Understanding magnetic instability in gapless superconductors:
loss of phase coherence driven by mismatch

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Warning: it is still very preliminary, and might be nonsense (hope not totally).

Based on hep-ph/0509177, not a formal paper.

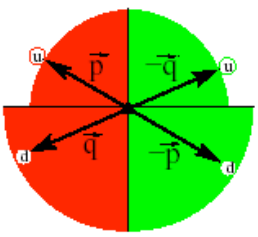
Many thanks to Prof. T. Hatsuda, Prof. Z. Tesanovic and Prof. Z.Y. Weng for discussion !!!

Key point: phase fluctuation plays the role of quantum disordering the superconducting phase when mismatch is large.

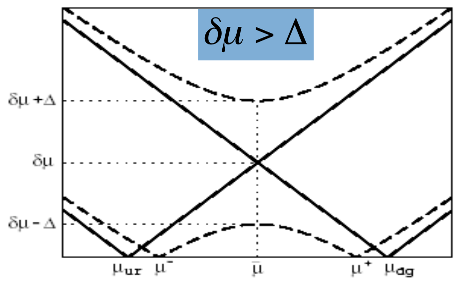
Content

I. A brief review on instabilities in gapless SCs

Pairing with mismatch



Gapless quasi-particles



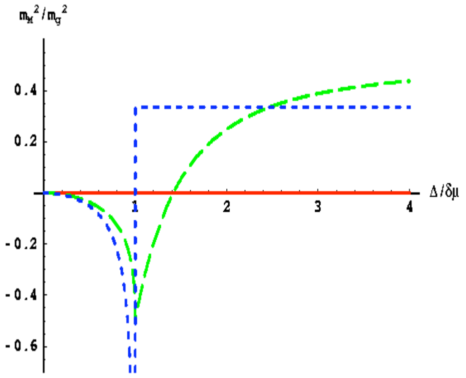
g2SC: Shovkovy, MH, PLB564:205,2003

gCFL: Alford, Kouvaris, Rajagopal, PRL92:222001,2004 — **Rajagopal's talk**

BP: Liu, Wilczek, PRL90:047002,2003; Gubankova, Liu, Wilczek, PRL91:032001,2003 — **Liu's talk**

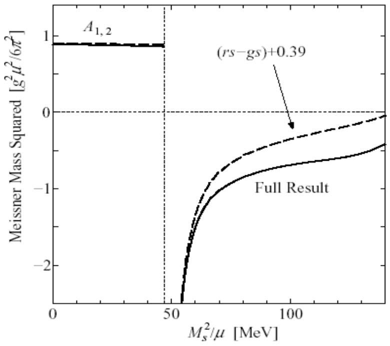
(Chromo)Magnetic instability

g2SC



MH, I.Shovkovy,
PRD70:051501,2004; 094030,2004

gCFL



Casalbuoni, et.al., PLB605:362-368,2005
Alford, Wang, J.Phys.G31:719-738,2005
K. Fukushima, hep-ph/0506080

BP: superfluid density is negative **Wu, Yip,** PRA67: 053603, 2003

Resolving magnetic instability

LOFF: $$< \bar{\psi}(\vec{r})\gamma_5\lambda_2\tau_2\psi_C(\vec{r}) >= \Phi e^{2i\vec{q}\cdot\vec{r}}$$ **(2SC)**

Giannakis, Ren, PLB611:137-146,2005; NPB723:255-280,2005

$$< \psi_{i\alpha}\, C\, \gamma_5\, \psi_{\beta j} >= \sum_{I=1}^{3} \Delta_I(\mathrm{r})\, \epsilon^{\alpha\beta I}\, \epsilon_{ijI}$$ **(CFL)**

Casalbuoni, Gatto, Ippolito, Nardulli, Ruggieri, hep-ph/0507247

Nardulli's talk

More discussion on LOFF, see Yang's talk.

Many ways go to the LOFF-like state

1. **Baryon current:** MH, hep-ph/0504235
2. **Goldstone current:** Hong's talk, or hep-ph/0506097
   Kryjevski, hep-ph/0508180; Schaefer's talk, or hep-ph/0508190
3. **Gluon condensate:** Gorbar, Hashimoto, Miransky, hep-ph/0507303
4. **More ...**

1 and 2 offer a Doppler shift superfluid velocity for the quasi-particles.

What's really happening? → **Rischke's remarks**
→ **phase decoherence (this talk)**

Previous treatment for g2SC, gCFL and BP:

At fixed mismatch, looking for the possibility of BCS Cooper pairing. This is a story of balance between energy gain and loss, e.g., Liu's talk.

Another way of thinking:

Starting from conventional BCS state without mismatch, asking how this BCS state will be eventually destroyed by increasing mismatch.

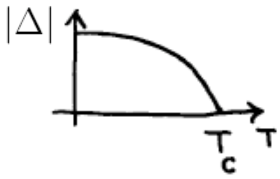
II. How a superconductor will be destroyed?

Firstly, what is a superconductor?

$$\Delta(x) = |\Delta| e^{i\varphi(x)}$$

magnitude — phase

Thermodynamic variable



Dynamic variable

$$\frac{1}{2}\rho_s(\nabla\varphi - e\vec{A})^2$$

$\downarrow$

stiffness

Two energy scales

V.J. Emery, S.A. Kivelson,
Nature 374(1995), 434

$|\Delta| \rightarrow E^{BCS}$ — **The energy scale for pairing established**

$\rho_s \rightarrow E^{phase}$ — **The energy scale for phase coherence established**

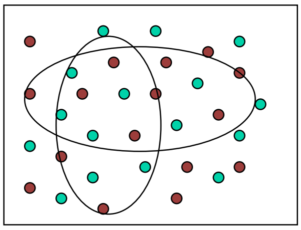
The system is governed by the lower energy scale

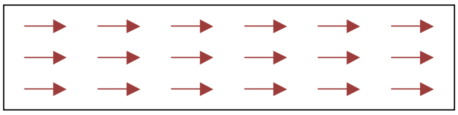
$$\mathbf{E} = min\{E^{BCS}, E^{phase}\}$$

e.g. if stiffness is small(soft) but gap magnitude is large, the system is governed by phase fluctuation.

BCS Superconducting phase: $<\Delta>\neq 0$

$$E^{BCS} < E^{phase}$$





Strongly coherent, ordered, rigid (large superfluid density), phase fluctuation is absent, BCS MF good

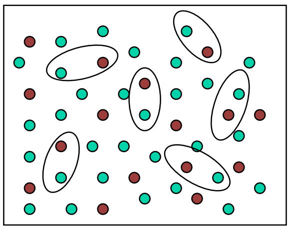
Pseudogap phase: long-range phase decoherence

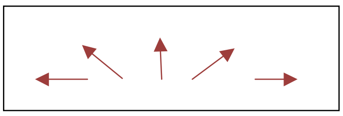
$$|\Delta| \neq 0, <\Delta>=|\Delta|<e^{i\varphi(x)}>=0$$

$$E^{BCS} > E^{phase}$$

"Doping" green fermions

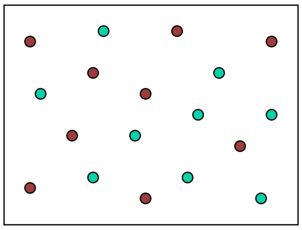
$n_g > n_r$





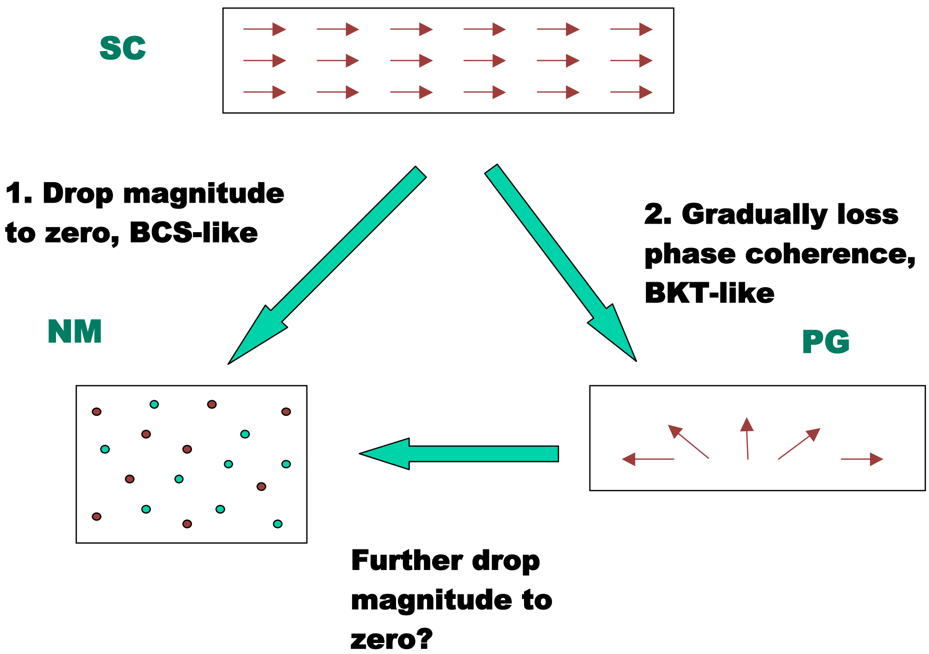
Loss of long-range phase coherence, quantum disordered, BCS MF cannot describe strong phase fluctuation

Normal phase: $|\Delta| = 0$



No order at all, very soft (zero superfluid density)

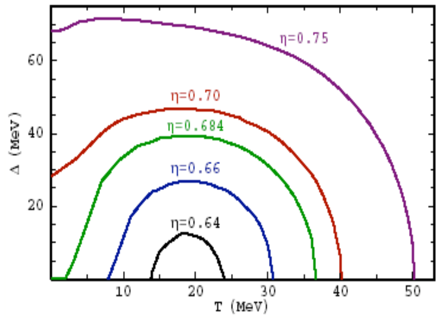
How a superconductor can be destroyed?



How a superconductor will be destroyed by mismatch?

1. Definitely non-BCS-like

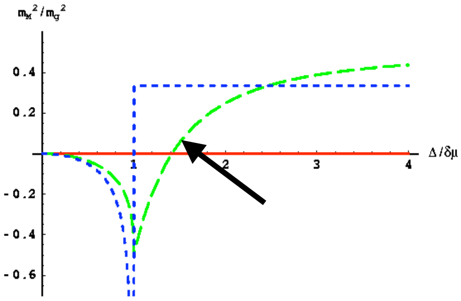
MH, I. Shovkovy,
Nucl.Phys.A729:835,2003



2. Possibly BKT-like

Berezinskii-Kosterlitz-Thouless

Mismatch increases,
superfluid density decreases,
phase fluctuation becomes
more important



III. The role of phase fluctuation

1. BCS at mean-field approximation

The minimal model for gapless phase

$$SU(2)_c \times U(1)_{EM} \times SU(2)_f$$

Hong, hep-ph/0506097

$$\mathcal{L}_q = \bar{q}\,(i\not{D} + \hat{\mu}\gamma_0)\,q + G_\Delta[\,(\bar{q}^C i\varepsilon\epsilon\gamma_5 q)(\bar{q} i\varepsilon\epsilon\gamma_5 q^C)\,]$$

$$\mu_u = \bar{\mu} - \delta\mu, \mu_d = \bar{\mu} + \delta\mu$$

$$D_\mu \equiv \partial_\mu - ieA_\mu$$

$q \in u, d$ **Original fermions**

Introducing auxiliary field, bosonization:

$$\mathcal{L}_q^b = \bar{q}(i\not{D}+\hat{\mu}\gamma^0)q - \frac{1}{2}\Delta[i\bar{q}\varepsilon\epsilon\gamma_5 q^C] - \frac{1}{2}[i\bar{q}^C\varepsilon\epsilon\gamma_5 q]\Delta^* - \frac{|\Delta|^2}{4G_\Delta}$$

$$\boxed{\Delta = |\Delta| e^{i\varphi}}$$

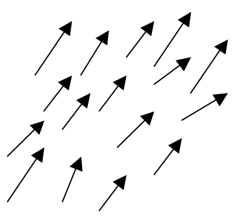
BCS MF: neglecting phase fluctuation $\Delta = |\Delta|$

Comments on BCS MF

1. **It is fine with small mismatch when the system is rigid;**
2. **It is not a good approximation for large mismatch when the system is "soft";**
3. **In all the papers regarding the instability in gapless or BP phases, phase fluctuation has been totally neglected.**

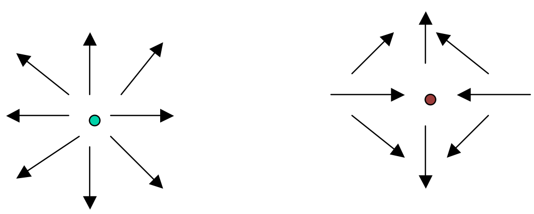
2. Formulating the role of phase fluctuation

A. Longitudinal phase fluctuation

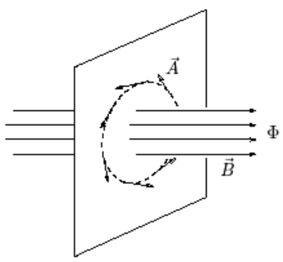


$$\nabla\times\nabla\varphi = 0$$

B. Transverse phase fluctuation



$$\nabla\times\nabla\varphi = 2\pi\hat{z}\delta(\vec{r}) \qquad \nabla\times\nabla\varphi = -2\pi\hat{z}\delta(\vec{r})$$



Topological defects in the phase order: Abrikosov-Nielsen-Olesen vortex(2D) / string(3D)

In order to couple the phase fluctuation to quasi-particles, one has to isolate the uncertain charge carried by q, similar to the "charge-spin separation" in HTSC.

We use: Franz-Tesanovic (FT) singular gauge transformation

M.Franz, Z. Tesanovic, Phys.Rev.Lett.87, 257003(2001); M.Franz, Z. Tesanovic, O.Vafek, Phys.Rev.**B66**(2002), 054535; A. Melikyan, Z. Tesanovic, cond-mat/0408344.

$$\bar{\psi}_u = e^{i\varphi_u}\bar{q}_u,$$
$$\bar{\psi}_d = e^{i\varphi_d}\bar{q}_d,$$
$$\varphi_u + \varphi_d = \varphi$$

ψ A new set of charge neutral quasi-particles

$$\nabla \times \nabla \varphi_{u(d)} = 2\pi \hat{z} \sum_i Q_i \delta(\vec{r} - \vec{r}_i^{u(d)})$$

Two emergent gauge fields:

$$v_\mu = \frac{1}{2}\partial_\mu \varphi, \quad \varphi_u + \varphi_d = \varphi$$
$$a_\mu = \frac{1}{2}(\partial_\mu \varphi_u - \partial_\mu \varphi_d)$$

v_μ Doppler gauge field / superflow field, couples with charges

a_μ Berry gauge field / Topological gauge field, couples with charge neutral quasiparticles (isospin)

Massive in SC, massless in PG state

$$\mathcal{L}_\psi = \mathcal{L}_\psi^{qp} + \mathcal{L}_0^{a,v},$$

$$\mathcal{L}_\psi^{qp} = \bar{\psi}\left(i\tilde{\not{D}} + \hat{\mu}\gamma_0\right)\psi - \frac{1}{2}|\Delta|[i\bar{\psi}\varepsilon\epsilon\gamma_5\psi^C] + c.c. - \frac{|\Delta|^2}{4G_\Delta}$$

$$\tilde{D} = (\partial_\mu + i2a_\mu) + i(v_\mu - eA_\mu)$$

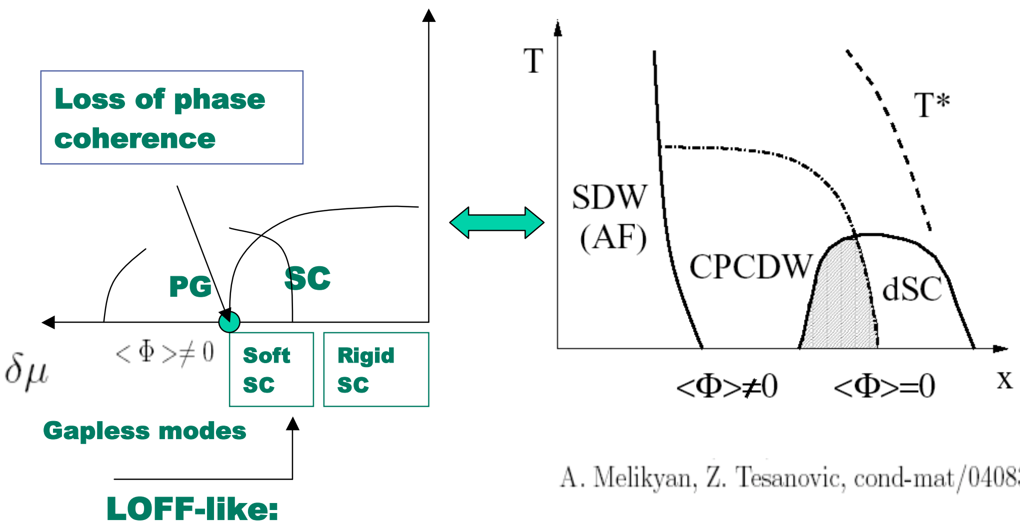
Contribution of gauge fields:

$$\mathcal{L}_0 \to \frac{K_\mu}{2}(\partial \times v)_\mu^2 + \frac{K_\mu}{2}(\partial \times a)_\mu^2$$

Another way, introducing dual disorder field:

$$\mathcal{L}_0[v,a] \to \frac{1}{4\pi^2|\Phi|^2}(\partial \times v)^2 + \frac{1}{4\pi^2|\Phi|^2}(\partial \times a)^2$$

IV. Expected phase diagram



A. Melikyan, Z. Tesanovic, cond-mat/0408344.

V. Conclusion and discussion

1. **When mismatch is large, the system cannot be described very well using BCS at MF;**
2. **With the increase of mismatch, the phase fluctuation plays more and more important role, it softens the superconducting phase.**
3. **At some critical mismatch, the strong phase fluctuation destroys the long-range phase coherence, turns the system to a phase decoherent pseudogap phase, while the gap amplitude is still finite.**
4. **Further increase of mismatch will drive the gap amplitude to zero (noraml phase) or other possible pairing state (spin-1).**

1. **The existence of the PG state is dependent on the assumption that at large mismatch, the amplitude fluctuation is not as important as phase fluctuation.**
2. **The topological defects might be different for different systems (g2SC,gCFL,BP).**
3. **Further studies are needed on the topological excitation.**

Open for any criticism,
comments, and suggestions !!!