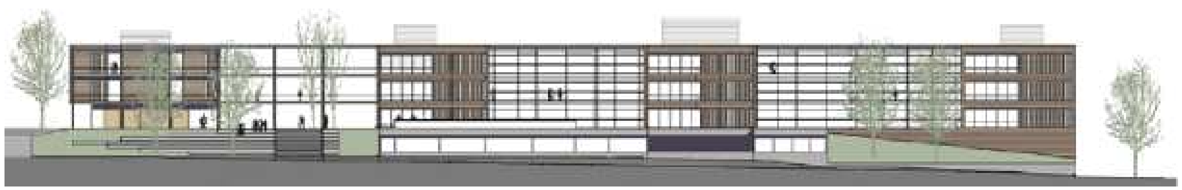
Neutrino trapping in a color superconductor

Igor A. Shovkovy



Frankfurt Institute for Advanced Studies
Johann W. Goethe-Universität
Frankfurt am Main, Germany

Collaborator(s)

- Stefan Rüster
- Verena Werth
- Dirk Rischke
- Michael Buballa

References

- S. B. Rüster, V. Werth, M. Buballa, I. A. Shovkovy and D. H. Rischke, `hep-ph/0509073`
- S. B. Rüster, V. Werth, M. Buballa, I. A. Shovkovy and D. H. Rischke, `hep-ph/0503184`, Phys. Rev. D **72** (2005) 034004

Outline

- Brief introduction
- Unconventional Cooper pairing in quark matter

- Effects of neutrino trapping
  - in two-flavor normal quark matter
  - in three-flavor normal quark matter
  - in the CFL phase
- Phase diagram in NJL model

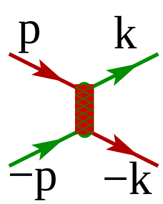
- Conclusions

Color superconductor

(i) Deconfined quarks ($\mu \gtrsim \Lambda_{QCD}$)

(ii) Pauli principle ($s = \frac{1}{2}$)

(iii) Attractive interaction

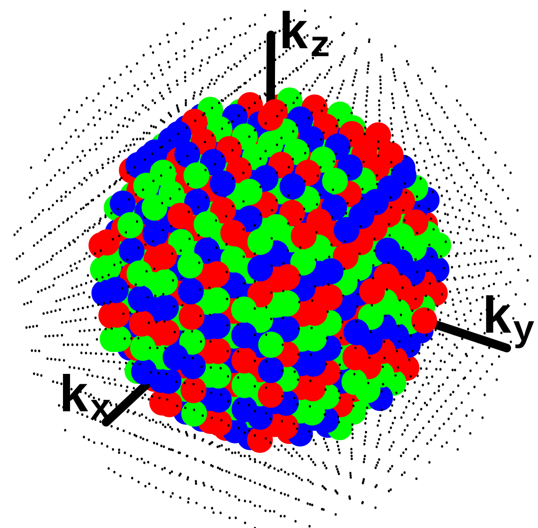


$$= \bar{3}_a + 6_s$$

Repulsive

Attractive

Note: pairing is *cross-flavor*,
e.g., $\langle u_r d_g \rangle$ and $\langle d_r u_g \rangle$



$\Downarrow$

Cooper instability

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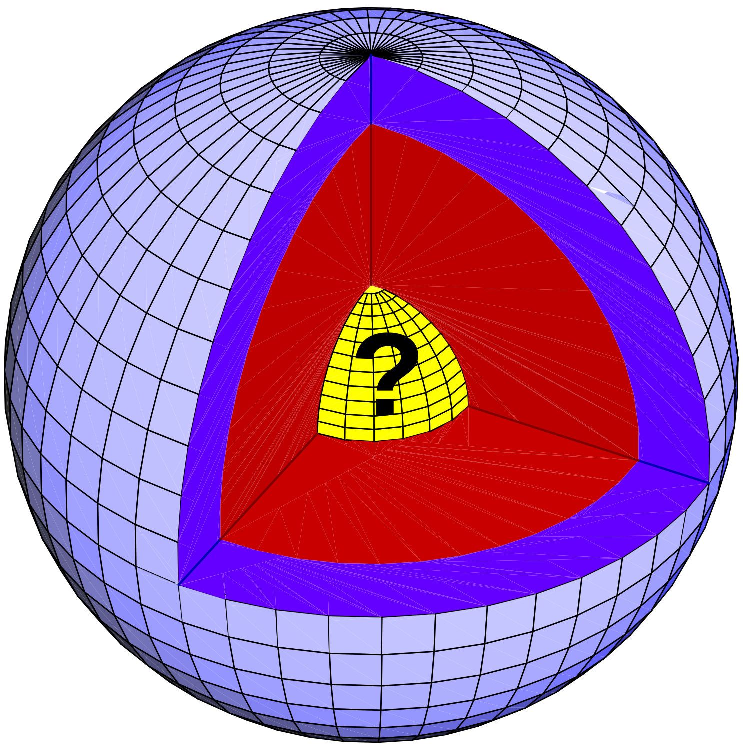
Color superconductivity

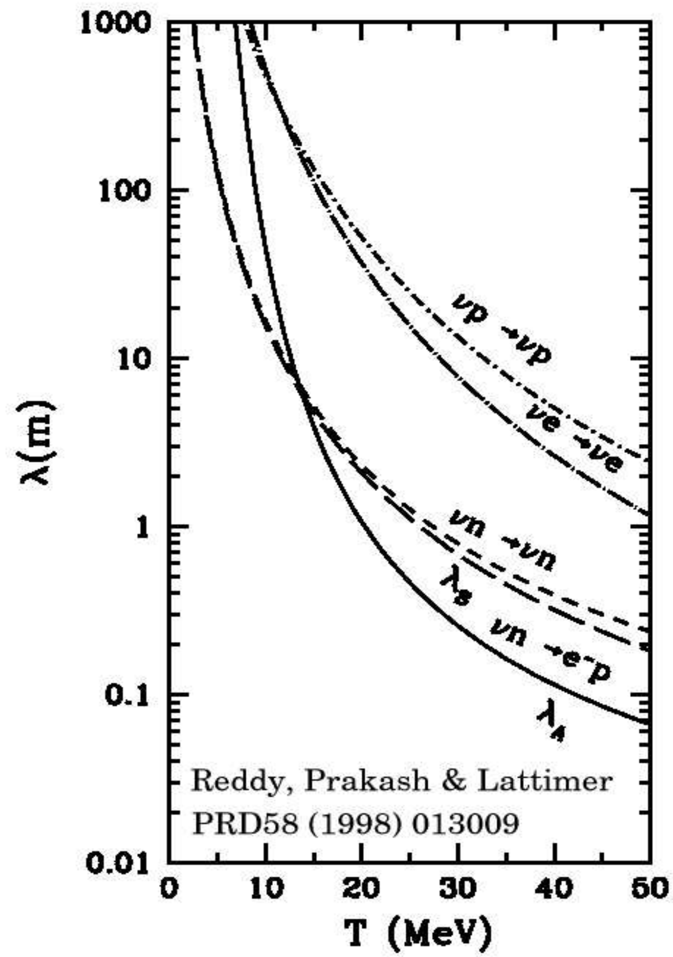
$$\langle \left(\bar{\Psi}^C\right)_i^\alpha \gamma_5 \Psi_j^\beta \rangle \neq 0$$

(Proto-)neutron stars

What is the state of matter at $\rho_c \gtrsim 5\rho_0$ and $T \lesssim 20$ MeV?



... also with neutrino trapping?



Unconventional Cooper pairing

- Wave function of a Cooper pair:

$$(|\textcolor{red}{\bullet}\textcolor{green}{\bullet}\rangle - |\textcolor{green}{\bullet}\textcolor{red}{\bullet}\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes \boxed{(|\mathrm{u}_{\mathbf{p}}, \mathrm{d}_{-\mathbf{p}}\rangle - |\mathrm{d}_{\mathbf{p}}, \mathrm{u}_{-\mathbf{p}}\rangle)_{1,\bar{3}}}$$

- In equilibrium, quarks have non-equal Fermi momenta:

$$p_F^{(u)} \neq p_F^{(d)} \neq p_F^{(s)}$$

$N_f = 2$: electric neutrality

$$\mu_d = \mu_u + \mu_e$$

$$\frac{2}{3} n_u - \frac{1}{3} n_d - n_e = 0$$

$$\delta\mu \equiv \frac{p_F^{(d)} - p_F^{(u)}}{2} = \boxed{\frac{\mu_e}{2}}$$

$N_f = 3$: color neutrality

$$m_s \gg m_u, m_d$$

CFL-locking: strange $\Leftrightarrow$ blue

$$\delta\mu \equiv \frac{p_F^{(bd)} - p_F^{(gs)}}{2} \approx \boxed{\frac{m_s^2}{2\mu}}$$

The mismatch $\delta\mu \neq 0$ tends to destroy Cooper pairing

β-equilibrium

Partition function

$$Z = \exp\left(-\frac{H + \sum_i \mu_i n_i}{T}\right)$$

where conserved charges n_i are

$$\begin{aligned} n_Q &= \psi^\dagger Q \psi - n_e - n_\mu \\ n_{L_e} &= n_e + n_{\nu_e}, \qquad n_{L_\mu} = n_\mu + n_{\nu_\mu} \\ n_q &= \psi^\dagger \psi, \qquad n_3 = \psi^\dagger T_3 \psi, \qquad n_8 = \psi^\dagger T_8 \psi \end{aligned}$$

Thus, electron, neutrino and quark chemical potentials are

$$\begin{aligned} \mu_e &= \mu_{L_e} - \mu_Q, \qquad \mu_{\nu_e} = \mu_{L_e} \\ \mu_{ab}^{\alpha\beta} &= \left(\mu \delta^{\alpha\beta} + \mu_Q Q_f^{\alpha\beta}\right) \delta_{ab} + \left[\mu_3 \left(T_3\right)_{ab} + \mu_8 \left(T_8\right)_{ab}\right] \delta^{\alpha\beta} \end{aligned}$$

Charge neutrality in 2-flavor case

Normal phase densities

$$n_{u,d} = \frac{\mu_{u,d}^3}{\pi^2}, \qquad n_e = \frac{\mu_e^3}{3\pi^2}, \qquad n_{\nu_e} = \frac{\mu_{\nu_e}^3}{6\pi^2}$$

Neutrality condition

$$\underbrace{2(1+\frac{2}{3}y)^3}_{u \text{ quarks}} - \underbrace{(1-\frac{1}{3}y)^3}_{d \text{ quarks}} - \underbrace{(x-y)^3}_{\text{electrons}} = 0 ,$$

where

$$x \equiv \frac{\mu_{L_e}}{\mu} \qquad \text{and} \qquad y \equiv \frac{\mu_Q}{\mu}$$

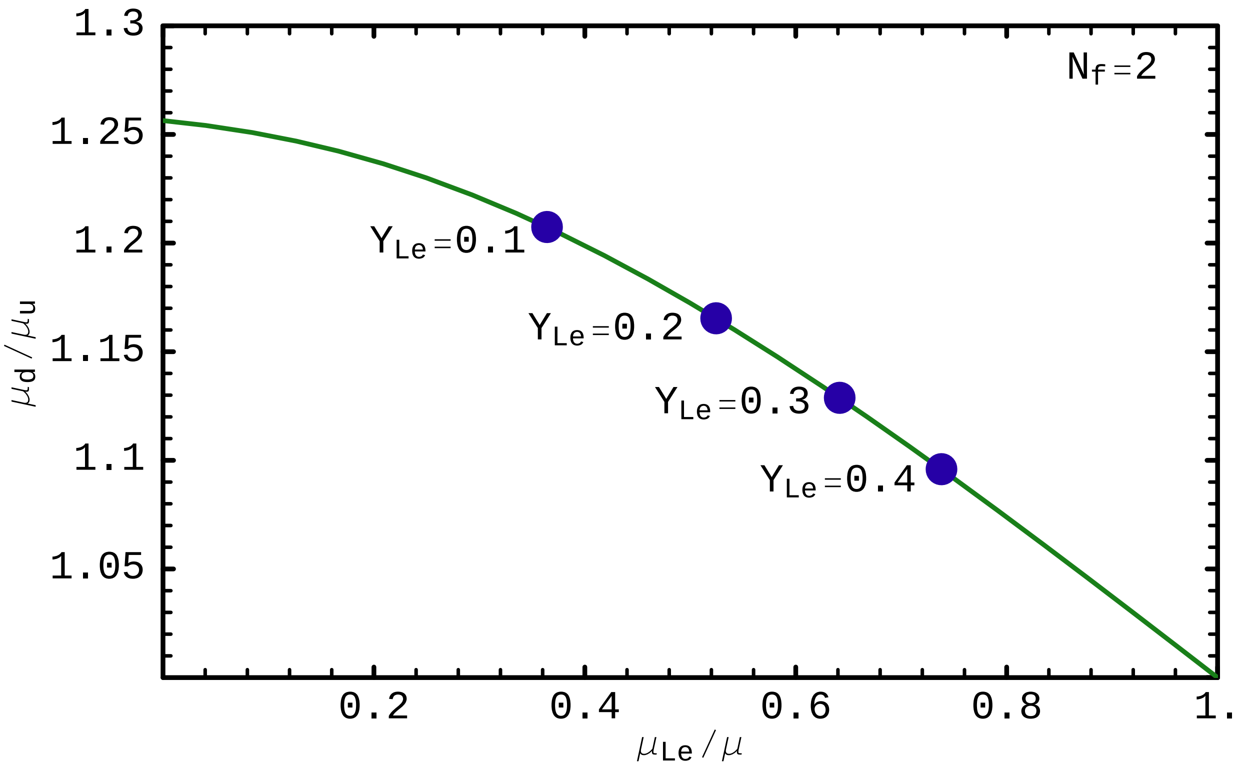
Without neutrino trapping ($\mu_{L_e} = 0$),

$$x = 0: \qquad y \approx -0.219 \quad \Rightarrow \quad \frac{n_d}{n_u} \approx \boxed{1.984} \quad \text{and} \quad \frac{n_e}{n_u} \approx \boxed{0.006}$$

Neutrino trapping in 2-flavor case

$\mu_{L_e} = 0$: $\mu_d/\mu_u \approx 1.256 \simeq 2^{1/3}$

$\mu_{L_e} = \mu$: $\mu_d/\mu_u = 1$



With increasing μ_{L_e}, cross-flavor Cooper pairing gets easier

i.e., $\mu_{L_e} > 0$ should favor 2SC-type pairing

Charge neutrality in 3-flavor case

Normal phase densities

$$n_{u,d,s} = \frac{\mu_{u,d,s}^3}{\pi^2}, \qquad n_e = \frac{\mu_e^3}{3\pi^2}, \qquad n_{\nu_e} = \frac{\mu_{\nu_e}^3}{6\pi^2}$$

Neutrality condition

$$\underbrace{2(1+\frac{2}{3}y)^3}_{u \text{ quarks}} - \underbrace{2(1-\frac{1}{3}y)^3}_{s \ \& \ d \text{ quarks}} - \underbrace{(x-y)^3}_{\text{electrons}} = 0 ,$$

where

$$x \equiv \frac{\mu_{L_e}}{\mu} \qquad \text{and} \qquad y \equiv \frac{\mu_Q}{\mu}$$

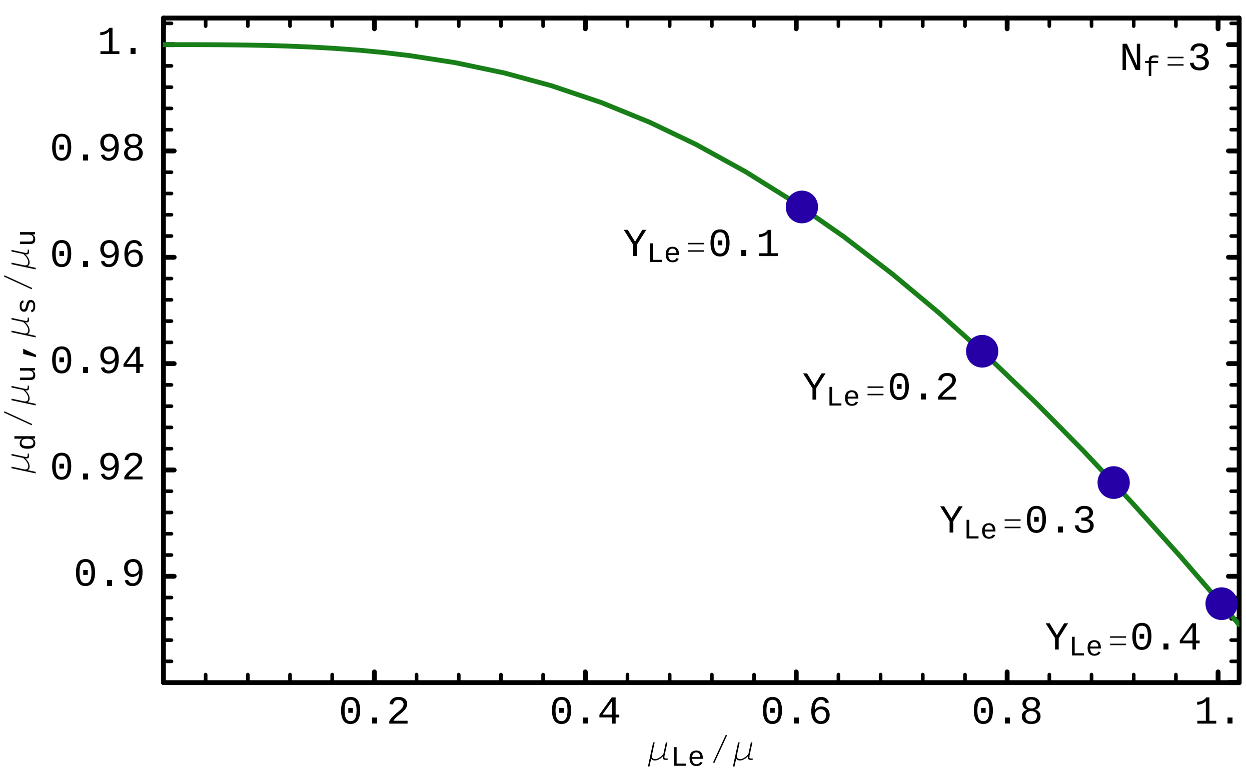
Without neutrino trapping ($\mu_{L_e} = 0$),

$$x = 0: \qquad y \approx 0 \quad \Rightarrow \quad \frac{n_d}{n_u} = \frac{n_s}{n_u} = \boxed{1} \quad \text{and} \quad n_e = \boxed{0}$$

Neutrino trapping in 3-flavor case

$\mu_{L_e} = 0$: $\mu_{d,s}/\mu_u = 1$

$\mu_{L_e} = \mu$: $\mu_{d,s}/\mu_u \approx 0.896$



With increasing μ_{L_e}, cross-flavor Cooper pairing gets harder

i.e., $\mu_{L_e} > 0$ should disfavor CFL-type pairing

Three flavor CFL-type pairing

	u_r	d_g	s_b	u_g	d_r	u_b	s_r	d_b	s_g
u_r		$-\Delta_3$	$-\Delta_2$						
d_g	$-\Delta_3$		$-\Delta_1$						
s_b	$-\Delta_2$	$-\Delta_1$							
u_g					Δ_3				
d_r				Δ_3					
u_b							Δ_2		
s_r						Δ_2			
d_b									Δ_1
s_g								Δ_1	

CFL plus neutrino trapping

Toy model:

$$p^{(\text{toy})} = \frac{1}{\pi^2}\sum_{a=1}^{3}\sum_{\alpha=1}^{3}\int_0^{p_{F,a\alpha}^{\text{common}}}(\mu_a^\alpha - p)\,p^2 dp + 3\frac{\mu^2\Delta^2}{\pi^2} + \frac{(\mu_{L_e}-\mu_Q)^4}{12\pi^2} + \frac{\mu_{L_e}^4}{24\pi^2}$$

where

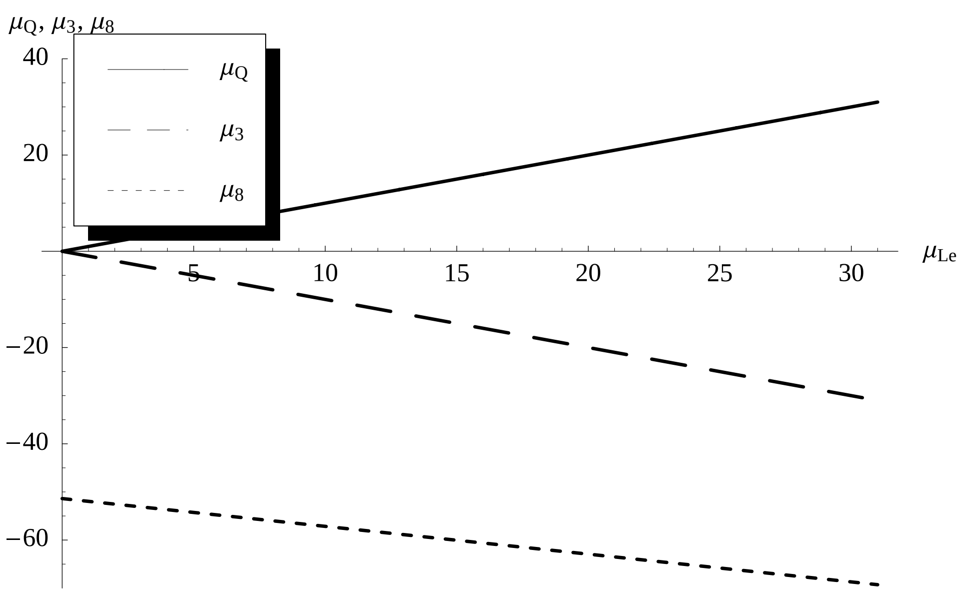
$$\begin{aligned}
p_{F,(ru,gd,bs)}^{\text{common}} &= \mu - \frac{M_s^2}{6\mu}\\
p_{F,(rd,gu)}^{\text{common}} &= \mu + \frac{\mu_Q}{6} + \frac{\mu_8}{2\sqrt{3}}\\
p_{F,(rs,bu)}^{\text{common}} &= \mu + \frac{\mu_Q}{6} + \frac{\mu_3}{4} - \frac{\mu_8}{4\sqrt{3}} - \frac{M_s^2}{4\mu}\\
p_{F,(gs,bd)}^{\text{common}} &= \mu - \frac{\mu_Q}{3} - \frac{\mu_3}{4} - \frac{\mu_8}{4\sqrt{3}} - \frac{M_s^2}{4\mu}
\end{aligned}$$

Solution to neutrality condition

$$n_Q - n_3 - \frac{n_8}{\sqrt{3}} \propto (\mu_Q - \mu_{L_e})^3 = 0 \quad \Rightarrow \quad \mu_Q = \mu_{L_e}$$

$$n_3 \propto (\mu_3 + \mu_Q) = 0 \quad \Rightarrow \quad \mu_3 = -\mu_{L_e}$$

$$n_8 = 0 \quad \Rightarrow \quad \mu_8 = -\frac{\mu_{L_e}}{\sqrt{3}} - \frac{M_s^2}{\sqrt{3}\mu}$$



CFL $\to$ gCFL due to neutrino trapping

Competition between Δ_i and $\delta\mu_i$:

$$\Delta_3 \quad vs. \quad \delta\mu_{(rd,gu)} \quad = \quad \frac{\mu_g^u - \mu_r^d}{2} = \mu_{L_e}$$

$$\Delta_2 \quad vs. \quad \delta\mu_{(rs,bu)} \quad = \quad \frac{\mu_b^u - \mu_r^s}{2} = \mu_{L_e} + \frac{M_s^2}{2\mu}$$

$$\Delta_1 \quad vs. \quad \delta\mu_{(gs,bd)} \quad = \quad \frac{\mu_b^d - \mu_g^s}{2} = \frac{M_s^2}{2\mu}$$

i.e., CFL $\to$ gCFL$'$ in which "rs–bu" quasiparticle mode is gapless

$$\mu_{L_e}^{(\mathrm{cr})} \approx \Delta_2 - \frac{M_s^2}{2\mu}$$

Lepton fraction in the CFL phase

Simple estimate

$$Y_{L_e} \approx \frac{1}{6}\left(\frac{\mu_{L_e}}{\mu}\right)^3$$

Taking into account that $\mu_{L_e} \lesssim \Delta - \frac{M_s^2}{2\mu} \lesssim 100$ MeV,

$$Y_{L_e} \lesssim 3 \times 10^{-3} \qquad (T = 0)$$

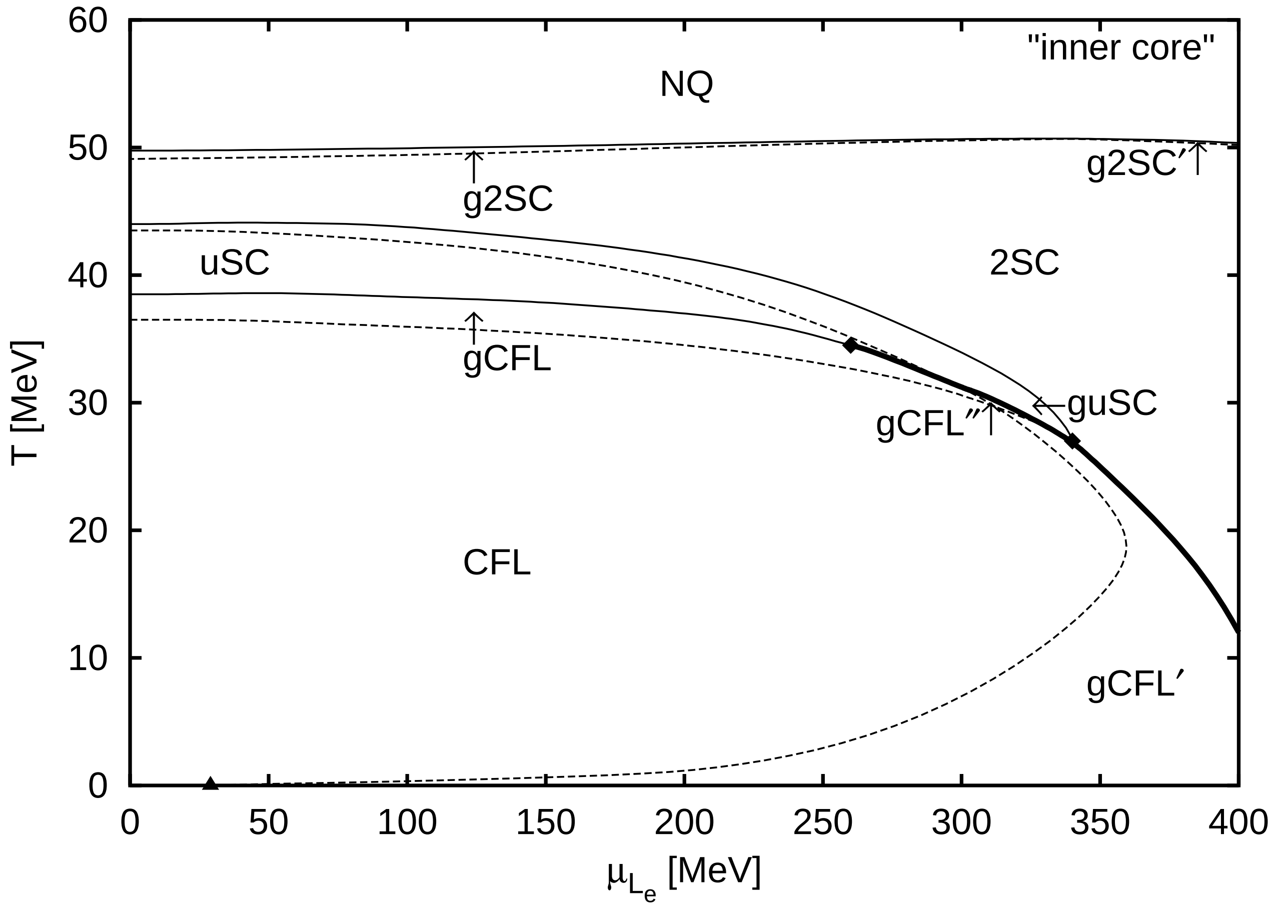
Numerical results at $T \lesssim 40$ MeV and $\mu \lesssim 500$ MeV

$$Y_{L_e} \lesssim 0.06$$

T–μ_{L_e} phase diagram, $\mu = 500$ MeV

("inner core case")

[Rüster, Werth, Buballa, Shovkovy & Rischke, hep-ph/0509073]



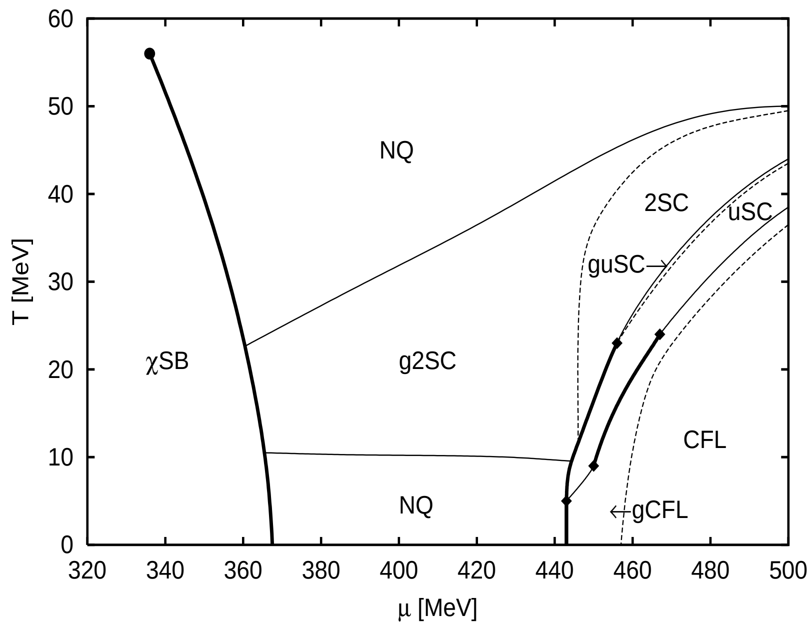
Definitions of gapless phases

Name	Gapless mode(s) $\epsilon(k) \sim \|k - k_F^{\rm eff}\|$	Diquark condensate(s)
g2SC	ru–gd, gu–rd	Δ_3
g2SC$'$	rd–gu, gd–ru	Δ_3
guSC	rs–bu	Δ_2, Δ_3
gCFL	gs–bd	Δ_1, Δ_2, Δ_3
gCFL$'$	rs–bu	Δ_1, Δ_2, Δ_3
gCFL$''$	gs–bd, rs–bu	Δ_1, Δ_2, Δ_3

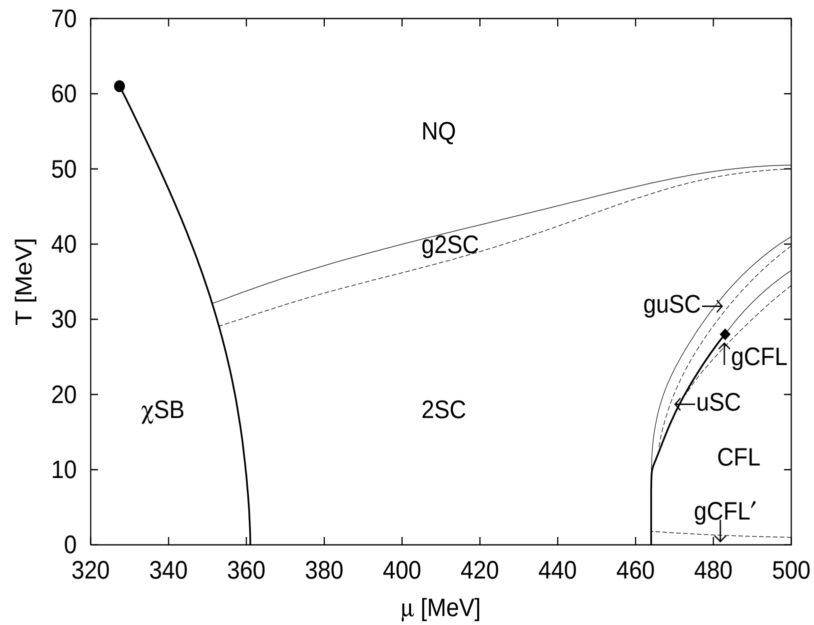
T–μ phase diagram

(without and with neutrino trapping)

[Rüster, Werth, Buballa, Shovkovy & Rischke, `hep-ph/0503184`; `hep-ph/0509073`]



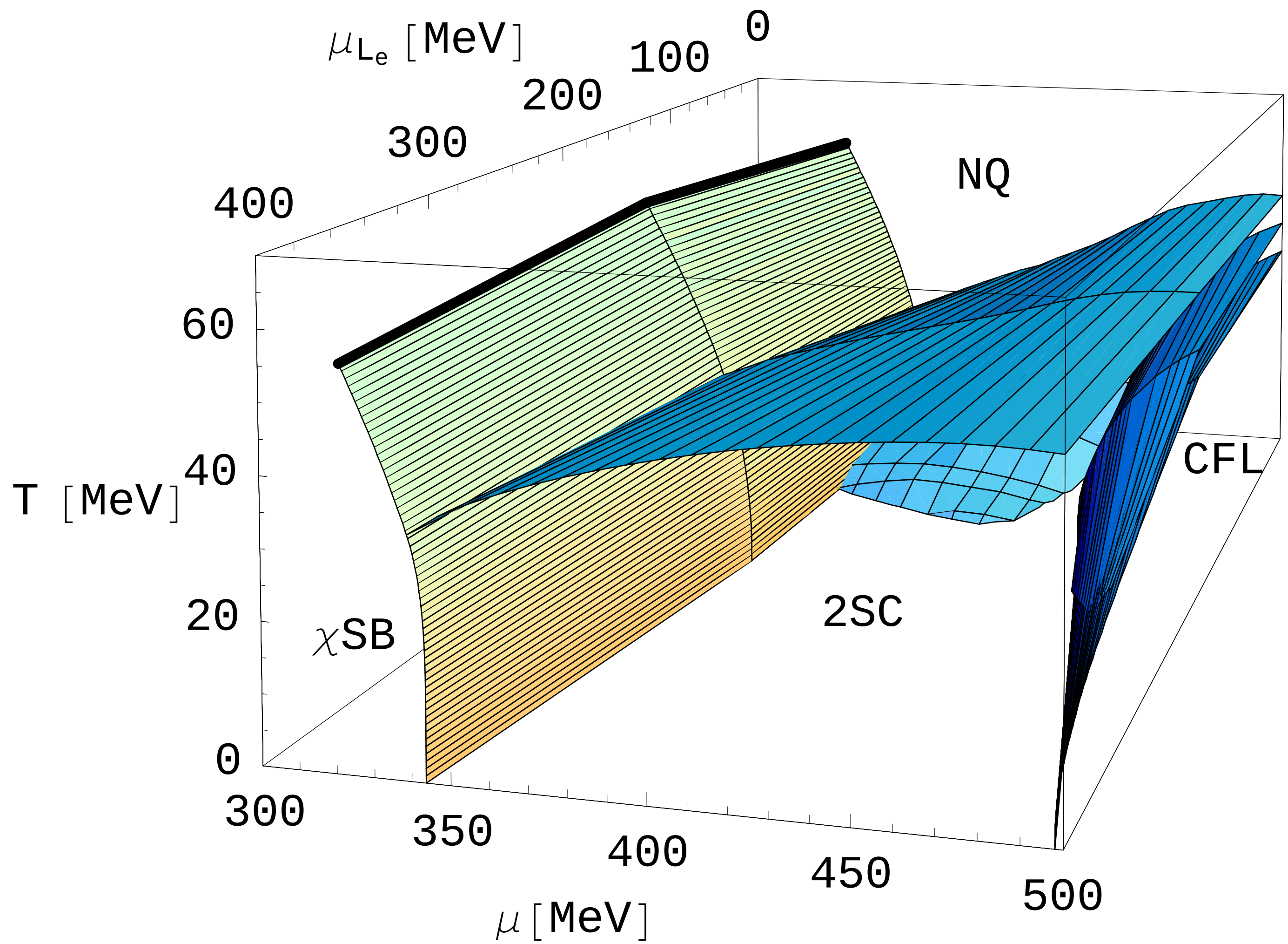
$\mu_{L_e} = 0$



$\mu_{L_e} = 200$ MeV

Note: $\mu_{L_e} > 0$ favors 2SC phase and disfavors CFL phase

3D phase diagram



Conclusions

- Neutrino trapping favors 2SC-type pairing by bringing the Fermi momenta of u and d quarks closer

- At $T = 0$, the CFL phase can withstand the stress due to μ_{L_e} if

$$\mu_{L_e} < \mu_{L_e}^{(\mathrm{cr})} \approx \Delta - \frac{M_s^2}{2\mu}$$

- Maximum lepton fraction in the CFL phase is about $Y_{L_e} \simeq 0.06$, i.e., the CFL phase is unlikely during early stages of protoneutron star

- Neutrino trapping may result in a rich dynamics inside cores of protoneutron stars

- Can 2SC $\rightarrow$ CFL transition re-ignite the supernova explosion ?