RG Analysis and Magnetic Instability in Gapless Superfluid

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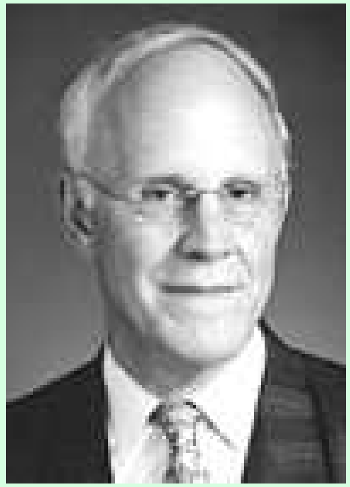
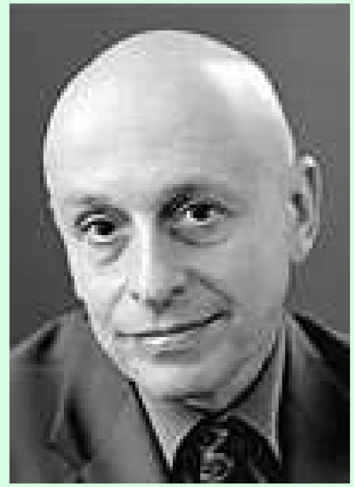
I. Introduction

- Asymptotic freedom of QCD (Gross-Politzer-Wilczek 73)

Nobel prize 04

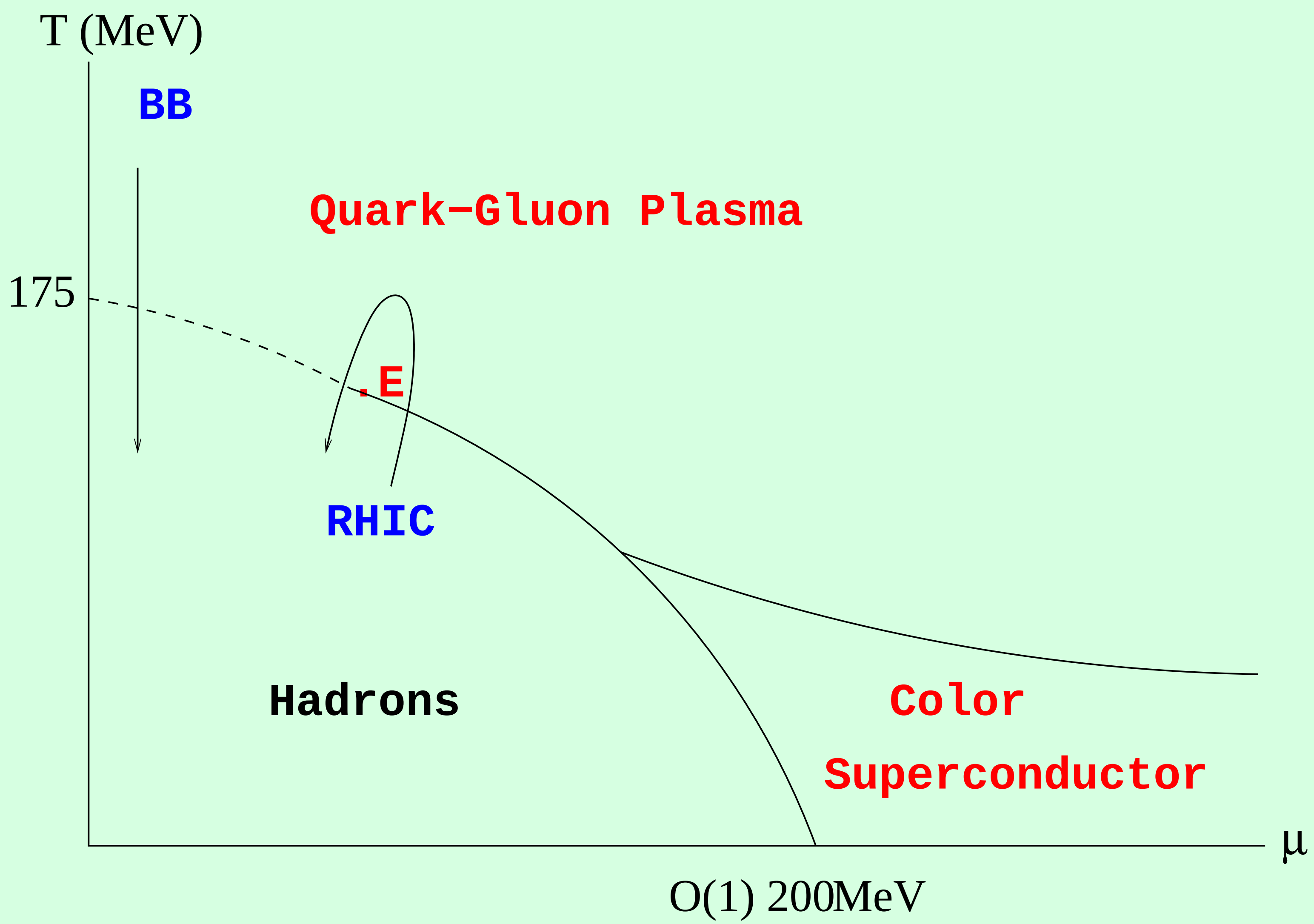





- Matter at extreme density is quark matter. (Collins-Perry 75)

- The ground state is a color superconductor. (Barrois 77)

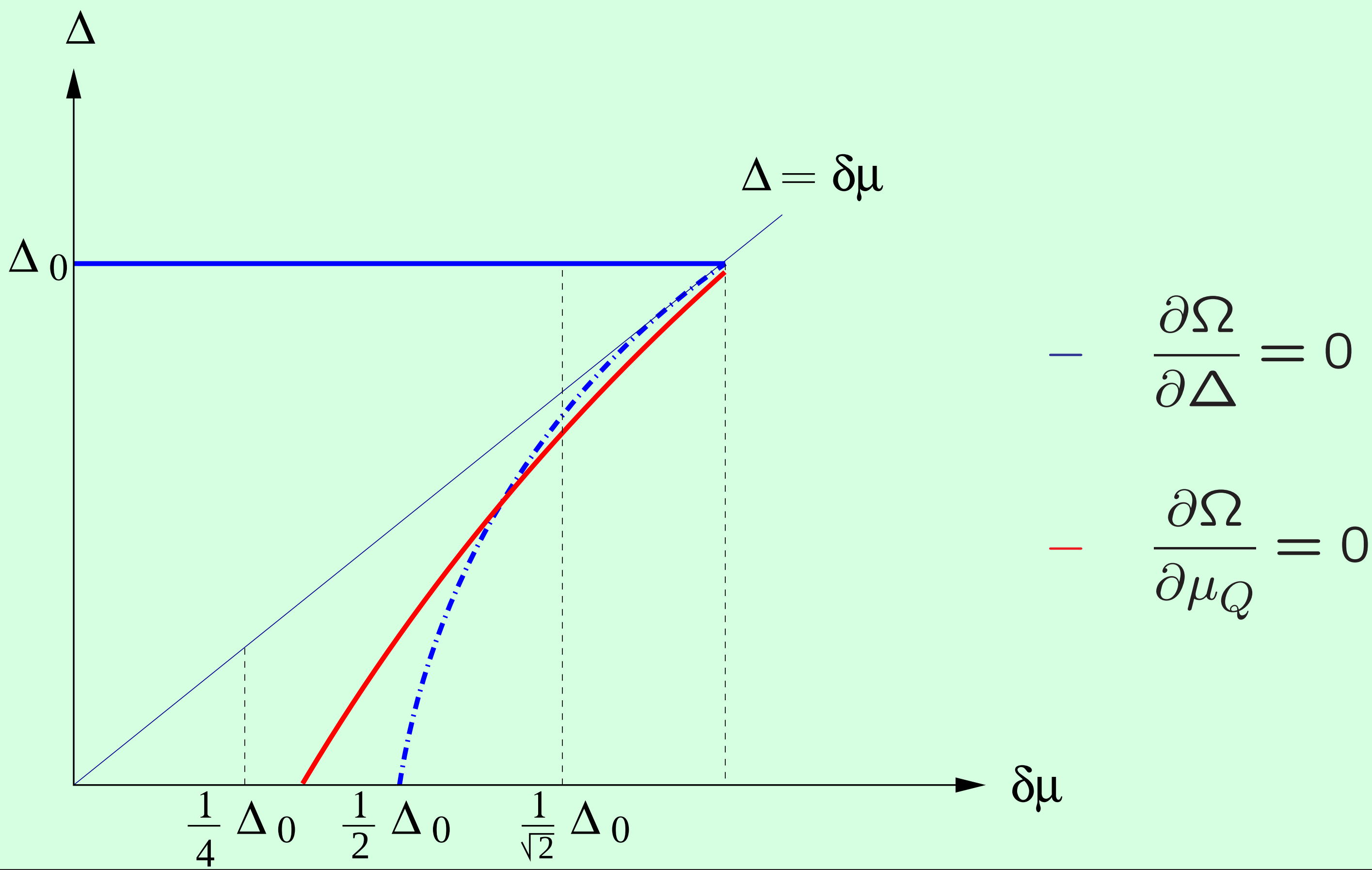
$$3 \otimes 3 \ni \bar{3} \longrightarrow \langle q_a(\vec{p}_F)\, q_b(-\vec{p}_F)\rangle = \epsilon_{ab3}\,\Delta \qquad (1)$$

QCD predicts phase transitions.



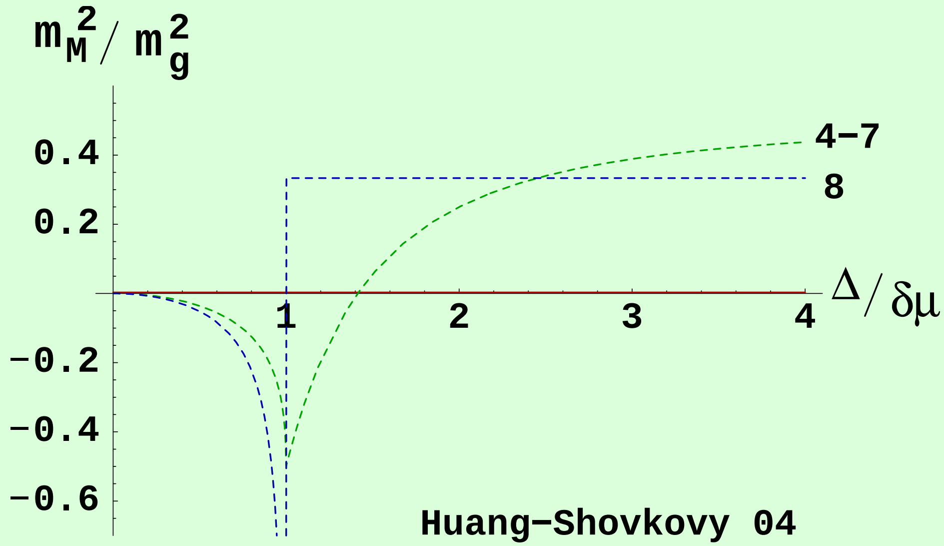
- Various phases of CSC due to stress (m_i, electroweak interaction) $\delta\mu \neq 0$

- Charge neutrality stabilizes the Sarma Phase ($\Delta < \delta\mu$): Huang-Shovkovy 03. (Cf. Bedaque-Caldas-Rupak 03: Forbes-Gubankova-Liu-Wilczek 05)



II. Meissner Mass and Instability

- Magnetic instability (Huang+Shovkovy 04, Wu+Yip, Casalbouni et al.):



$$m_M^2 = \frac{1}{3}\delta^{ij}\left.\frac{\partial^2 V(A)}{\partial A_i \partial A_j}\right|_{A=0} < 0.$$

- Effective potential

$$V(A) = V_0 - \frac{1}{2}m_D^2\, A_0^2 + \frac{1}{2}m_M^2\, \vec{A}^{\,2} + \cdots . \tag{2}$$

- Two-point correlation functions (HDET):

1. Free fermions, $V^\mu = (1, \vec{v}_F)$:

$$\Pi_{\mu\nu}(p) = -g^2 \frac{p_F^2}{\pi} \sum_{\vec{v}_F} V_\mu V_\nu \left[1 - \frac{p_0 + \vec{v}_F \cdot \vec{p}}{p_0 - \vec{v}_F \cdot \vec{p}}\right] + \delta\Pi_{\mu\nu} \qquad (3)$$

$$\Pi_{00}(0) = {m_D}^2, \quad \Pi_{ij}(0) = 0 \qquad (4)$$

2. BCS superconductor

$$\Pi_{ij}(0) = \frac{1}{3}\delta_{ij} \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{4\Delta^2}{\left[l_4^2 + \left(\vec{v}_F \cdot \vec{l}\right)^2 + \Delta^2\right]^2} = \frac{1}{3}\delta_{ij}\, m_D^2 \qquad (5)$$

3. Gapless superfluid (Goldstone field, φ)

$$\Pi_{\mu\nu}(p) = \tilde{\Pi}_{\mu\nu}(p_0 + i\partial_t\varphi, \vec{p} - i\vec{\nabla}\varphi) \qquad (6)$$

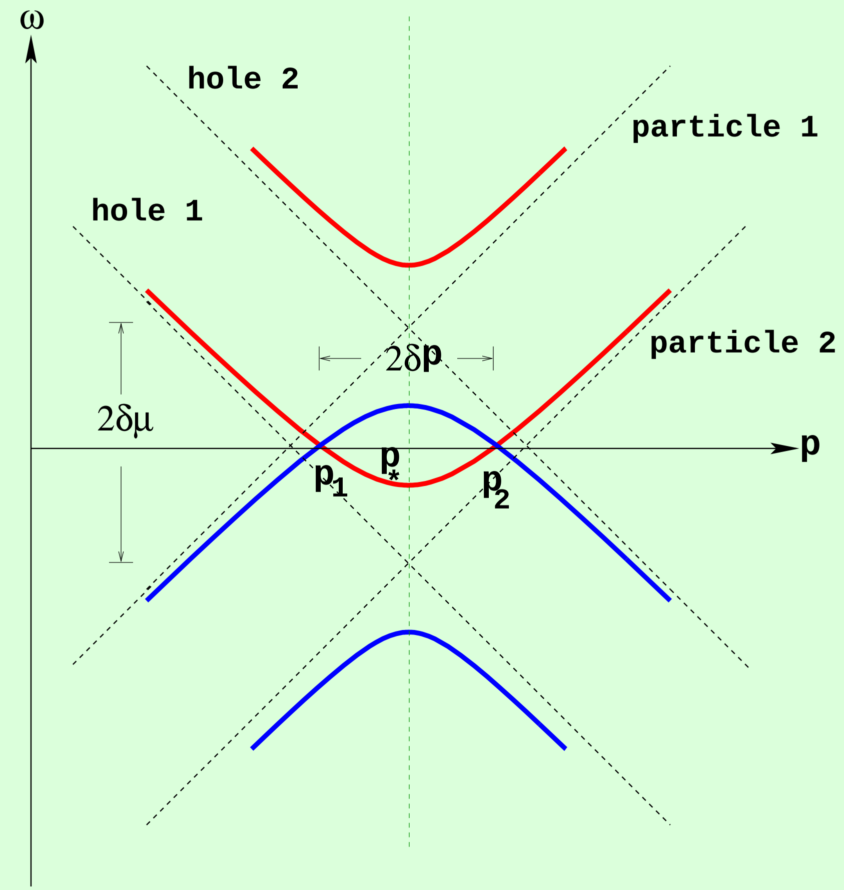
- Energy dispersion relation:

$$\omega(\vec{p}) = \pm\left(\delta\mu \pm \sqrt{\epsilon^2(\vec{p}) + \Delta^2}\right)$$

- Gapless modes near the Fermi surface:

$$\omega(\vec{p}) \simeq \begin{cases} \eta\, \vec{v}_i \cdot \vec{l}, & \text{if } \left|\vec{v}_i \cdot \vec{l}\right| < \delta p \\ \frac{(\vec{v}_i \cdot \vec{l})^2}{2\delta\mu}, & \text{if } \delta p < \left|\vec{v}_i \cdot \vec{l}\right| < \delta\mu \\ \vec{v}_i \cdot \vec{l}, & \text{if } \bar{\mu} \gg \left|\vec{v}_i \cdot \vec{l}\right| > \delta\mu \end{cases}$$

$$(\delta p = \sqrt{\delta\mu^2 - \Delta^2},\ \eta = \delta\mu/\delta p)$$



- When $\delta\mu \approx \Delta$ or $\delta p \approx 0$, the gapless modes are quadratic all the way down to the Fermi surface.

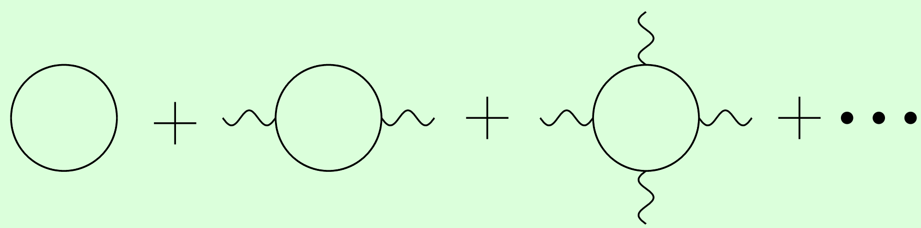
- The instability is due to the IR divergences by gapless modes.

- Longitudinal mode suffers IR divergences = Magnetic Instability

- One-loop Coleman-Weinberg effective potential

$$V(A) = \frac{1}{2} \mathrm{N_g} \,\mathrm{Tr}\, \ln \left[p_4^2 + \left(\delta\bar{\mu} - \sqrt{\bar{\epsilon}_1 \bar{\epsilon}_2 + \Delta^2} \right)^2 \right] + [- \to +] \qquad (7)$$

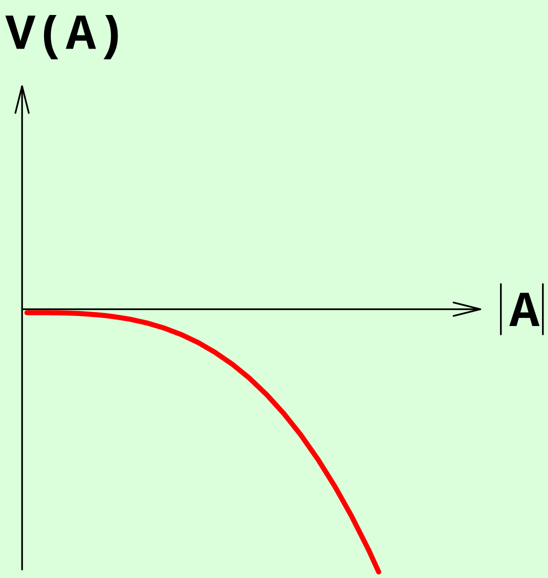
where $\delta\bar{\mu} = \delta\mu - \delta q\, A_0$ and $\bar{\epsilon}_i = E(\vec{p} + q_i \vec{A}) - \bar{\mu} - \bar{q}\, A_0$.

$V(A) =$ 

- When $\delta\mu \approx \Delta$, the gapless modes have quadratic dispersion relation, which is problematic:

1. The effective potential is unbounded from below, $V_* = (1, \vec{v}_*)$:

$$V = -\frac{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{1}{4}\right)}{3\sqrt{\pi}\,2\delta\mu}\int_{\vec{v}_*}\frac{\nu_*|e|^3}{|\vec{v}_*|}\left|q_1 V_*\cdot A - \delta q A_0\right|^{\frac{3}{2}}\left|q_2 V_*\cdot A + \delta q A_0\right|^{\frac{3}{2}} + \cdots . \quad (8)$$



⋆ Under a scaling $E \mapsto s\,E$, $\vec{A} \to s^{1/2}\,\vec{A}$:

$$V(A) \sim \left|\vec{A}\right|^3 . \quad (9)$$

(cf. Liu-Wilczek-Zoler 04)

2. ABS construction of K theory on Fermi surface shows the gapless modes must be linear. (Horava 05)

III. RG Analysis and Secondary Gap

- Effective Lagrangian for quadratic gapless modes (cf. HDET):

$$\mathcal{L}_{\rm eff}=\sum_{\vec{v}_*}\Psi^\dagger(\vec{v}_*,x)\left[i\partial_t+\frac{(\vec{v}_*\cdot\vec{\nabla})^2}{2\,\delta\mu}\right]\Psi(\vec{v}_*,x)+\frac{\kappa}{2}\left(\Psi^\dagger\Psi\right)^2+\cdots \quad (10)$$

- Under the scaling $E\mapsto s\,E$, the action $S=\int \mathrm{d}t\mathrm{d}^3\mathrm{l}\,\mathcal{L}_{\rm eff}$ is invariant:

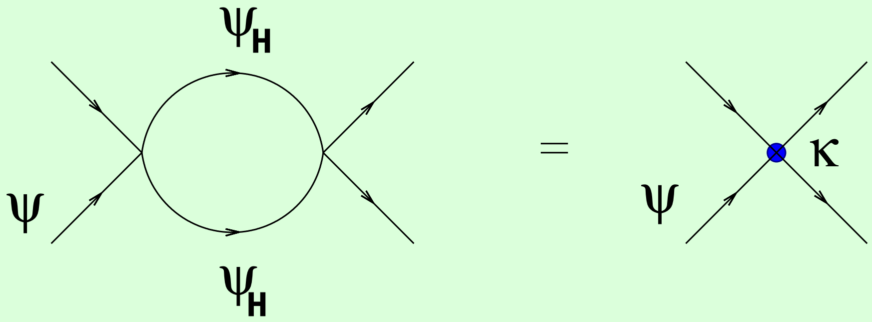
$$\vec{l}_{||}\mapsto s^{1/2}\,\vec{l}_{||},\quad \psi_{\vec{v}_*}(t,\vec{l}\,)\mapsto s^{-1/4}\psi_{\vec{v}_*}(t,\vec{l}\,) \quad (11)$$

- The four Fermi-interaction is relevant for incoming fermions with opposite momenta:

$$\kappa\mapsto s^{-1/2}\,\kappa \quad (12)$$

(Cf. If not opposite, the interaction is marginal.)

- At tree level the interaction between the gapless modes and the gapped modes (Ψ_H) are repulsive. However the repulsive interaction induces attraction among the gapless modes upon integrating out the gapped modes (cf. Kohn-Luttinger theorem):



$$\kappa = \frac{\pi}{4} G_s^2 \frac{\nu_*}{v_*} \qquad (13)$$

- The Cooper-pair gap eq. for the secondary gap is given as

$$\Delta_s = \kappa \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\Delta_s}{l_4^2 + \left[\frac{(\vec{l}\cdot\vec{v}_*)^2}{2\delta\mu}\right]^2 + \Delta_s^2}. \qquad (14)$$

- The secondary gap is power-suppressed in κ or $g \equiv [\ln(2\bar{\mu}/\Delta)]^{-1}$,

$$\Delta_s \simeq 6.85\,\kappa^2 \left(\frac{\nu_*}{v_*}\right)^2 \delta\mu = 4.2 \left(\frac{G_s}{G}\right)^2 g^4\,\delta\mu\,, \tag{15}$$

  (cf. Alford-Wang 05)

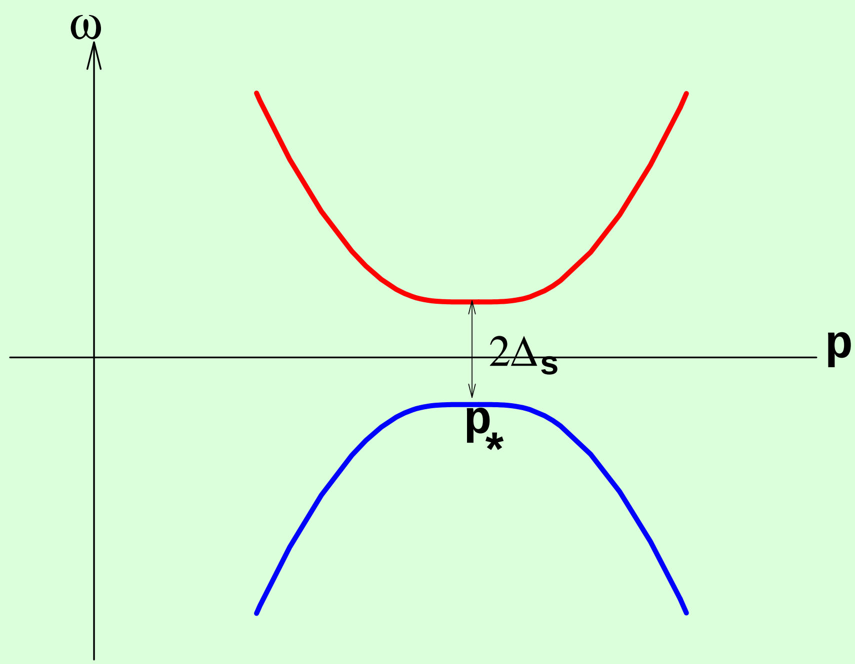
- For g2CSC, it is spin-1 and color-antitriplet pairing while for gCFL it has to be color-sextet pairing like $\langle \Psi_L C\gamma_0\gamma_5\Psi_R\rangle$. (cf. Alford et al. 02)

- Meissner mass due to the secondary gap (Hong-Sohn, to appear):

$$V(A)|_{\text{quadratic}} = \frac{N_g}{2} \mathrm{Tr} \ln \left\{ l_4^2 + \frac{1}{(2\delta\mu)^2} \left[\vec{v}_* \cdot \left(\vec{l} + \vec{A} \right) \right]^4 + \Delta_s^2 \right\} \quad (16)$$

$$\longrightarrow \quad m_M^2 \Big|_{\text{quadratic}} \simeq \left(\frac{\pi}{2} - \frac{2}{3} \right) m_M^2 \Big|_{\text{BCS}}$$

(N.B. The Meissner mass is independent of δq.)



IV. New Stable Gapless Superfluids

- For a large mismatch the secondary gap does not open, since the RG running is not long enough for

$$\Lambda^{\rm IR}_{\rm quadratic} > \Delta_s \quad \text{or} \quad \frac{1}{2\delta\mu}\left(\delta\mu^2 - \Delta^2\right) > \Delta_s\,. \tag{17}$$

  Instead, the condensate of Goldstone currents develops.

- The system is then dominated by linearly gapless modes at low energy, $\omega < (\delta\mu^2 - \Delta^2)/(2\delta\mu)$.

- Integrating out modes with $\omega \gtrsim \delta\mu$, we may write the effective potential Eq. (7) as

$$V(A) = -\frac{N_g}{2}\int_\Lambda \frac{{\rm d}^4 p_E}{(2\pi)^4}\ln\left[p_4^2 + \left(\sqrt{\bar\epsilon_1\bar\epsilon_2 + \Delta^2} - \delta\bar\mu\right)^2\right] + \\ +C_0(\Lambda) + \frac{C_2(\Lambda)}{2}\vec{A}^2 - \frac{B_2(\Lambda)}{2}{A_0}^2 + \cdots \tag{18}$$

- Integrating further out the gapless modes till they become linearly gapless, we get with $V_1^\mu = (\eta\bar{q} + \delta q, \eta\bar{q}\,\vec{v}_1)$ and $V_2^\mu = (\eta\bar{q} - \delta q, \eta\bar{q}\,\vec{v}_2)$.

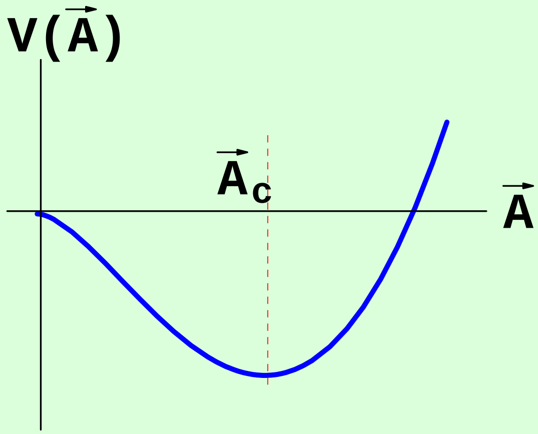
$$V(A) \;=\; -\nu_1\,\eta^{-1}\int\frac{\mathrm{d}\Omega_{\vec{v}_1}}{4\pi}(eV_1\cdot A)^2\left[\ln\frac{\Lambda^2}{(eV_1\cdot A)^2}+1\right]+(1\to 2)$$
$$+C_0(\Lambda)+\frac{C_2(\Lambda)}{2}\vec{A}^{\,2}-\frac{B_2(\Lambda)}{2}{A_0}^2+\cdots \qquad (19)$$

- Counter-terms are fixed by the renormalization conditions,

$$\left.\frac{\partial^2 V}{\partial A_0^2}\right|_{A_0=M} = -m_D^2\,, \quad \frac{1}{3}\,\delta_{ij}\left.\frac{\partial^2 V}{\partial A_i\partial A_j}\right|_{\vec{A}^{\,2}=M^2} = m_M^2\,, \qquad (20)$$

- The effective potential becomes ($\vec{A} \mapsto \vec{A} - \vec{\nabla}\varphi$)

$$V(A) = \frac{1}{2}m_M^2\vec{A}^{\,2} - \frac{\eta}{3}\left(\nu_1 v_1^2 + \nu_2 v_2^2\right) e^2\bar{q}^{\,2}\,\vec{A}^{\,2}\left[\ln\left(\frac{M^2}{\vec{A}^{\,2}}\right) + 3\right] \\ -\frac{1}{2}m_D^2 A_0^2 - \frac{1}{\eta}\left(\nu_1 + \nu_2\right)\left(\eta\bar{q} + \delta q\right)^2 e^2 {A_0}^2\left[\ln\left(\frac{M^2}{{A_0}^2}\right) + 3\right] \quad (21)$$



★ Since the linear modes are scale-invariant, it is logarithmically divergent.

- The minimum occurs at

$$\left\langle\left(\vec{A} - \vec{\nabla}\varphi\right)^2\right\rangle \simeq \delta\mu^2 \exp\left[2 - \frac{4\,\nu_*\,v_*^2}{\eta\left(\nu_1 v_1^2 + \nu_2 v_2^2\right)}\right]. \quad (22)$$

(Cf. Huang, 05)

★ This slide is added to answer some questions raised during the talk.

- The renormalization condition Eq. (20) fixes the curvature of the potential, Eq. (21), at $A = M$. Since the potential is independent of M, the Meissner mass at different scale M' is related to the one at M as

$$m_M^2(M') = m_M^2(M) - c_M^2 \ln\left(\frac{M^2}{M'^2}\right) \tag{23}$$

where $c_M^2 = \frac{2\eta}{3}\left(\nu_1 v_1^2 + \nu_2 v_2^2\right) e^2 \bar{q}^2$.

- As M' approaches zero, the log becomes large and one has to sum all the log's to get

$$m_M^2(M') = m_M^2(0) + \frac{m_M^2(M) - m_M^2(0)}{1 + b \ln\left(\frac{M^2}{M'^2}\right)} \tag{24}$$

where $b = c_M^2/[m_M^2(M) - m_M^2(0)]$.

- The ground state breaks the rotational symmetry. (Cf. FFLO state)

- The ground state, however, does not carry any net current, which agrees with the London theorem:

$$\left\langle \vec{J} \right\rangle \equiv -\left.\frac{\partial V(A)}{\partial \vec{A}}\right|_{\vec{A}=\vec{A}_c} = 0 \tag{25}$$

- Under an external field $\vec{A}$, the current is given as

$$J_i(x) = -\left.\frac{\partial^2 V(A)}{\partial A_i \partial A_j}\right|_{\vec{A}=\vec{A}_c} \left(A_j - \partial_j \varphi\right). \tag{26}$$

  Taking the curl of the current, we get

$$\nabla^2 \vec{B} = c_M^2 \left(\vec{B} - \vec{B}\cdot\hat{A}_c\,\hat{A}_c\right), \quad c_M^2 = \frac{2\eta}{3}\left(\nu_1 v_1^2 + \nu_2 v_2^2\right) e^2 \bar{q}^{\,2} \tag{27}$$

- The new state is stable and has a non-negative, but directional, Meissner mass.

V. Conclusion

- The magnetic instability of gapless superfluids is studied in terms of effective field theories.

- The instability arises due to the IR divergences associated with gapless modes.

- The unstable gapless superfluids make phase transition to

  1. a fully gapped phase, opening a secondary gap, if $\delta\mu \approx \Delta$ or $\delta\mu - \Delta < \Delta_s$.

  2. Gapless superfluids with condensed supercurrents, if $\delta\mu \gtrsim \Delta$ or $\delta\mu - \Delta > \Delta_s$.