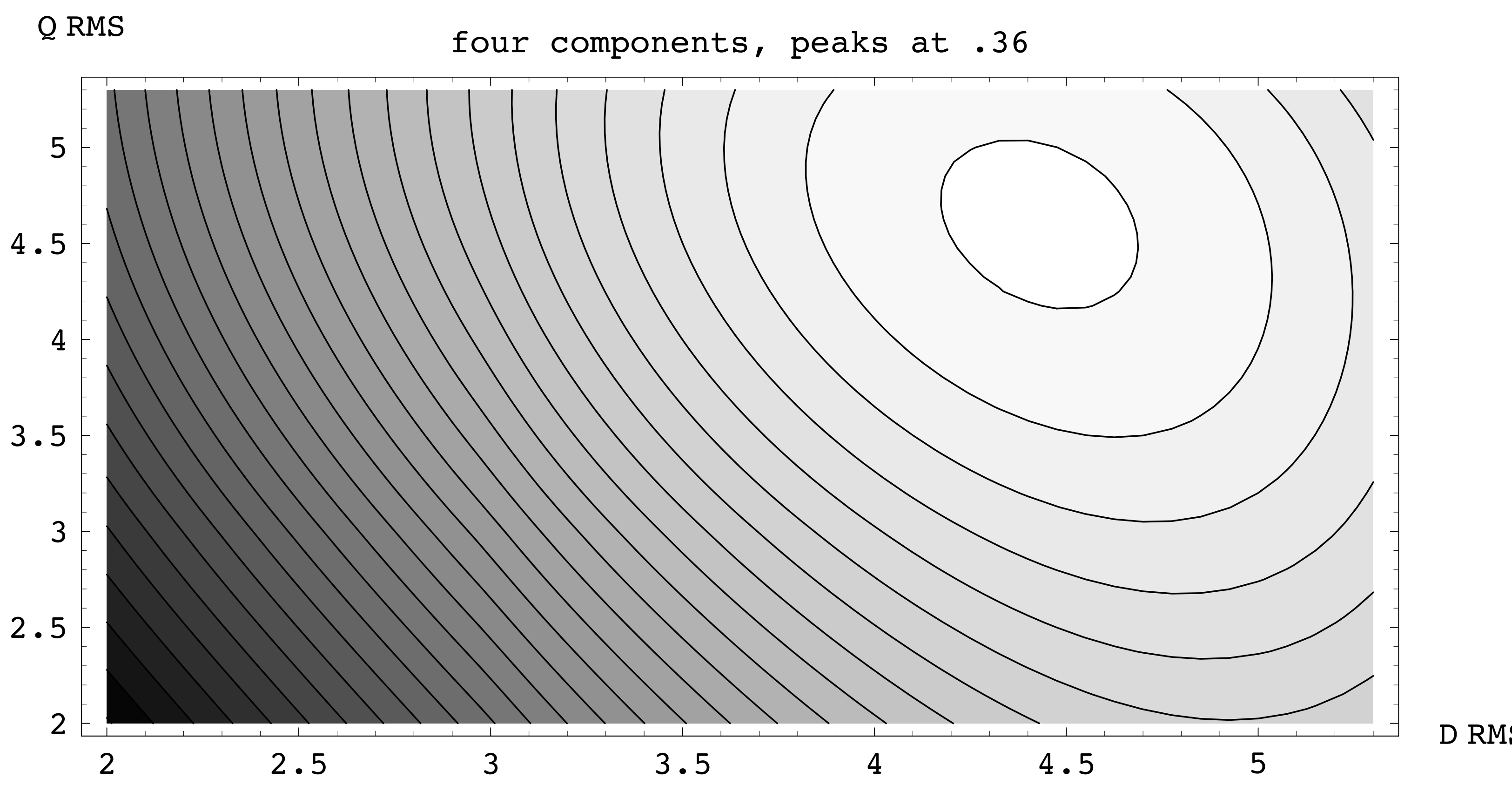
- Quenched Wilson, 16^3x32 lattices
- $\beta = 6.0, \ \kappa = 0.1530$
- source interpolating field motivated by diquark model of nucleon:

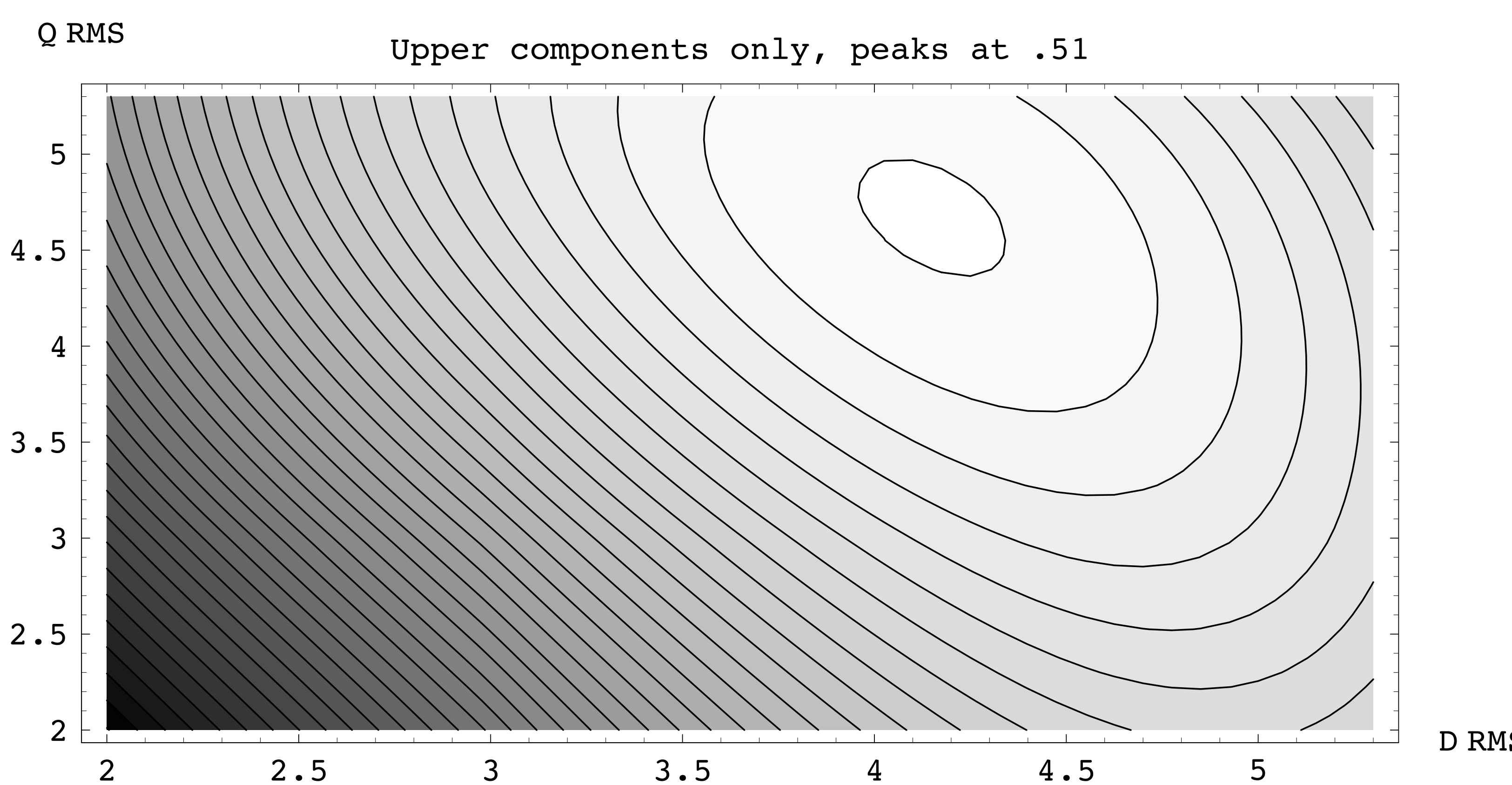
$$\mathrm{J} = (\ \mathrm{U}_{s1} \ \mathrm{C}\gamma_5 \ \mathrm{D}_{s1} \ ) \ \mathrm{U}_{s2}$$

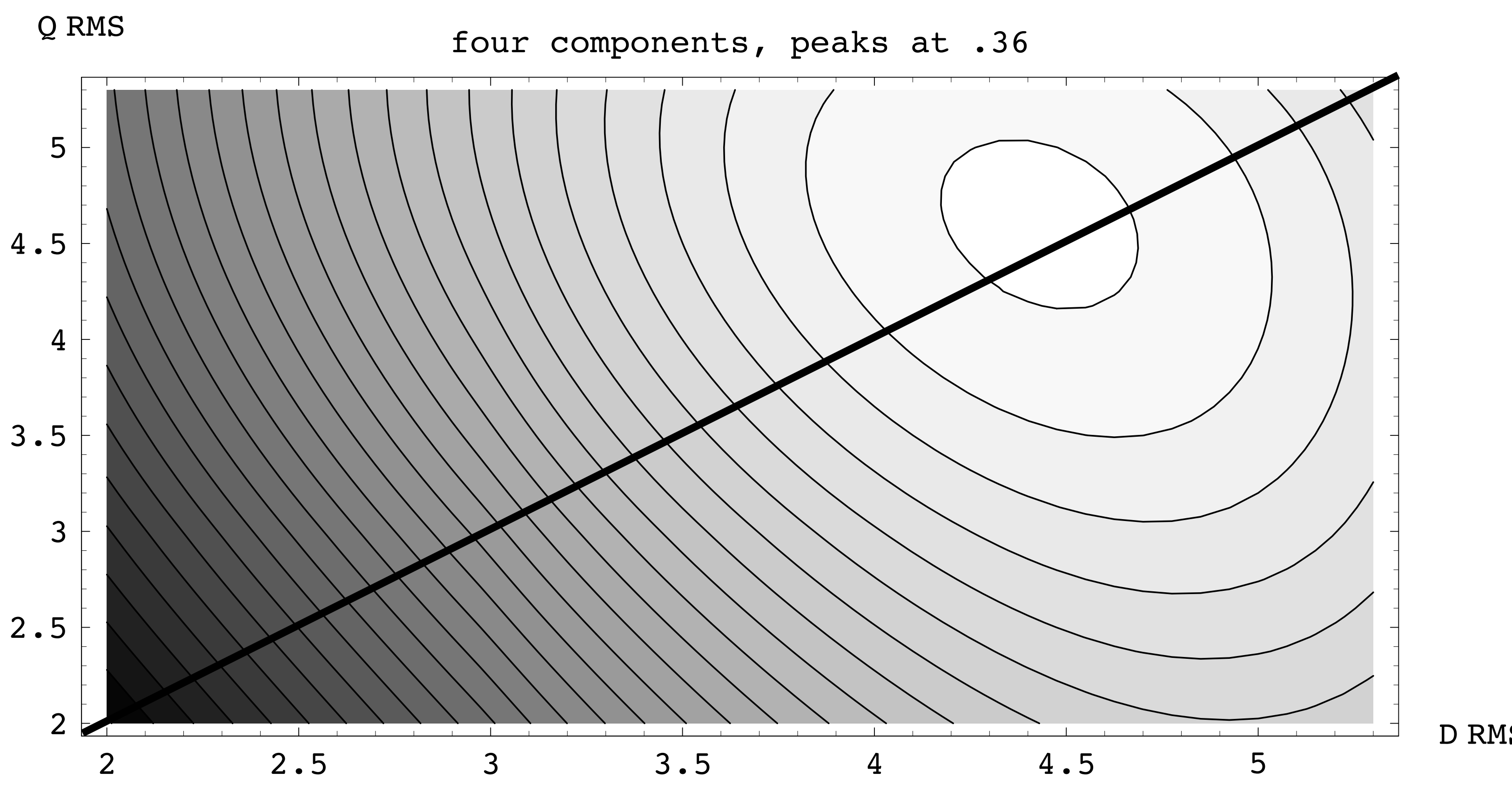
- “diquark” and “lone quark” can have different RMS radii (gauge invariant smearing)
- want to find the maximum overlap in this two-dimensional parameter space

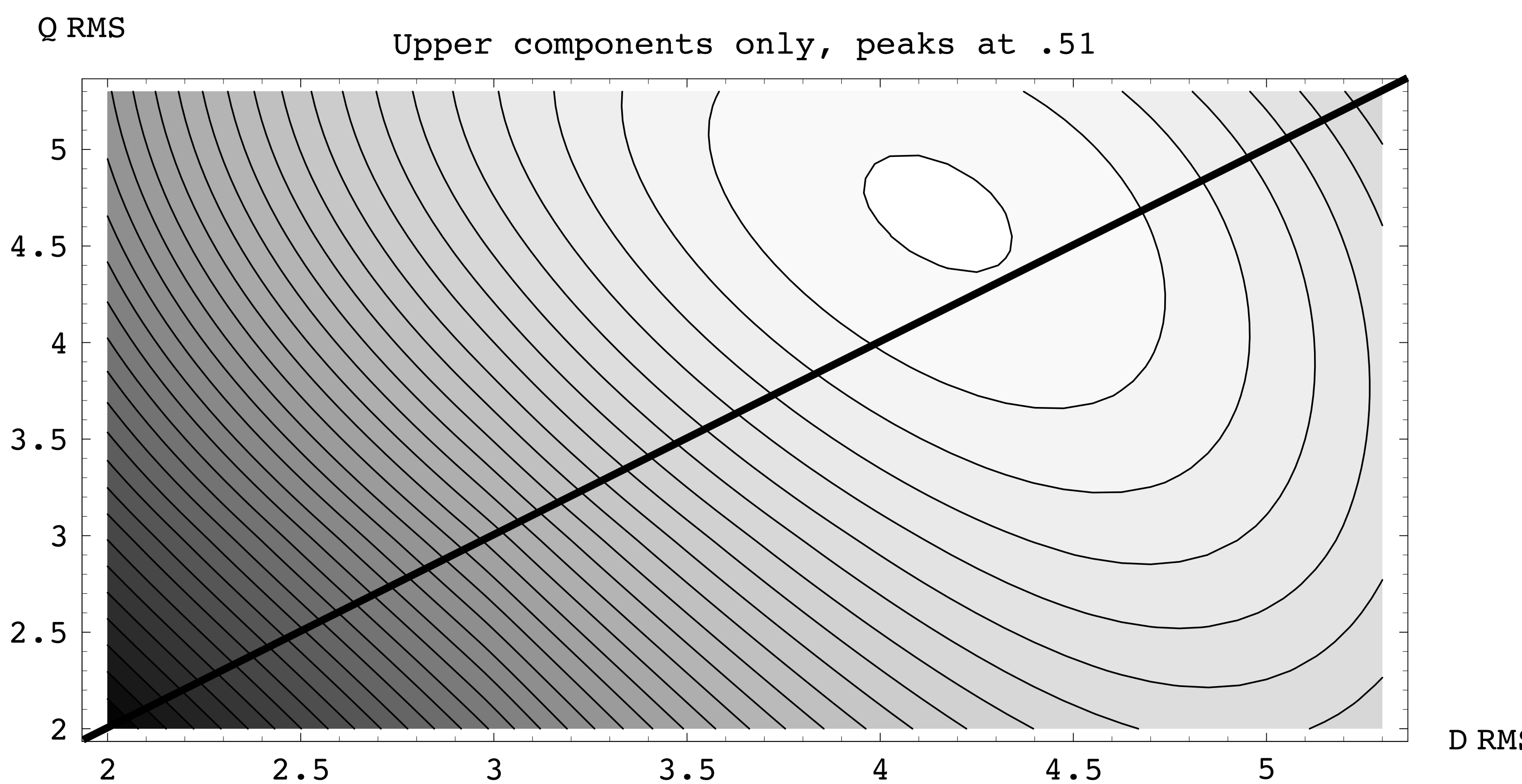
- two-component versus four-component spinors




Q RMS
four components, peaks at .36
5
4.5
4
3.5
3
2.5
2
2
2.5
3
3.5
4
4.5
5
D RMS


Q RMS
Upper components only, peaks at .51
5
4.5
4
3.5
3
2.5
2
2
2.5
3
3.5
4
4.5
5
D RMS


Q RMS
four components, peaks at .36
5
4.5
4
3.5
3
2.5
2
2
2.5
3
3.5
4
4.5
5
D RMS


Q RMS
Upper components only, peaks at .51
5
4.5
4
3.5
3
2.5
2
2
2.5
3
3.5
4
4.5
5
D RMS

Adding APE Smearing

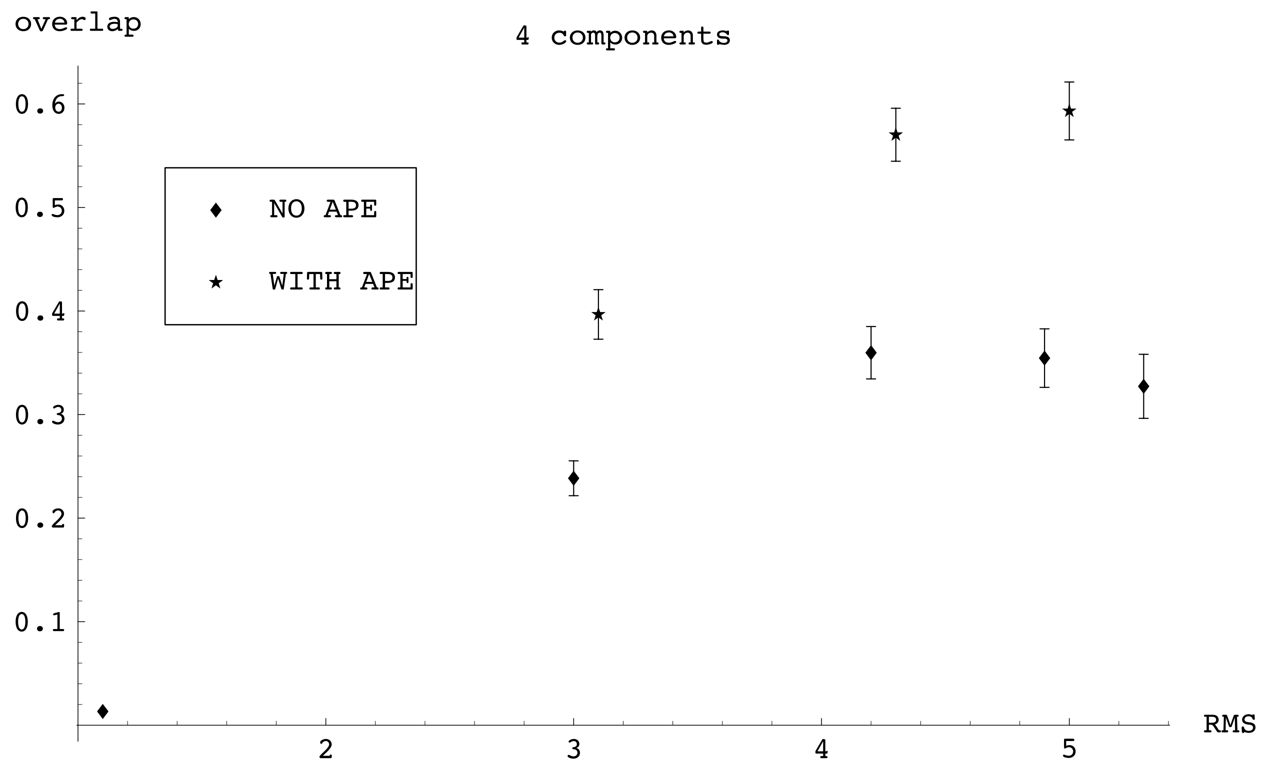
- smear gauge links used in constructing the source:

$$\Big(\quad \Big| \rightarrow (1-c) \Big| \quad + c \sum \Big[\quad \Big)^N$$

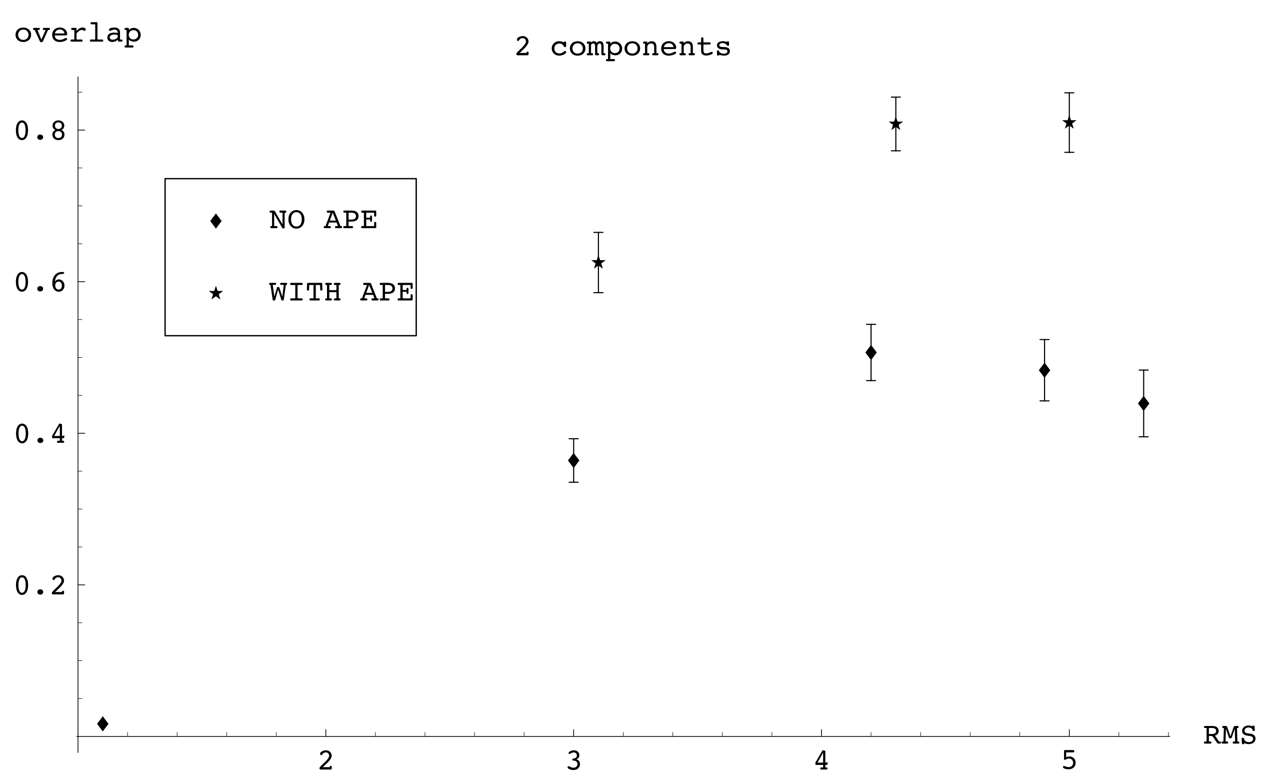
- parameters used:

c = 0.35,   N = 25

- improves overlaps!



overlap
4 components
0.6
0.5
0.4
0.3
0.2
0.1
NO APE
WITH APE
RMS
2
3
4
5



overlap
2 components
0.8
0.6
0.4
0.2
NO APE
WITH APE
RMS
2
3
4
5

Try displacing quark....

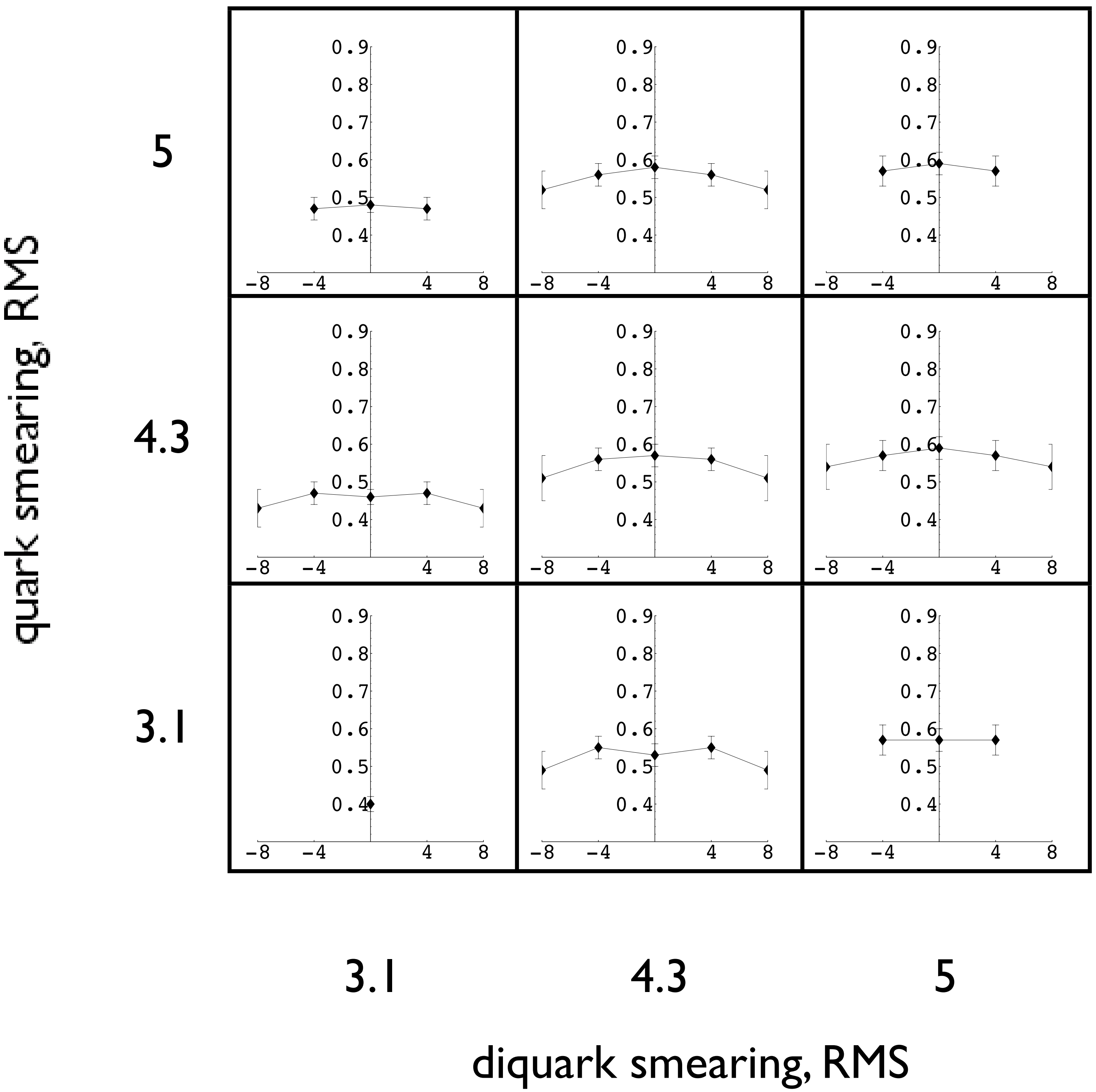
- write down a source with quark, diquark separated:

$$\mathrm{J} = (\ \mathrm{U}_{s1}(x)\ \mathrm{C}\gamma_5\ \mathrm{D}_{s1}(x)\ )\ \mathrm{U}_{s2}(y)$$

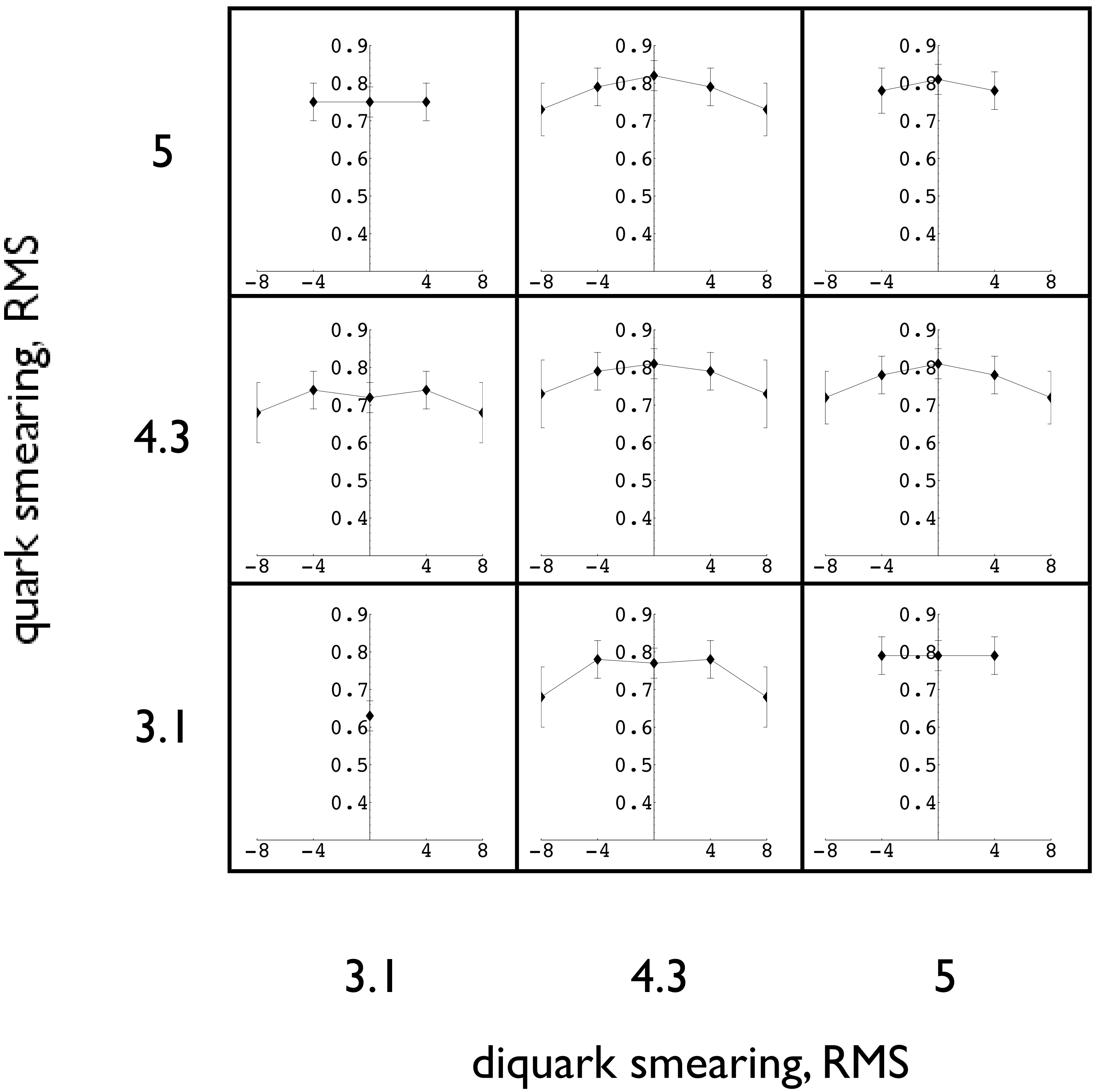
- use symmetric sink:

$$C(t) = \sum_{\vec{r}} \sum_{\hat{j}} \langle\, (\bar{u}(\vec{r},t) C\gamma_5 \bar{d}(\vec{r},t))\bar{u}(\vec{r}+\ell\hat{j},t)\ (u(0,0)C\gamma_5 d(0,0))u(\ell\hat{x},0)\,\rangle$$

four component spinors



upper components only



Summary

- what can we say about the optimal nucleon source?
  - 2- vs. 4-component
  - APE vs. non-APE
  - displaced quark
- diquarks? (caveat)