The Triton and Three-Nucleon Force in Nuclear Lattice Simulations

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INT 06-1 Workshop

Outline

Introduction

- Lattice field theory allows nonperturbative treatment of QCD
- Modern lattice QCD simulations work with lattices not much larger than size of single nucleon
- In forseeable future: lattice QCD simulations not suited to obtain direct results in few- and many-body nuclear physics (beyond lightest nuclei)

Introduction

- Lattice field theory allows nonperturbative treatment of QCD
- Modern lattice QCD simulations work with lattices not much larger than size of single nucleon
- In forseeable future: lattice QCD simulations not suited to obtain direct results in few- and many-body nuclear physics (beyond lightest nuclei)
- Employ effective field theory on the lattice:
  pions and nucleons are point particles on lattice sites,
  external (axial-) vector fields live on links

Chiral effective field theory

- QCD exhibits (approximate) SU(2)$_L$ $\times$ SU(2)$_R$ chiral symmetry, broken down spontaneously to SU(2)$_V$
  $\Rightarrow$ 3 Goldstone bosons (π^+, π^-, π^0) with small masses

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- At low energies: chiral perturbation theory, the effective field theory of QCD, is successful in describing interactions among mesons and baryons.

  Green's functions are expanded in Goldstone boson masses and small momenta $\Rightarrow$ chiral counting scheme

  ChPT is model independent, effective field theory of QCD

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- Pions can be summarized in matrix $U(x) \in \mathrm{SU}(2)$

$$U(x) = \exp\left(\frac{i}{f_\pi}\,\pi(x)\right)$$

$f_\pi \simeq 93$ MeV,
pion decay constant

$$\mathcal{L} = \mathcal{L}(U, \partial U, \partial^2 U, \ldots, \mathcal{M}) \,, \qquad \mathcal{M} = \mathrm{diag}(m_u, m_d)$$

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- Nucleons (p, n) can be included

Lattice χEFT

- Lattice ChPT with mesons has been investigated in
  - Myint & Rebbi (1994); Levi, Lubicz & Rebbi (1997)
  - Shushpanov & Smilga (1999)
- Extension to baryonic sector
  - Lewis & Ouimet (2001)
  - Borasoy, Lewis & Ouimet (2002) (2004)
- Studies on a finite lattice
  - Borasoy & Lewis (2004)
- Multi-nucleon effective field theory
  - Chandrasekharan, Pepe, Steffen & Mazur (2003)
  - Lee, Borasoy & Schäfer (2004), Lee & Schäfer (2005),
  - Borasoy, Krebs, Lee & Meißner (2005)

Lattice χEFT

Euclidean SU(2) chiral Lagrangian in mesonic sector

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots$$

$$\mathcal{L}_2 = \frac{f_\pi^2}{4} \left\langle \nabla_\mu^{(+)} U^\dagger \nabla_\mu^{(+)} U \right\rangle - \frac{B f_\pi^2}{2} \left\langle \mathcal{M}(U + U^\dagger) \right\rangle$$

$$\mathcal{L}_4 = -\frac{1}{4} l_1 \left\langle \nabla_\mu^{(\pm)} U^\dagger \nabla_\mu^{(\pm)} U \right\rangle^2 - \frac{1}{4} l_2 \left\langle \nabla_\mu^{(\pm)} U^\dagger \nabla_\nu^{(\pm)} U \right\rangle \left\langle \nabla_\mu^{(\pm)} U^\dagger \nabla_\nu^{(\pm)} U \right\rangle + \ldots$$

with pion fields

$$U(x) = \exp\left(\frac{i\tau^a \pi^a(x)}{f_\pi}\right)$$

Lattice χEFT

- Use of *nearest-neighbor* covariant derivative in leading order Lagrangian avoids unphysical states

$$\nabla_\mu^{(+)} U(x) = \frac{1}{a}\left[R_\mu(x) U(x+a_\mu) L_\mu^\dagger(x) - U(x)\right]$$

with external fields

$$L_\mu(x) = \exp\left[-ia\ell_\mu(x)\right] = \exp\left[-ia(V_\mu(x) - A_\mu(x))\right]$$
$$R_\mu(x) = \exp\left[-iar_\mu(x)\right] = \exp\left[-ia(V_\mu(x) + A_\mu(x))\right]$$

Triton and three-nucleon force

- Work in SU(4)-symmetric (Wigner) limit of pionless effective field theory: isospin- and spin-symmetric

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m} \right) \psi - \frac{C_0}{2} \left(\psi^\dagger \psi \right)^2 - \frac{D_0}{6} \left(\psi^\dagger \psi \right)^3$$

- ψ includes four nucleon states with mass m

$$\psi = \begin{bmatrix} p_\uparrow \\ p_\downarrow \\ n_\uparrow \\ n_\downarrow \end{bmatrix}$$

- Renormalized two-body interaction C_0 is directly related to two-body scattering length a_2

$$C_0 = \frac{4\pi a_2}{m}$$

Continuum regularization

- Two-body binding energy B_2

$$B_2 = \frac{1}{ma_2^2}$$

- SU(4)-symmetric limit: ${}^1S_0$ and ${}^3S_1$ two-body sectors are degenerate. We choose SU(4)-symmetric two-nucleon binding energy $B_2 = 1$ MeV
- Three-body interaction D_0 not fixed

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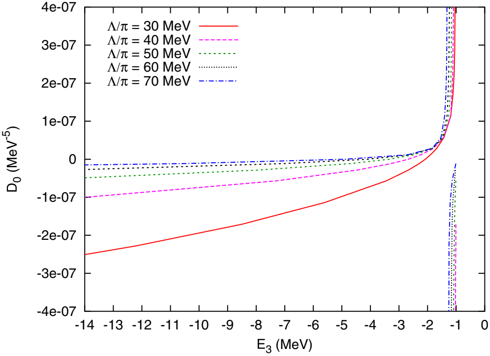
- Solve homogeneous S-wave bound state equation for dimer-nucleon system with cutoff momentum Λ

  $\Rightarrow$ strong dependence of D_0 on Λ

Continuum regularization

Three-body interaction D_0 as function of triton binding energy E_3



- Nontrivial dependence of D_0 on Λ is nonperturbative effect, no finite set of diagrams reproduces ultraviolet divergence (Bedaque, Hammer & van Kolck ('99))

Triton and three-nucleon force

- D_0 scales roughly as Λ^{-2} for $E_3 \ll -1$ MeV
- Pole in $D_0(E_3)$ close to continuum threshold for dimer plus nucleon
- Pole location decreases for larger Λ
- As Λ is increased, new deeper bound states appear which are outside range of validity of EFT
- Triton is identified with shallowest bound state

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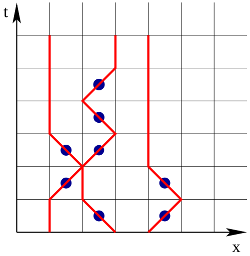
- Singular behavior of three-body system could lead to different cutoff dependence of D_0 in different regularization scheme

Triton and three-nucleon force in nuclear lattice simulations

- Path integral is evaluated by computing Monte Carlo sample of world lines



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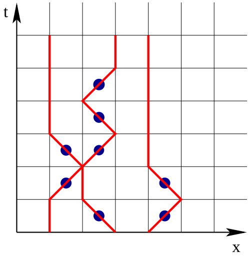
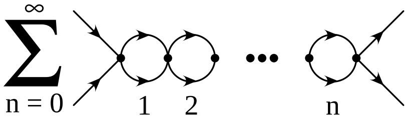


- Strength of two-body coefficient C_0 is matched to deuteron binding energy B_2 (we take 1 MeV)
- Three-body interaction D_0 not fixed

Two-body interaction

- Two-body coupling C_0 determined by summing nucleon-nucleon bubble diagrams on the lattice



- Tune C_0 to reproduce deuteron binding energy B_2

a^{-1}(MeV)	a_t^{-1}(MeV)	C_0(MeV^{-2})
40	16	-1.83×10^{-4}
50	25	-1.39×10^{-4}
60	36	-1.13×10^{-4}
70	49	-0.94×10^{-4}

Contributions to path integral

- Hopping parameter: $h = \frac{\alpha_t}{2m}$, $\alpha_t = \frac{a_t}{a}$
- *Single fermion worldline*:

  hop to a neighboring lattice site during time step: h
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- Factor $e^{-C_0\alpha_t}$ for each pairwise interaction
- Contribution from three-body interaction for three different fermions: $e^{-D_0\alpha_t}(1-6h)^3$

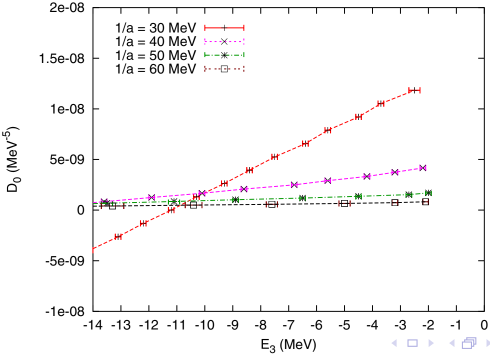
Triton and three-nucleon force in nuclear lattice simulations

- Compute lattice approximation for

$$\langle 0,0,0|\exp\left[-\beta H\right]|0,0,0\rangle$$

$|0,0,0\rangle$: state with three nucleons, each of different kind and zero momentum

$\Rightarrow$ measurement of triton ground state energy E_3



Triton and three-nucleon force in nuclear lattice simulations

- Coupling D_0 scales as $\sim a^2$ for fixed E_3 (modulo shift)
- Energy region $E_3 \sim -1$ MeV has not been computed since there are many dimer plus nucleon continuum states near energy threshold $\hookrightarrow$ difficult to extract E_3

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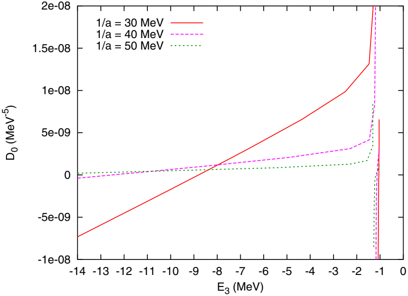
- Results have been confirmed using Hamiltonian lattice with Lanczos method

Hamiltonian lattice

- Compare with Hamiltonian lattice using Lanczos method



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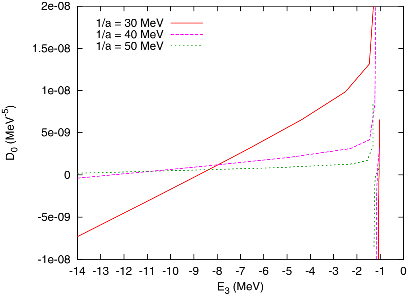

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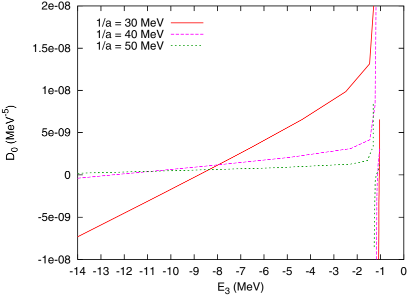

- D_0 scales roughly as a^2 (modulo shift)
- Hamiltonian approach restricted to small volumes and small # of particles
- Euclidean lattice method can be generalized to larger nucleon numbers and more complicated forces amongst nucleons

Triton and three-nucleon force in nuclear lattice simulations

To be done:

- More nucleons
- Inclusion of four-body force
- Effects due to breaking of Wigner symmetry
- Higher chiral orders in effective Lagrangian
- Inclusion of pions

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- Effects due to breaking of Wigner symmetry
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- Important step towards future many-body simulations with arbitrary number of nucleons
- Neutron matter with pions (Lee, Borasoy & Schäfer ('04))

  $\hookrightarrow$ results for hot neutron matter, $T \approx 20 - 40$ MeV, and densities twice below nuclear matter density

Lattice action positivity

- Cold dilute neutron matter: pionless EFT should provide adequate description of low-energy physics
- Implementation on the lattice with a positive semi-definite action via Hubbard-Stratonovich transformation
  $\hookrightarrow$ efficient Monte Carlo simulations

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- Implementation on the lattice with a positive semi-definite action via Hubbard-Stratonovich transformation
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- Cold dilute nuclear matter with small proton fraction: 3-nucleon force is required for consistent renormalization
- 3-nucleon force could spoil positivity of lattice action
- For triton binding energies of about 8 MeV and assuming four-nucleon force to be zero (or small) condition for lattice action positivity [Chen, Lee & Schäfer ('04)] is satisfied:
  no sign problem

Concluding remark:

Lattice simulations of chiral effective field theory are a promising tool to investigate few-body nuclear physics

Nuclear matter

A central goal of nuclear physics:

understand properties of strongly interacting matter at finite density and temperature

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Experiments

- new data generated by RHIC, Brookhaven
- upcoming heavy ion facility planned at GSI, Darmstadt
- high energy frontier: ALICE @ LHC

Nuclear matter

Astrophysical interest

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Lattice QCD

- connects QCD to observed phenomenology
- finite temperature ok.
- finite density, i.e. chemical potential

  $\Rightarrow$ determinant of quark Dirac matrix becomes complex

  $\Rightarrow$ highly oscillatory

Simulations of nuclear matter

- Study of neutron matter with nuclear lattice simulations

  first step (w/ pions): Lee, Borasoy & Schäfer (2004)
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- Better situation than in finite density lattice QCD

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Why?

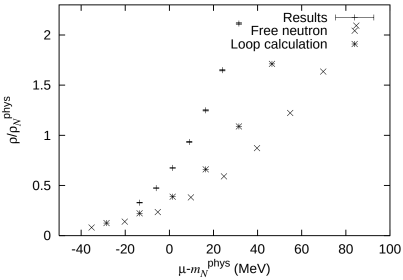
- nucleons and pions give a simpler representation of the essential physics in the hadronic phase
- nucleons are much heavier than up and down quarks

Simulations of nuclear matter

- Results of numerical simulation at weak coupling agree with results from perturbation theory for neutron & pion self-energies, shift in average energy
- Neutron-neutron contact interaction coupling C determined by S-wave scattering phase shifts on lattice at zero temperature and density

Results

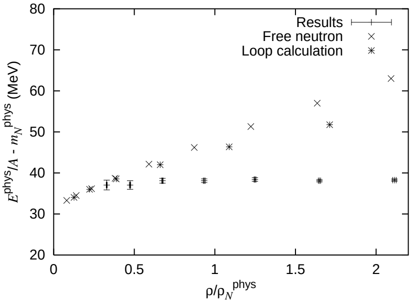
Density versus chemical potential



$T^{phys} = 25.0$ MeV, $a^{-1} = 150$ MeV

Results

Energy per neutron



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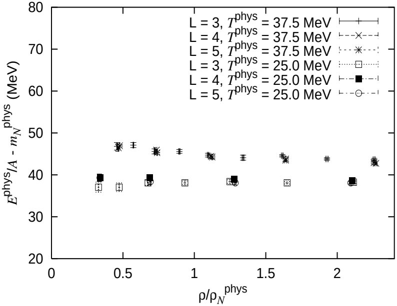
Results

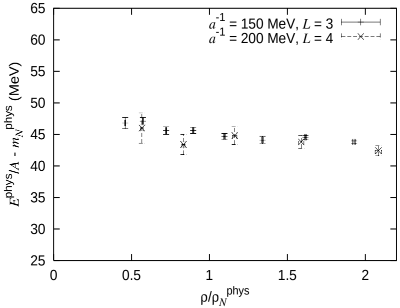
Different lattice volumes



T^{phys} =25 MeV and 37.5 MeV , $a^{-1} = 150$ MeV

Results

Different lattice spacings



T^{phys} =37.5 MeV, a^{-1} = 150 MeV and 200 MeV

Conclusions

- Probe **larger volumes, lower temperatures, greater nuclear densities** than lattice QCD
- Coupling C is determined by fitting to NN scattering data
- Cutoff dependence is absorbed into C
- *Realistic* simulation of many-body nuclear phenomena with
  - no free parameters
  - a systematic expansion
  - clear theoretical connection to QCD
- Include protons, charged pions, higher orders, $3N$ forces etc.

Concluding remark:

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