The Quark Propagator in Superconducting Phases

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Pairing in Fermionic Systems: Beyond the BCS Theory
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Outline

Outline

Motivation

cold and dense quark matter expected to be a color superconductor
$\Rightarrow$ investigations in

- NJL-type models

  (M. Alford, K. Rajagopal, F. Wilczek, 1998; R. Rapp, T. Schäfer, E. Shuryak, M. Velkovsky, 1998)

- weak coupling at asymptotically large densities

  (T. Schäfer, F. Wilczek, 1999; R. Pisarski, D. Rischke, 1999; D. Hong, V. Miransky, I. Shovkovy, L. Wijewardhana, 1999)

quark propagator provides information about

- dynamical symmetry breaking
- pressure, i.e. thermodynamical potential
- excitation spectrum
- ...

$\Rightarrow$ Extend successful truncation scheme of DSE's in vacuum to finite densities!

Dyson-Schwinger equation for quark propagator

our approach:

- approximate gluon propagator and quark-gluon vertex separately

other approaches:

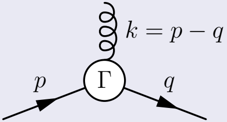
- weak coupling expansion

  (T. Schäfer, F. Wilczek, 1999; R. Pisarski, D. Rischke, 1999; D. Hong *et al.*, 1999)
- phenomenological coupling

  (C. Roberts, S. Schmidt, 2000)

Approximation of quark-gluon vertex

abelian vertex construction



$$\Gamma^a_\mu(p, q; k = p - q) \quad \longrightarrow \quad i\, g\, \Gamma(k^2)\, \gamma_\mu\, \frac{\lambda^a}{2}$$

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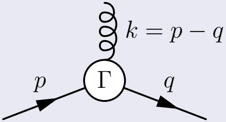


$$\Gamma^a_\mu(p, q; k = p - q) \longrightarrow i\, g\, \Gamma(k^2)\, \gamma_\mu \, \frac{\lambda^a}{2}$$

$\Gamma(k^2)$ determined by

DSE studies including Yang-Mills sector in Landau gauge:

chosen such that

- propagator multiplicative renormalizable
- correct anomalous dimension of mass function

analysis of lattice QCD data for propagators in Landau gauge:

invert DSE for $\Gamma(k^2)$ for given quark and gluon propagator

Approximation of gluon propagator

vacuum gluon propagator in Landau gauge

$$D^{ab\,vac}_{\mu\nu}(k) = \frac{Z(k^2)}{k^2}\,T_{\mu\nu}(k)\,\delta^{ab}$$

Approximation of gluon propagator

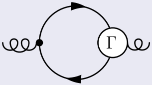
vacuum gluon propagator in Landau gauge

$$D_{\mu\nu}^{ab\,vac}(k) = \frac{Z(k^2)}{k^2} T_{\mu\nu}(k)\, \delta^{ab}$$

incorporate medium effects of particle-hole excitations

add medium polarisation of bare quarks to inverse vacuum propagator:

$$D_{\mu\nu}^{ab\,-1} = D_{\mu\nu}^{ab\,vac\,-1} +$$



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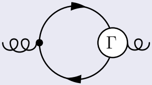


medium gluon propagator in Landau gauge

$$D^{ab}_{\mu\nu}(k) \;=\; \left(\frac{Z(k^2)}{k^2+G(k)}\,P^{T}_{\mu\nu}(k) + \frac{Z(k^2)}{k^2+F(k)}\,P^{L}_{\mu\nu}(k)\right)\delta^{ab}$$

Truncated Dyson-Schwinger equation

$$\Rightarrow S^{-1}(p) = Z_2 S_0^{-1}(p) + \frac{Z_2}{3\pi^3}\int d^4q\, \gamma_\mu S(q)\, \gamma_\nu \left(\frac{\alpha_s(k^2)}{k^2+G(k)} P^T_{\mu\nu} + \frac{\alpha_s(k^2)}{k^2+F(k)} P^L_{\mu\nu}\right)$$

- similar to HDL
- strong running coupling

$$\alpha_s(k^2) = \frac{Z_{1F}}{Z_2^2}\, g^2\, Z(k^2)\, \Gamma(k^2)$$

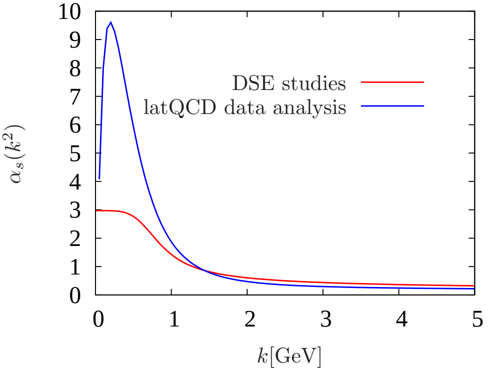
- screening and damping included through

$$m_g(k^2)^2 = \frac{N_f\, \mu^2\, \alpha_s(k^2)}{\pi}$$

$$F(k) = 2\, m_g(k^2)^2 + \dots$$

$$G(k) = \frac{\pi}{2}\, m_g(k^2)^2\, \frac{k_4}{|\vec{k}|} + \dots$$

Strong running coupling α_s



- with abelian vertex construction DSE studies underestimate chiral symmetry breaking in vacuum ($M_q \approx 170\mathrm{MeV}$)
  (C.S. Fischer, R. Alkofer, 2003)
- lattice studies cannot constrain deep infrared
  (M.S. Bhagwat, M.A. Pichowsky, C.D. Roberts, P.C. Tandy, 2003)

Outline

1. Motivation and introduction
2. The quark propagator in the chirally unbroken phase
3. The quark propagator in the 2SC/CFL phase for massless quarks
4. The quark propagator in the 2SC/CFL phase for massive quarks
5. Summary and outlook

The unbroken phase - I

quasiparticle propagator in chiral limit:

$$
\begin{aligned}
S^{-1} &= S^{+\,-1}\gamma_4\Lambda^+ + S^{-\,-1}\gamma_4\Lambda^- \\
S^{+\,-1} &= -ip_4 + \mu - |\vec{p}| + \Sigma^+ \\
&= -ip_4\left(1 - \frac{\mathrm{Im}\Sigma^+}{p_4}\right) + \mu - |\vec{p}| + \mathrm{Re}\Sigma^+
\end{aligned}
$$

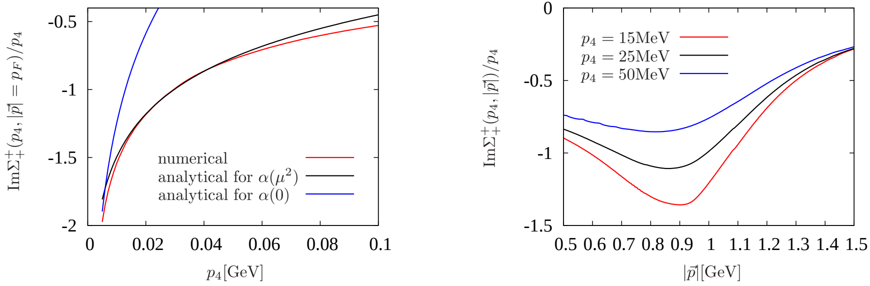
Static magnetic gluon exchange renders unbroken phase a non-Fermi liquid. leading non-analytic contribution:

$$
\begin{aligned}
\frac{\mathrm{Im}\Sigma^+}{p_4} &\simeq -\frac{4}{3\pi}\int_{k>|p-p_F|} dk \frac{\alpha_s(k^2)k}{k^2 + \frac{\pi}{2}m(k^2)^2|p_4|} \\
&\underset{p=p_F}{\longrightarrow} \frac{4}{9}\frac{\alpha_s({\Lambda_1}^2)}{\pi}\ln\left(\frac{|p_4|}{\Lambda_2}\right)
\end{aligned}
$$

$\Lambda_1 \propto |p_4|^{\frac{1}{3}}$ dominating scale of contributions

The unbroken phase - II

for coupling from DSE studies and $\mu = 1\mathrm{GeV}$:



- logarithmic divergence constrained to small energies (temperatures)
- sensitive to infrared behavior of strong coupling
- dependence on $\vec{p}$ included

Outline

Nambu-Gor'kov formalism

In Nambu-Gor'kov space with bispinors $\Psi = \begin{pmatrix} \psi \\ \psi^c \end{pmatrix}$, the quark DSE

$$\mathcal{S}^{-1} = Z_2 \mathcal{S}_0^{-1} + Z_{1F} \Sigma$$

includes the gap-equation

$$\Sigma = \begin{pmatrix} \Sigma^+ & \Phi^- \\ \Phi^+ & \Sigma^- \end{pmatrix} = -\int \frac{d^4q}{(2\pi)^4} \Gamma^{\mu}_{0\,a} \mathcal{S}(q) \Gamma^{\nu}_{b}(p,q) D^{ab}_{\mu\nu}(p-q).$$

$\rightarrow$ room for diquark condensation (through finite Φ's)

$\rightarrow$ apply same truncations as in the unbroken case

Dirac and Color-Flavor structure in chiral limit

Dirac structure

T- and χ- symmetric, even-parity and color-flavor symmetric

(R. Pisarski, D. Rischke, 1999)

$$\begin{aligned}\Sigma_i &= -i\,\vec{p}\cdot\vec{\gamma}\,\Sigma_{A,i} - i\,\omega_p\gamma_4\,\Sigma_{C,i} &&= \gamma_4\left(\Sigma_i^+\Lambda^+ + \Sigma_i^-\Lambda^-\right)\\ \phi_i &= \left(\phi_{C,i} + \gamma_4\,\hat{p}\cdot\vec{\gamma}\,\phi_{A,i}\right)\gamma_5 &&= \gamma_5\left(\phi_i^+\Lambda^+ + \phi_i^-\Lambda^-\right)\end{aligned}$$

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color-flavor structure of gap functions (similar for Σ^+)

2SC with 3 flavors: $$\Phi^+ = \frac{Z_2}{Z_{1F}}\,\phi_{2SC}\,\lambda_2\otimes\tau_2$$

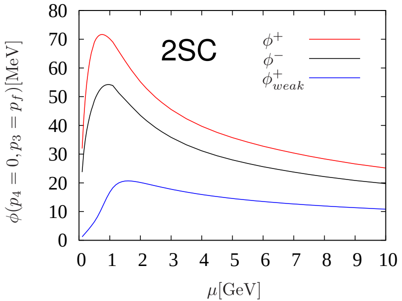
CFL: $$\Phi^+ = \frac{Z_2}{Z_{1F}}\left(\phi_{\bar{3}}\sum_A \lambda_A\otimes\tau_A + \phi_6\sum_S \lambda_S\otimes\tau_S\right)$$

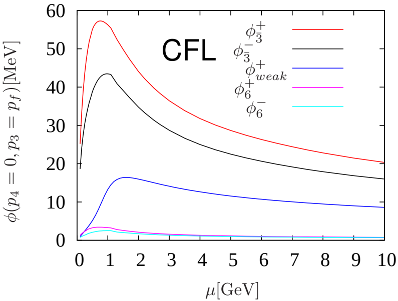
Gap-functions on the Fermi surface - I

analytical result in weak coupling (Q. Wang, D. Rischke, 2001):

$$\phi^+_{weak} = 512\,\pi^4 \left(\frac{2}{N_f g^2}\right)^{\frac{5}{2}} e^{-\frac{\pi^2+4}{8}}\,\mu\, e^{-\frac{3\pi^2}{\sqrt{2}g}} \times \begin{cases} 1 & \text{2SC} \\ 2^{-1/3} & \text{CFL} \end{cases}$$

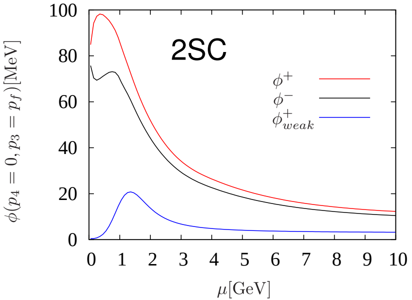
. . . with α_s from DSE studies:

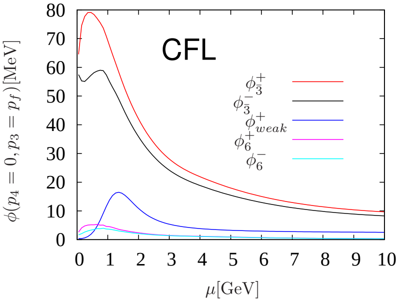




Gap-functions on the Fermi surface - II

...with α_s from lattice QCD studies:

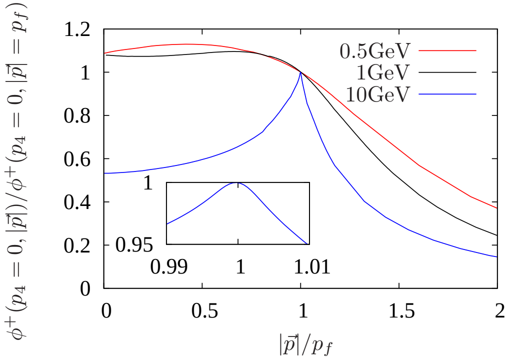




- huge deviations from extrapolated weak coupling result!
  $\phi^+_{2SC}(\mu \approx 400\text{MeV}) > 60\text{MeV}!$
- ratio of $\phi^+_{\bar{3}}$ to ϕ^+_{2SC} similar to weak coupling result
- large values for anti-quasiparticle pairing

Momentum dependence of gap-functions

relative dependence of ϕ^+_{2SC} on $|\vec{p}|$ on Fermi surface ($p_4 = 0$):



- weak coupling regime: gap function concentrated around Fermi momentum (colinear scattering dominates)
- strong coupling regime: no scale separation ($\Lambda_{QCD} \approx p_F$)

Spectral functions via Maximum Entropy Method

dispersion relation for massless fermions:

$$S^{+}(|\vec{p}|, p_4) = -\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\rho(|\vec{p}|, \omega)}{-ip_4 + \mu - \omega}$$

Fredholm equation (1st kind):
$\rightarrow$ need positivity of ρ for inversion
$\rightarrow$ MEM

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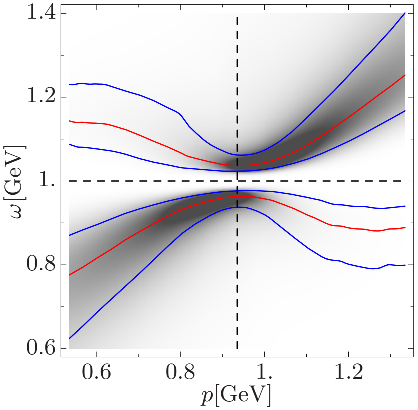
for gapped 2SC at $\mu = 1\mathrm{GeV}$:



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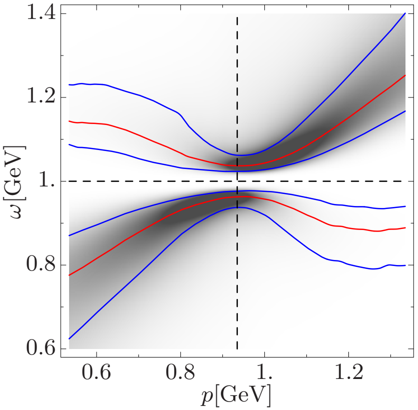

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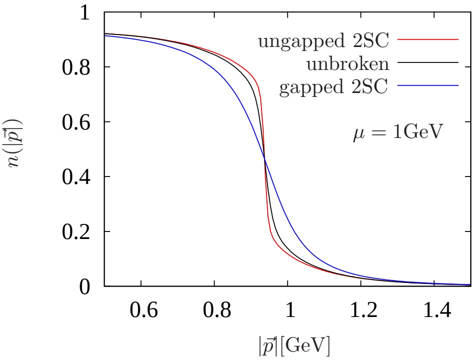
$\rightarrow$ need positivity of ρ for inversion
$\rightarrow$ MEM

- access to dispersion relation and width of quasiparticles
- particle/holes vanishing below/above Fermi momenta
- small group velocity ($\approx .5c$)

Occupation number and number densities

density given by

$$\rho = \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left(Z_2\gamma_4 S^+(p)\right) = \sum_i \frac{g_i}{(2\pi)^3} \int d^3p \; n_i(p) =: \sum_i \frac{g_i}{6\pi^2} p_i^3$$



- unbroken phase non-Fermi liquid
- Luttinger's theorem not valid, i.e. $p_i \neq p_F$

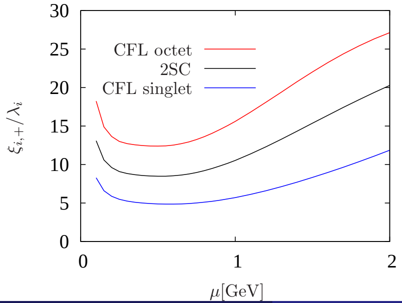
Coherence lengths

extract coherence length from diquark correlator

$$\langle\langle \psi(x)^T C M_i \psi(y) \rangle\rangle = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} M_i \sum_{e=\pm} T^+_{i,e}(p) \Lambda^e_{\vec{p}}$$

by

$$\xi^2_{i,e} = \frac{\int d^4p \, |\nabla_{\vec{p}} T^+_{i,e}(p)|^2}{\int d^4p \, |T^+_{i,e}(p)|^2}$$



- $\xi_+/\lambda \sim O(10)$
  $\rightarrow$ BEC-BCS transition?
- even smaller with coupling from lattice studies

UV-behavior of gap-functions

For $\mu, \phi, \Lambda_{QCD} \ll p$ the gap-equation takes the form

$$\sum_i \phi^+_{C,i}(p) M_i \simeq -\frac{3}{16\pi} \sum_i \lambda_a^T M_i \lambda_a \left(\frac{\alpha_s(p^2)}{p^2} \int^{p^2} dq^2\, \phi^+_{C,i}(q) + \int_{p^2} dq^2\, \frac{\alpha_s(q^2)\phi^+_{C,i}(q)}{q^2} \right)$$

with regular asymptotic form

$$\phi^+_{C,i}(p) \propto \frac{1}{p^2} \left(\ln\left(\frac{p^2}{\Lambda^2} \right) \right)^{\gamma_\phi - 1}$$

and

$$\gamma_\phi = \begin{cases} \frac{\gamma_m}{2} & \text{attractive channel} \\ -\frac{\gamma_m}{4} & \text{repulsive channel} \end{cases},$$

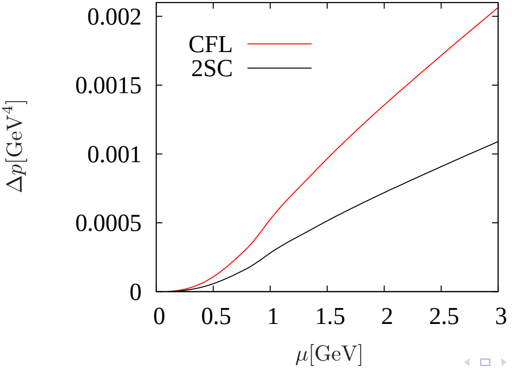
where $\gamma_m = 12/(33 - 2N_f)$.

Which is the preferred phase?

effective potential, i.e. the negative pressure, via "Cornwall-Jackiw-Tomboulis" formalism:

$$\mathcal{A}[\mathcal{S}] \quad = \quad -\frac{1}{2}\mathrm{Tr}_{p,D,c,f,N}\mathrm{Ln}\mathcal{S}^{-1} + \frac{1}{4}\mathrm{Tr}_{p,D,c,f,N}\left(1 - \mathcal{S}_0^{-1}\mathcal{S}\right)$$

pressure difference between 2SC and CFL to unbroken phase:



Outline

Dirac and Color-Flavor structure with massive strange quarks

Dirac struture

self-energy and gap-function in T- symmetric, even-parity and color-flavor symmetric phase parametized by (R. Pisarski, D. Rischke, 1999):

$$\begin{aligned}\Sigma_i &= -i\vec{p}\cdot\vec{\gamma}\,\Sigma_{A,i} - i\gamma_4\,(p_4 + i\mu)\,\Sigma_{C,i} + \Sigma_{B,i} + \gamma_4\,\vec{p}\cdot\vec{\gamma}\,\Sigma_{D,i} \\ \phi_i &= (\gamma_4\,\hat{p}\cdot\vec{\gamma}\,\phi_A + \gamma_4\,\phi_B + \phi_C + \hat{p}\cdot\vec{\gamma}\,\phi_D)\,\gamma_5\end{aligned}$$

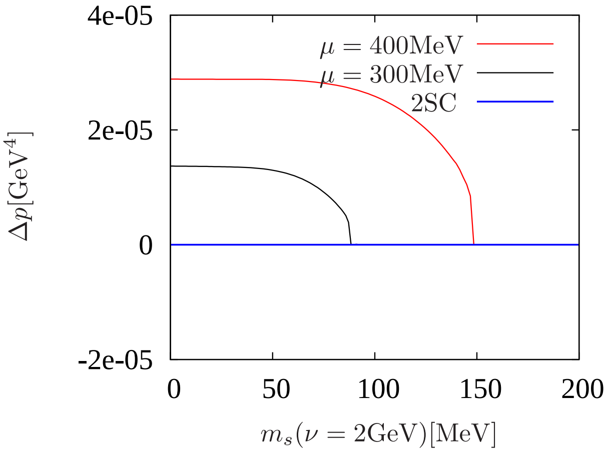
$\rightarrow$ selfconsistent, dynamical treatment of mass function!

$\rightarrow$ is there a simple physical interpretation for this functions?

color-flavor struture

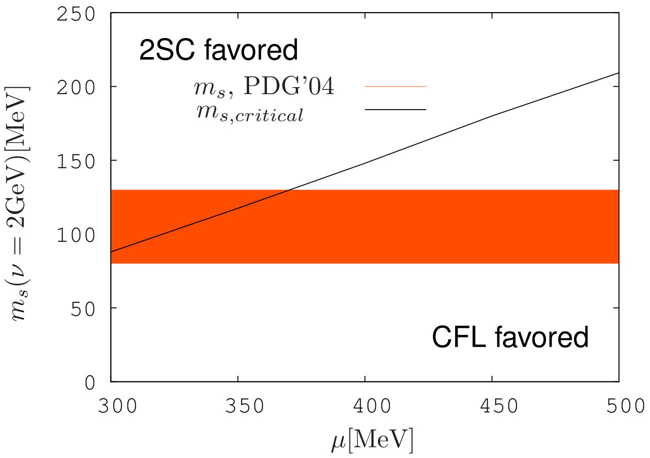
extend ansatz from (M. Alford, J. Berges, K. Rajagopal, 1999) to become selfconsistent

Dependence of pressure difference on m_s



- first order phase transition at critical strange quark mass

Critical strange quark mass



- light quark screen interaction also in strange quark sector
  → only small dynamical chiral symmetry breaking (different to NJL)!!!
  → 2SC never favored?

Outline

Summary and outlook

Summary

- selfconsistent solution of DSE by approximating gluon propagator, incorporating medium effects
- access to exitation spectrum via MEM
- huge deviations from extrapolated weak coupling results
- 2SC phase for physical strange quark mass?

Outlook

- spin-1 phases
- temperature
- neutrality
- incorporate Meissner effect
  (quasiparticle RPA, fully selfconsistent incl. of Yang-Mills sector)

Color-Flavor structure with massive strange quarks

Extend ansatz from (M. Alford, J. Berges, K. Rajagopal, 1999):

$$\Sigma = \begin{pmatrix} b+e & b & c1 & & & & & & \\ b & b+e & c1 & & & & & & \\ c2 & c2 & d & & & & & & \\ & & & e & & & & & \\ & & & & e & & & & \\ & & & & & f & & & \\ & & & & & & g & & \\ & & & & & & & f & \\ & & & & & & & & g \end{pmatrix}, \Phi = \begin{pmatrix} b+e & b & c1 & & & & & & \\ b & b+e & c1 & & & & & & \\ c2 & c2 & d & & & & & & \\ & & & & e & & & & \\ & & & e & & & & & \\ & & & & & & f1 & & \\ & & & & & f2 & & & \\ & & & & & & & & f1 \\ & & & & & & & f2 & \end{pmatrix}$$

where rows and columns correspond to

$$(color, flavor) = (r,u),(g,d),(b,s),(r,d),(g,u),(r,s),(b,u),(g,s),(b,d).$$