

Lattice QCD with mixed actions

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using Lattice QCD”

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Outline

- Mixed action Lattice QCD: What? Why? Does it make sense?
- Brief overview: Status of mixed action QCD
 - Numerical simulations
 - (Mixed) Chiral Perturbation Theory
- Some issues with mixed action QCD
 - Size of the cut-off effects
 - Quark mass matching
 - Partial quenching effects
- What I would like to see next

Mixed action Lattice QCD: What is it ?

Fermions enter a numerical lattice calculation twice

1. generating the configurations
(sea quarks)

$$e^{-S_{eff}} = \det(D + m)e^{-S_g}$$

2. computing quark propagators
(valence quarks)

$$(D + m)^{-1}$$

these are two separate steps

→ Use different Dirac operators
for sea and valence quarks

$$e^{-S_{eff}} = \det(D_S + m_S)e^{-S_g}$$

“Mixed action Lattice QCD”

$$(D_V + m_V)^{-1}$$

Mixed action Lattice QCD: What is it ?

Mixed action QCD is a generalization of partially quenched QCD !

- Partially quenched QCD $D_V = D_S$ $m_V \neq m_S$
- Mixed action QCD $D_V \neq D_S$ $m_V \neq m_S$

Theoretical formulation: sea and valence quarks + valence ghosts
(similarly to PQQCD)

Difference: No full (unquenched) subsector in mixed action QCD

$m_V = m_S$  unquenched QCD

It is even not entirely obvious what $m_V = m_S$ means (later more)

Does it make sense ?

Any “good” lattice Dirac operator satisfies

$$D(p) = ip_\mu \gamma_\mu + \mathcal{O}(ap^2)$$

1. All “good” D give the same continuum limit $a \rightarrow 0$
2. Differences are $\mathcal{O}(a)$ and vanish in the continuum limit

$$D_S - D_V = \mathcal{O}(a)$$

Naive expectation: Using different Dirac operators for sea and valence quarks results in errors of $\mathcal{O}(a)$ which vanish in the continuum limit

Some lattice Dirac operators

- Wilson fermions (twisted or untwisted)
- Staggered fermions
- Ginsparg-Wilson fermions (satisfying the Ginsparg-Wilson relation)
 - Overlap fermions
 - Domain wall fermions
- Approximate GW fermions
 - Chirally improved (CI) fermions
 - Fixed-point (FP) fermions

Note: Using different approximations of the same type of fermions is also a mixed action theory !

Example: Domain wall fermions with $L_{5,S} \neq L_{5,V}$

Why mixed action QCD ?

Theoretically “ideal”

lattice simulation: Dynamical Ginsparg-Wilson fermions

- But
- Too expensive (right now)
 - Algorithmic issues (changing the top. sector)

Cost efficient compromise: Mixed simulation with

- cheap sea quarks (Wilson, twisted mass Wilson, staggered ...)
- Ginsparg-Wilson valence quarks

Why mixed action QCD ?

Advantages:

- Cost efficient alternative to expensive full GW simulations
 - Exact chiral symmetry in the valence sector
- Beneficial for calculation of weak matrix elements ($K \rightarrow \pi\pi, \dots$)
- Use other people's configs \longrightarrow \sim cost of a quenched GW simulation

Drawbacks:

- Unitarity is lost at non-zero lattice spacing
- Afflicted with all “diseases” known in PQQCD
- No unquenched sector ($m_V = m_S$) at non-zero lattice spacing

Status - Numerical simulations

Staggered sea quarks (MILC configurations):

- LHP collaboration, using domain-wall valence quarks
Edwards et.al.
 - Pion and nucleon form factors
Phys.Rev.D72:054506,2005
 - Nucleon axial charge
Phys.Rev.Lett.96:052001,2006
 - Moments of parton distributions
Nucl.Phys.Proc.Suppl.140:255-260,2005
- NPLQCD collaboration, using domain-wall valence quarks
Beane et.al.
 - $I=2$ $\pi\pi$ scattering length
hep-lat/0506013
 - Nucleon-Nucleon scattering length
hep-lat/0602010
- UKQCD collaboration, using overlap valence quarks
Bowler et.al.
 - Light hadron spectrum, decay constants, nucleon masses
JHEP 08 (2005) 003

Exploratory study !!! Only 10 configurations

Status - Numerical simulations

- In all these simulations:
 - One lattice spacing $a \approx 0.125 fm$
No continuum extrapolation
 - Continuum ChPT is used for the chiral extrapolation
Error due to neglected lattice spacing artifacts

Status - Numerical simulations

Wilson sea quarks:

Still to come ...

Mixed Chiral Perturbation Theory

Step I: Construct Symanzik effective theory (continuum theory)

$$\longrightarrow \mathcal{S}_{\text{Sym}} = \mathcal{S}_4 + a\mathcal{S}_5 + a^2\mathcal{S}_6 + \dots$$

- \mathcal{S}_{Sym} : Most general expression compatible with locality and symmetries
- \mathcal{S}_4 : Continuum PQQCD
- $\mathcal{S}_5, \mathcal{S}_6$: Higher dimensional operators of dim 5, and dim 6
- Simplest example: Wilson fermions (explicit chiral symmetry breaking)

$$\longrightarrow \mathcal{S}_5 = c \bar{\psi}_S \sigma_{\mu\nu} G_{\mu\nu} \psi_S \quad \text{Pauli term}$$

Mixed Chiral Perturbation Theory

Step 2: Symmetry group of PQQCD (massless limit)

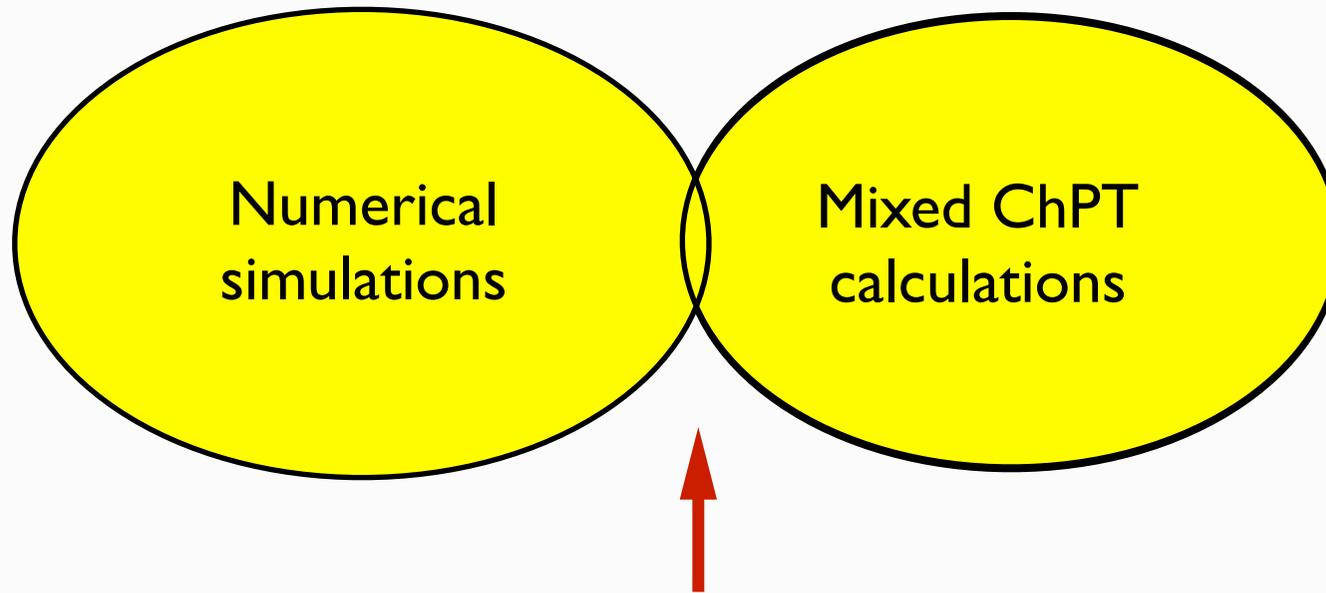
$$G_{\text{PQQCD}} = SU(N_S + N_V | N_V)_L \otimes SU(N_S + N_V | N_V)_R$$

- Assumption: spontaneously broken to vector subgroup
 - light pseudo scalar Goldstone bosons
 - described by a chiral Lagrangian
- Follow standard procedure to construct the chiral Lagrangian
 - Spurion analysis for mass and symmetry breaking terms in \mathcal{S}_5 and \mathcal{S}_6
 - chiral Lagrangian with explicit a dependence

Status - Mixed ChPT

- Staggered sea quarks:
 - Pseudoscalar masses and decay constants
OB, Bernard, Shores, Rupak 2005
 - $I=2$ $\pi\pi$ scattering length
Chen et.al. 2005
 - Baryon masses
Tiburzi 2005
 - Vector meson masses
Grigoryan, Thomas 2005
 - Scalar correlator
Prelovsek 2005
- Wilson sea quarks
 - Pseudoscalar masses
OB, Shores, Rupak 2003
 - Baryon masses
Tiburzi 2005
 - Nucleon properties (magnetic moments, ...)
Beane, Savage 2003; Arndt, Tiburzi 2004
 - Vector meson masses
Grigoryan, Thomas 2005
 - The role of the double pole
Golterman, Izubuchi, Shamir 2005

Status summary



Unfortunately very little overlap

- Almost no results to assess the mixed action approach !

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Symanzik action for the mixed theory

$$S_{\text{Sym}} = S_4 + aS_5 + a^2S_6 + \dots$$

- S_4 : Continuum PQQCD
- S_5 : Operators with sea fields only (for Wilson sea quarks)
- Three types of operators of $\mathcal{O}(a^2)$
 - Type 1: Involve sea fields only
 - Type 2: Involve valence fields only
 - Type 3: Involve both

Mixed 4 - fermion operators

General structure of mixed 4 - fermion operators

OB, Bernard,
Rupak, Shores 2005

$$O_{\text{Mix}}^{(6)} = \bar{\psi}_S (\gamma_{Spin} \otimes t_{Color}^a) \psi_S \bar{\psi}_V (\gamma_{Spin} \otimes t_{Color}^a) \psi_V$$

- Allowed operators are products of a sea-sea and a val-val bilinear
- In total there are four types of these operators

γ_{Spin} : vector or axial vector

t_{Color}^a : color group generator or identity

- For staggered sea quarks: The sea-sea bilinear is trivial in taste
→ form is the same as for four Wilson sea quarks

Direct sea - valence coupling at $\mathcal{O}(a^2)$

LO chiral Lagrangian

$$\mathcal{L}_\chi = \frac{f^2}{8} \langle \partial_\mu \Sigma \partial_\mu \Sigma^\dagger \rangle - \frac{f^2 B}{4} \langle \Sigma M^\dagger + M \Sigma^\dagger \rangle + \frac{m_0^2}{6} \langle \Phi \rangle^2 + a^2 \mathcal{V}$$

M : quark mass matrix

- Singlet term explicitly left in the Lagrangian
(for convenience, later we take $m_0 \rightarrow \infty$)
- \mathcal{V} : potential terms (no derivative) proportional to a^2
- \mathcal{V} enters LO Lagrangian

Underlying assumption: $m \approx a^2 \Lambda_{\text{QCD}}^3$

This assumption can and should be checked (see later)

The potential

The potential can be written as a sum of two terms*

$$\mathcal{V} = \mathcal{U}_S + \mathcal{U}_V$$

● \mathcal{U}_S contains sea fields only due to P_S : projector on sea fields

● \mathcal{U}_S contains the same terms as

○ staggered ChPT (for staggered sea quarks)
Lee, Sharpe, Bernard, Aubin

example: $C_1 \langle \hat{\xi}_5 P_S \Sigma \hat{\xi}_5 P_S \Sigma^\dagger \rangle$ $\hat{\xi}_5$: taste matrix

○ Wilson ChPT (for Wilson sea quarks)
Sharpe, Singleton, Rupak, Shoreh, OB, Aoki

$$W_0^2 W_7' \langle P_S (\Sigma - \Sigma^\dagger) \rangle^2$$

● \mathcal{U}_S does not contribute to val-val masses and decay constants at one loop

*for staggered fermions sometimes written as: $\mathcal{V} = \mathcal{U}_S + \mathcal{U}'_S + \mathcal{U}_V$

The term \mathcal{U}_V

$$\mathcal{U}_V = -C_{\text{Mix}} \langle \tau_3 \Sigma \tau_3 \Sigma^\dagger \rangle$$

$$\tau_3 = (1_S, -1_V)$$

- The potential \mathcal{U}_V
 - stems from the mixed 4 fermion operators
 - contains only one term associated with one low-energy constant C_{Mix}
 - preserves the SU(4) taste symmetry for staggered sea quarks
(expected since the mixed 4-fermion operators are trivial in taste space)

- \mathcal{U}_V is present only in mixed theories

Reason: smaller symmetry group of the mixed theory

$$G_{\text{Mixed}} = G_{\text{Sea}} \otimes G_{\text{Val}} \neq G_{\text{PQQCD}}$$

→ Mixed ChPT has more unknown low-energy constants (C_{Mix} at LO)

LO masses

Notation: Sea quark flavors: S, S'
 Valence quark flavors: V, V'

Val-Val: $m_{VV'}^2 = B(m_V + m_{V'})$

- Vanishes for zero quark mass because of exact chiral symmetry

LO masses

Sea-Sea:
(staggered)

$$m_{SS',b}^2 = B(m_S + m_{S'}) + a^2 \Delta(\xi_b)$$

ξ_b :Taste label

Taste splittings $\Delta(\xi_b)$
are the same as in
Staggered ChPT

Sea-Sea:*
(Wilson)

$$m_{SS'}^2 = B(m_S + m_{S'}) + 2a^2 c_2$$

c_2 same as in
Wilson ChPT

*Quark masses contain shift linear in a ('shifted masses')

LO masses

Sea-Val: $m_{SV}^2 = B(m_S + m_V) + a^2 \Delta_{\text{Mix}}$

- Mass does not vanish in the massless limit (no symmetry argument)
- Mixed meson mass depends on the new low-energy constant: $\Delta_{\text{Mix}} \equiv \frac{16C_{\text{Mix}}}{f^2}$
- Mass shift is proportional to a^2 and C_{Mix}
 - direct measure for size of the cut-off artifacts
- Mass can be directly measured from the propagator of a mixed meson
- This mass will enter the 1-loop expression for the decay constant (see later)

Lessons I

- Mixed ChPT has more unknown low-energy constants than ‘ordinary’ ChPT
- Mixed lattice QCD has more observables (e.g. mixed meson mass)

→ Not necessarily less predictive

- Mixed meson mass is a direct measure for the size of the lattice spacing artifacts

→ Measure it to decide the appropriate power counting

$$m \approx a^2 \Lambda_{\text{QCD}}^3 \quad \text{means} \quad B(m_S + m_V) \approx a^2 \Delta_{\text{Mix}}$$

Quark mass matching

Quark mass matching: Required to reach unquenched QCD in the continuum limit

$$m_V = m_S \quad \xrightarrow{\text{e.g.}} \quad m_{VV'}^2 = m_{SS'}^2$$

- Note: Matching is not unique
 - other choices possible
 - different matching conditions differ by $\mathcal{O}(a^2)$

Quark mass matching

Suppose we want: $m_{VV'}^2 = m_{SS'}^2$ for $\begin{matrix} S \rightarrow u \\ S' \rightarrow d \end{matrix}$ (degenerate)

- Wilson sea quarks

$$m_{SS'}^2 = m_{\pi^\pm}^2 = m_{\pi^0}^2$$

- Twisted mass Wilson sea quarks

$$m_{SS'}^2 = m_{\pi^\pm}^2 \neq m_{\pi^0}^2$$

- Staggered sea quarks

$$m_{SS',b}^2 = m_{\pi_5^\pm}^2 \neq m_{\pi_{\mu 5}^\pm}^2 \neq m_{\pi_{\mu\nu}^\pm}^2 \neq m_{\pi_\mu^\pm}^2 \neq m_{\pi_I^\pm}^2$$

Question: Is there a preferred way to match ?*

Does mixed ChPT give us a hint?

*Staggered sea quarks: This question is independent of the 4th root trick!

Staggered sea quarks: 1-loop pion mass

OB, Bernard, Rupak, Shoresh

$$\frac{(m_{VV}^{\text{NLO}})^2}{2Bm_V} = 1 + \frac{1}{16\pi^2 f^2} \frac{2}{3} \left(R_1^{[2,2]}(\{\mathcal{M}_{X,I}^{[2]}\}; \{\mu_I^{[2]}\}) \tilde{l}(m_X^2) \right. \\ \left. + \sum_{j=1}^2 D_{j,1}^{[2,2]}(\{\mathcal{M}_{X,I}^{[2]}\}; \{\mu_I^{[2]}\}) \ell(m_j^2) \right) \\ + \text{analytic}$$

$l(m^2), \tilde{l}(m^2)$: chiral logs

R, D : residue functions
ratios of products
involving LO masses

Example:

$$R_1^{[2,2]} = \frac{(m_{U_I}^2 - m_{VV}^2)(m_{S_I}^2 - m_{VV}^2)}{m_{VV}^2 - m_{\eta_I}^2}$$

Question

Can we choose the quark masses such that the result resembles the full (unquenched) theory ?

Can we bring the coefficients of the chiral logs to continuum form ?

Answer: Yes !

This is achieved by choosing
(at this order)

$$m_{VV}^2 = m_{\pi_I^\pm}^2$$

Matching to the taste singlet pion

Explicit result after matching

$$\frac{(m_{\pi_{VV}^+}^{\text{NLO}})^2}{2Bm_V} = 1 + \frac{1}{16\pi^2 f^2} \left(l(m_{\pi_I^0}^2) - \frac{1}{3} l(m_{\eta_I}^2) \right) \\ + \frac{16B}{f^2} (2L_8 - L_5) (2m_V) + \frac{32B}{f^2} (2L_6 - L_4) (2m_S + m_{S'}) + a^2 \mathcal{C}$$

- Simplified residues, but still not unquenched

The mixed result has always some remnant of partial quenching

But: correct result in the naive continuum limit

- Masses in the chiral logs are taste singlet masses (sea -sea)

Decay constant

OB, Bernard, Rupak, Shoresh 2005

$$\frac{f_{\pi_{VV}^+}^{\text{NLO}}}{f} = 1 + \frac{1}{16\pi^2 f^2} \left[-2l(m_{S_V}^2) - l(m_{S'_V}^2) \right] \\ + \frac{8B}{f^2} L_5(2m_V) + \frac{16B}{f^2} L_4(2m_S + m_{S'}) + a^2 \mathcal{F}$$

- No residue functions (cancellations for degenerate valence quark masses)
 - ➔ no obviously preferred way to define a ‘full’ pion
- **Mixed meson masses** in the chiral logs
 - ➔ log behaviour is different compared to the pion mass result
(also true for Wilson sea quarks)

Double pole effects with Wilson sea quarks

Golterman, Izubuchi, Shamir 2005

$$\mathcal{L}_\chi[a^2] = -a^2 W_0^2 \left(W_7' \langle P_S (\Sigma - \Sigma^\dagger) \rangle^2 + \dots \right)$$

W_0, W_7' : low-energy constants

This term contributes to the
flavor neutral two-point function:

P_S : projector on the sea fields

$$G_{ij}(p) = \left(\delta_{ij} - \frac{1}{N_S} \right) \frac{1}{p^2 + m_{VV}^2} - \frac{R}{(p^2 + m_{VV}^2)^2}$$

Residue of the double pole:

$$R = \frac{m_{SS}^2 - m_{VV}^2}{N_S} + \frac{32}{f^2} a^2 W_0^2 W_7'$$

Double pole effects with Wilson sea quarks

Golterman, Izubuchi, Shamir 2005

We have two choices for the quark mass matching

- Tune m_V such that $R = 0 \longrightarrow m_{VV}^2 \neq m_{SS}^2$

Sign of W'_7 determines whether $m_{VV} > m_{SS}$ or $m_{VV} < m_{SS}$

- Tune m_V such that $m_{VV}^2 = m_{SS}^2 \longrightarrow R \neq 0$

\longrightarrow double pole (= partial quenching) effects in various quantities for example

- $I=0$ $\pi\pi$ scattering
Bernard, Golterman 1996
- a_0 propagator
Bardeen et.al. 2002
- nucleon - nucleon potential
Beane, Savage 2002

Lessons 2

- Mixed lattice QCD suffers from partial quenching effects
- The size of these effects depends
 - on the quark mass matching
 - on the size of the lattice spacing ($\approx a^2$)
 - on the observable



Study the size of these effects in actual simulations !

Example: The scalar propagator

Connected

scalar correlator

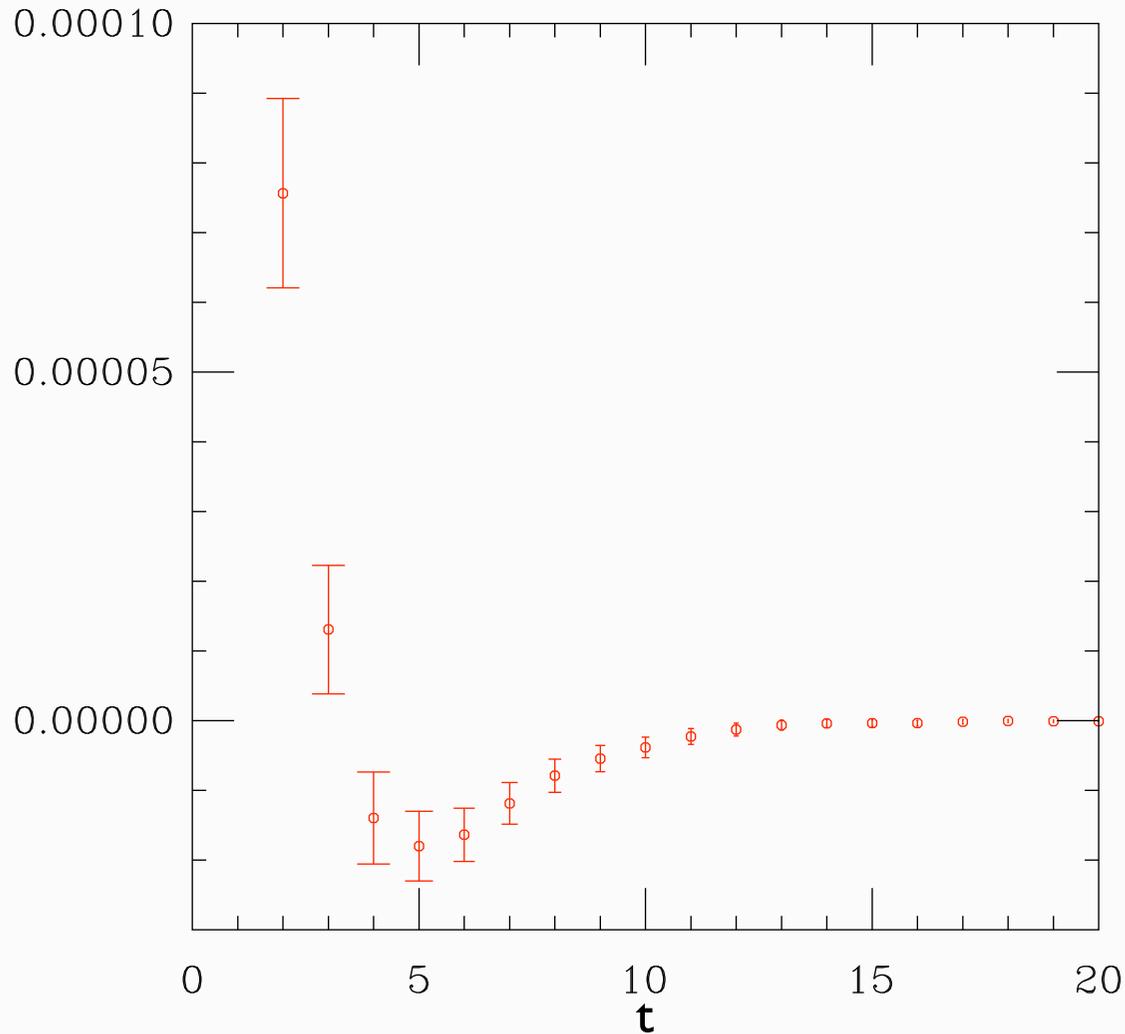
$$C(t) = \sum_{\vec{x}} \langle 0 | \bar{d}u(\vec{x}, t) \bar{u}d(\vec{0}, 0) | 0 \rangle$$
$$= A e^{-m_{a_0} t} + B(t) + \dots$$

- $B(t)$: contribution from two-pseudoscalar states ($\pi\eta, KK, \pi\eta'$)
... : excited scalar states, multi-hadron states (less important)
- A LO continuum PQ ChPT analysis shows
Prelovsek et.al. 2004
 - $B(t)$ = sum of exponentials for $m_V = m_S$
 - $B(t)$ gives a negative contribution for large t if $m_V < m_S$
 - The scalar correlator can become negative for $m_V < m_S$

A negative scalar correlator is a signal for partial quenching !

The scalar propagator with staggered sea quarks

LHP collaboration



plot by K. Orginos

Quark mass matching using
the Goldstone pion

$$m_{VV}^2 = m_{\pi_5^\pm}^2$$

→ negative propagator

clear sign of
partial quenching !

What happens for

$$m_{VV}^2 = m_{\pi_I^\pm}^2$$

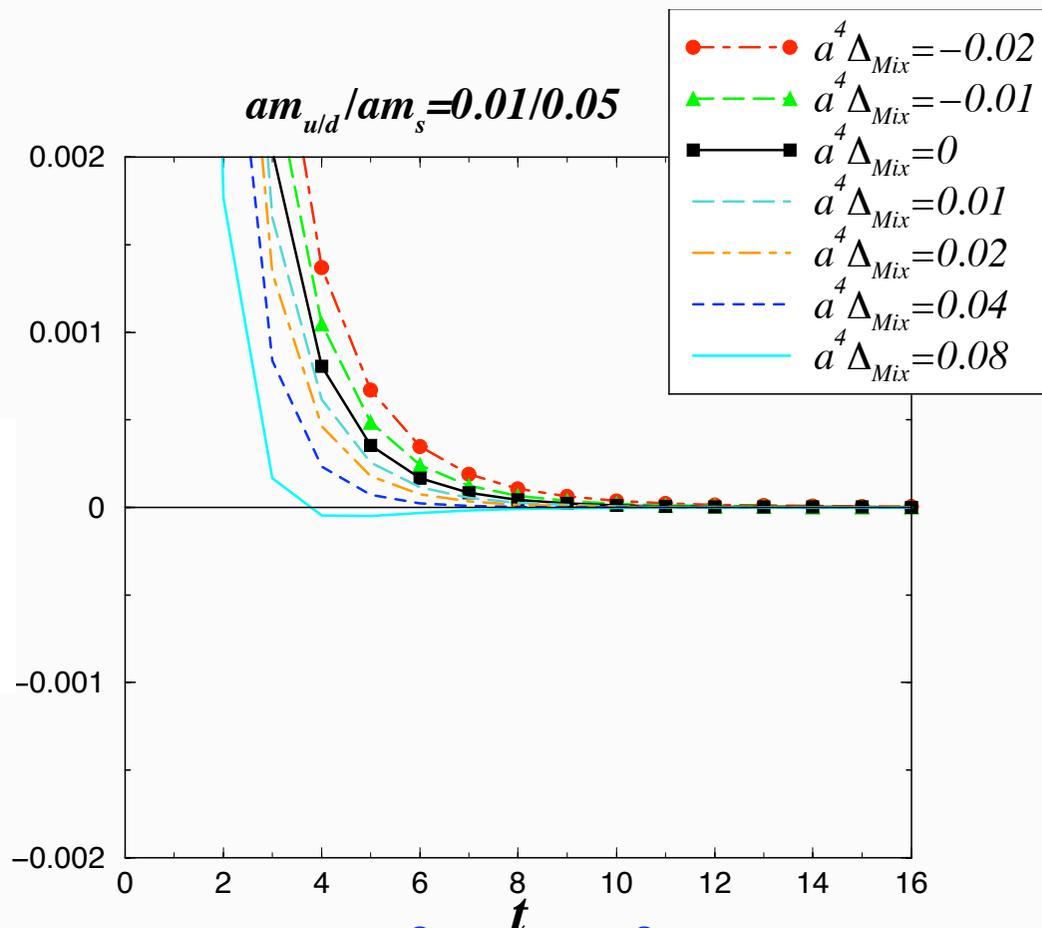
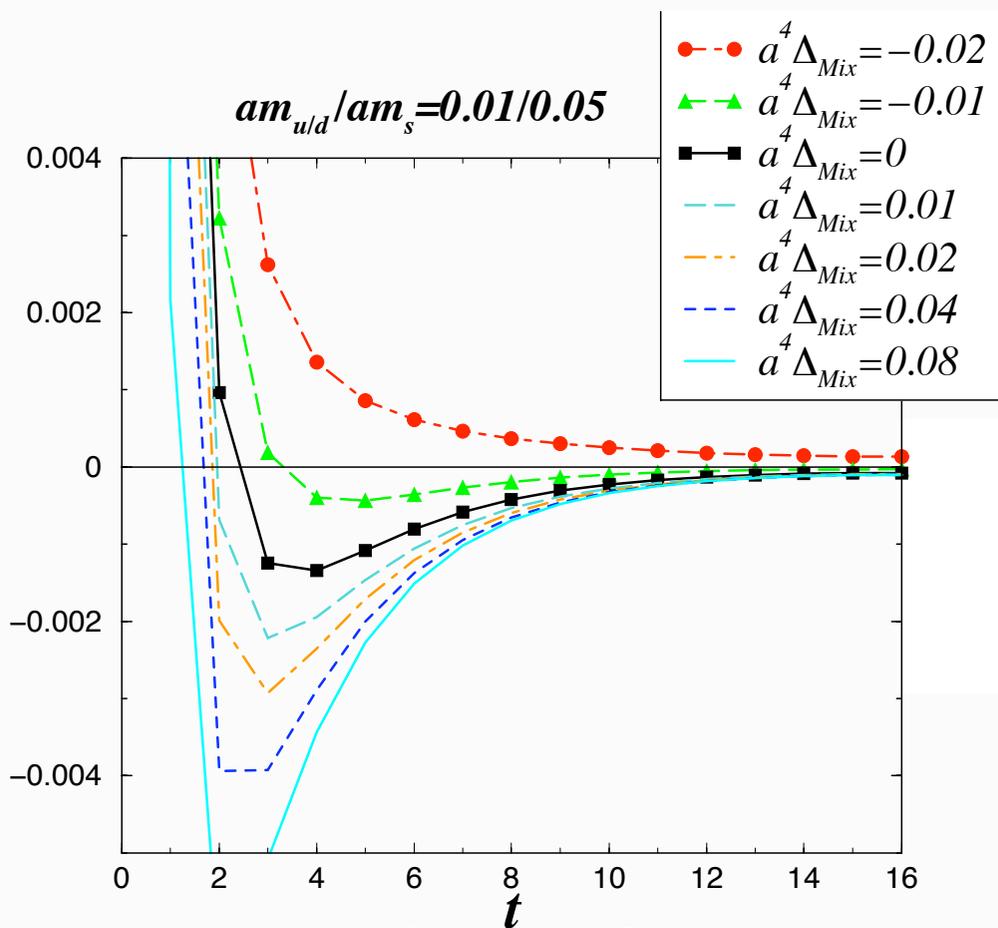
?

$B(t)$ in mixed ChPT

Prelovsek 2005

plots from
hep-lat/0510080

$B(t)$ depends on C_{Mix} \longrightarrow here free parameter



Matching:

$$m_{VV}^2 = m_{\pi_5^\pm}^2$$

$$m_{VV}^2 = m_{\pi_I^\pm}^2$$



Effective theory predicts negative contribution $B(t)$
for the matching with the Goldstone pion

Lessons 3

- Mixed lattice QCD has certain ‘diseases’ (partial quenching effects)
 - BUT: Mixed ChPT - provided it is the correct low-energy theory - must reproduce these sicknesses
- Exploit the characteristic signatures of the diseases to test the validity of the effective theory
- Similar idea: check for curvature due to chiral logs
- Provided we have established that the effective description is correct :
- Use the effective theory to account for the disease and extract the physical result
- Example: Scalar correlator and extraction of m_{a_0}
Prelovsek et.al. 2004

What I would like to see next

- Measure the mass of the mixed meson and check the size of the a^2 effects

Useful measure (?)
$$R = \frac{m_{SV}^2}{m_{VV}^2}$$

- Measure the pion decay constant and
 - take the continuum limit and compare to the unquenched result
 - compare to mixed ChPT

Can we see the chiral log with the mixed meson mass ?

- Measure other 'simple' quantities (scalar correlator, ...)
- Move on to more complicated observables