Phase Transition in Asymmetrical Fermion Superfluids

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Motivation:

In the last few years there has been a great interest, both theoretically and experimentally, in asymmetrical fermionic systems!

Possible examples of systems exhibiting pairing between particles with different Fermi surfaces:

- Hyperfine States in a Trap
- Ultracold systems (atoms of different elements: ^{40}K or ^{6}Li - atomic traps)
- QCD (quarks of different species: up, down, etc): color superconductivity.

We have studied these asymmetrical fermion superfluids at zero T:

- Paulo Bedaque, Heron Caldas, Gautam Rupak, Phys.Rev.Lett. 91: 247002, 2003.
- Heron Caldas, Phys.Rev. A 69: 063602, 2004.

Since in real experiments the temperature is always non zero, some questions naturally arise:

- Does the particle's masses and chemical potentials asymmetry alter the phase transition between the superfluid and normal states?

- How does the critical temperature of an asymmetric system depend on its asymmetry?

Phase Transitions at Fixed Chemical Potentials

The (non-relativistic) dilute system:

Particles **a**: m_a , m_a , $P_F^a = \sqrt{2 m_a m_a}$ Spin $\uparrow$

Particles **b**: m_b , m_b , $P_F^b = \sqrt{2 m_b m_b}$ Spin $\downarrow$

? We investigate:

$$P_F^a = P_F^b \; with \; m_a \neq m_b \; and \; m_a \neq m_b$$

$$P_F^a \neq P_F^b \; pairing \; around \; P_F \equiv \sqrt{2Mm} = \sqrt{2\frac{m_a m_b}{m_a + m_b}\left(m_a + m_b\right)}$$

The model:
The system is described by the following Hamiltonian:

$$H - \sum_{i=a,b} m_i \, n_i =$$

$$\sum_k e_k^a a_k^* a_k + e_k^b b_k^* b_k - g \sum_{k,k'} a_{k'}^* b_{-k'}^* b_{-k} a_k$$

$$e_k^i = \frac{k^2}{2m_i} - m_i$$

$$-g < 0 \rightarrow attraction!$$

From *H* we obtain

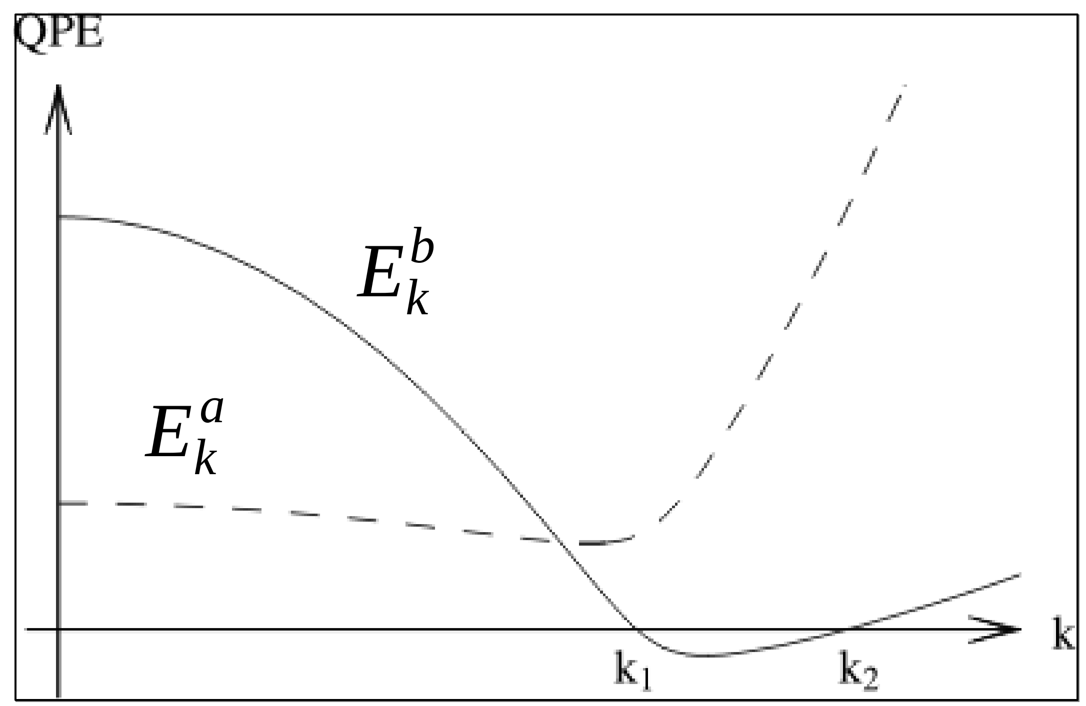
$$\Omega = \frac{\Delta^2}{g} + \sum_{\substack{K<K_1 \\ K>K_2}} \left[e_k^+ - \sqrt{{e_k^+}^2 + \Delta^2} - T\ln\left(e^{-b\,E_k^a} + 1\right) - T\ln\left(e^{-b\,E_k^b} + 1\right) \right]$$

$$+ \sum_{K_1}^{K_2} \left[e_k^b - T\ln\left(e^{-b\,|e_k^b|} + 1\right) \right]$$

$$E^k_{a,b} = \mp\,\varepsilon^k_- + \sqrt{{\varepsilon^k_+}^2 + \nabla^2}$$

$$e^- = \frac{e_k^a - e_k^b}{2} \qquad e^+ = \frac{e_k^a + e_k^b}{2}$$

Depending on the relative magnitudes of $P_F^{a,b}$ and $m_{a,b}$, $E_k^{a,b}$ (QPE) can be < 0.
We take $P_F^b > P_F^a$ and $m_b > m_a$ such that only $E_k^b < 0$ for $k_1 < k < k_2$.



Roots of E_k^b :

$$k_{1,2}^2 = \frac{1}{2} dP_F^+ \mp \frac{1}{2}\left[dP_F^{-\,2} - 16 m_a m_b \Delta^2 \right]^{1/2}$$

$$dP_F^+ \equiv P_F^{b\,2} + P_F^{a\,2} \qquad dP_F^- \equiv P_F^{b\,2} - P_F^{a\,2}$$

$$k_{1,2}\ is\ real\ \ if\ \ \Delta \le \Delta_c = \frac{\left| dP_F^- \right|}{4\sqrt{m_a m_b}}\ \ or\ if\ \ P_F^b \neq P_F^a$$

$$\equiv \Delta_S \qquad\qquad Sarma\ phase$$

The Sarma phase would, *supposedly*, have its energy lowered if the space between k_1 and k_2 is filled with **b** particles.

The state which minimizes the energy can be written

$$\left|\Psi\right\rangle = \prod_{\substack{k<k_1 \\ k>k_2}} \left[u_k + v_k a_k^+ b_{-k}^+\right] \prod_{k_1}^{k_2} b_k^+ \left|0\right\rangle$$

For $\Delta \geq \Delta_c$ or $P_F^a = P_F^b$ Ω turns out to be

$$\Omega^{assym.}_{\substack{fully\\gapped}} =$$

$$\frac{\Delta^2}{g} + \sum_K \left[e_k^+ - \sqrt{e_k^{+2} + \Delta^2} - T \ln\left(e^{-b\,E_k^a} + 1\right) - T \ln\left(e^{-b\,E_k^b} + 1\right) \right]$$

From which we get the (thermal) gap equation

$$1 = g \sum_k \frac{1}{2E_k} \left(1 - \frac{1}{e^{E_k^a/T} + 1} - \frac{1}{e^{E_k^b/T} + 1} \right)$$

$$E_k^{a,b} \xrightarrow{\Delta \to 0} e_k^{a,b}$$

From the gap equation we obtain a non linear equation for T_c

$$T_c = \frac{s\ \Delta_0}{2p}\, e^{-\frac{1}{2}F\left(\frac{1}{2T_c}\frac{m_b m_b - m_a m_a}{m_a + m_b}\right)}$$

$$s = \frac{\sqrt{m_a m_b}}{m_a + m_b}$$

$$F(x) \equiv \Psi\left(\frac{1}{2} + \frac{i\,x}{p}\right) + \Psi\left(\frac{1}{2} - \frac{i\,x}{p}\right)$$

Since $P_F^a = P_F^b$ we get $F(0) = -2[g - \ln(4)]$

$$\frac{T_c}{\Delta_0} = 2s\,\frac{e^g}{p}\;*$$

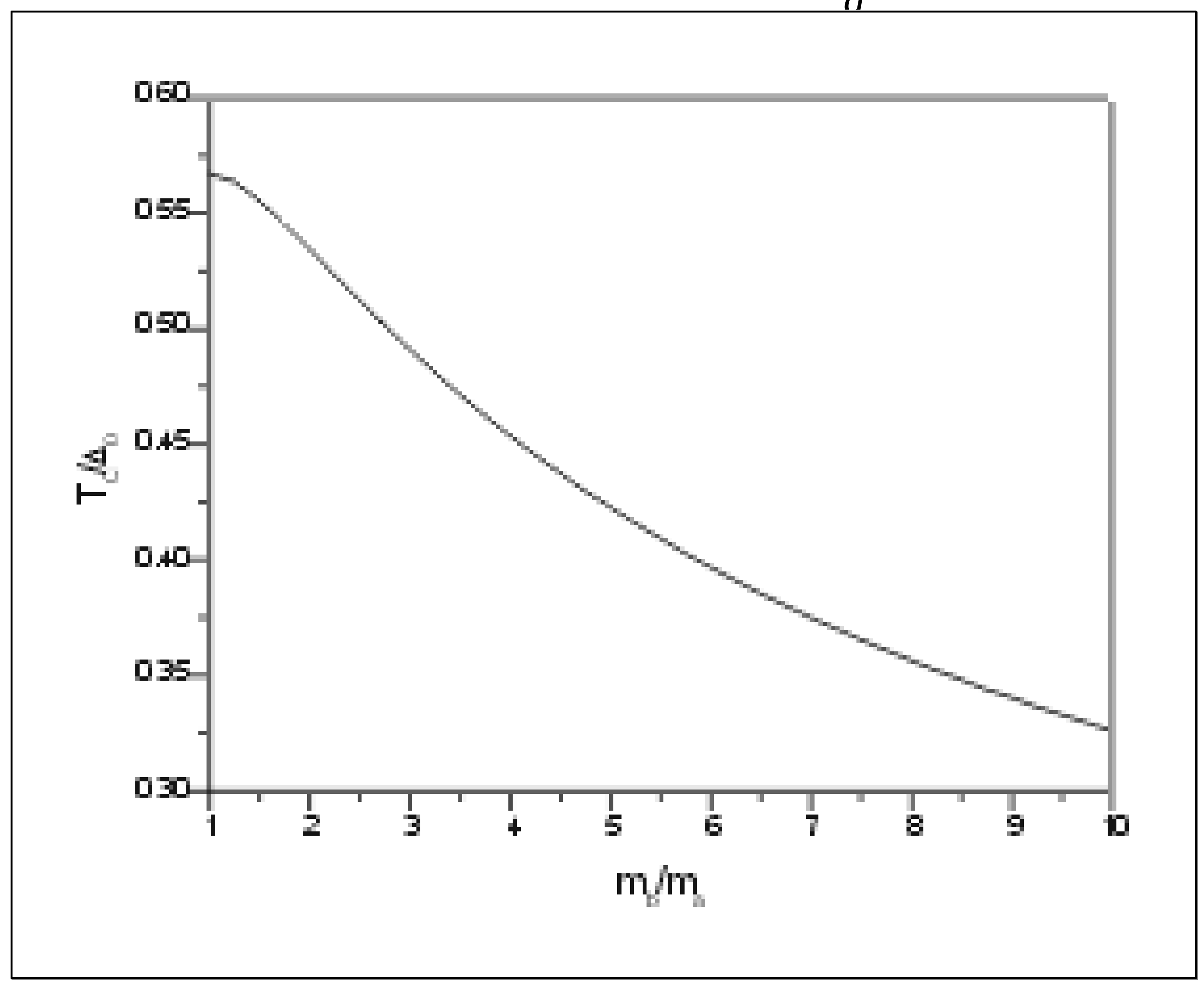
$$s = \frac{\sqrt{m_a m_b}}{m_a + m_b} = \frac{\sqrt{m_a m_b}}{m_a + m_b} \xrightarrow{m_a = m_b,\, m_a = m_b} \frac{1}{2}$$

* No cutoff dependence! Also, *a la* Sakita:

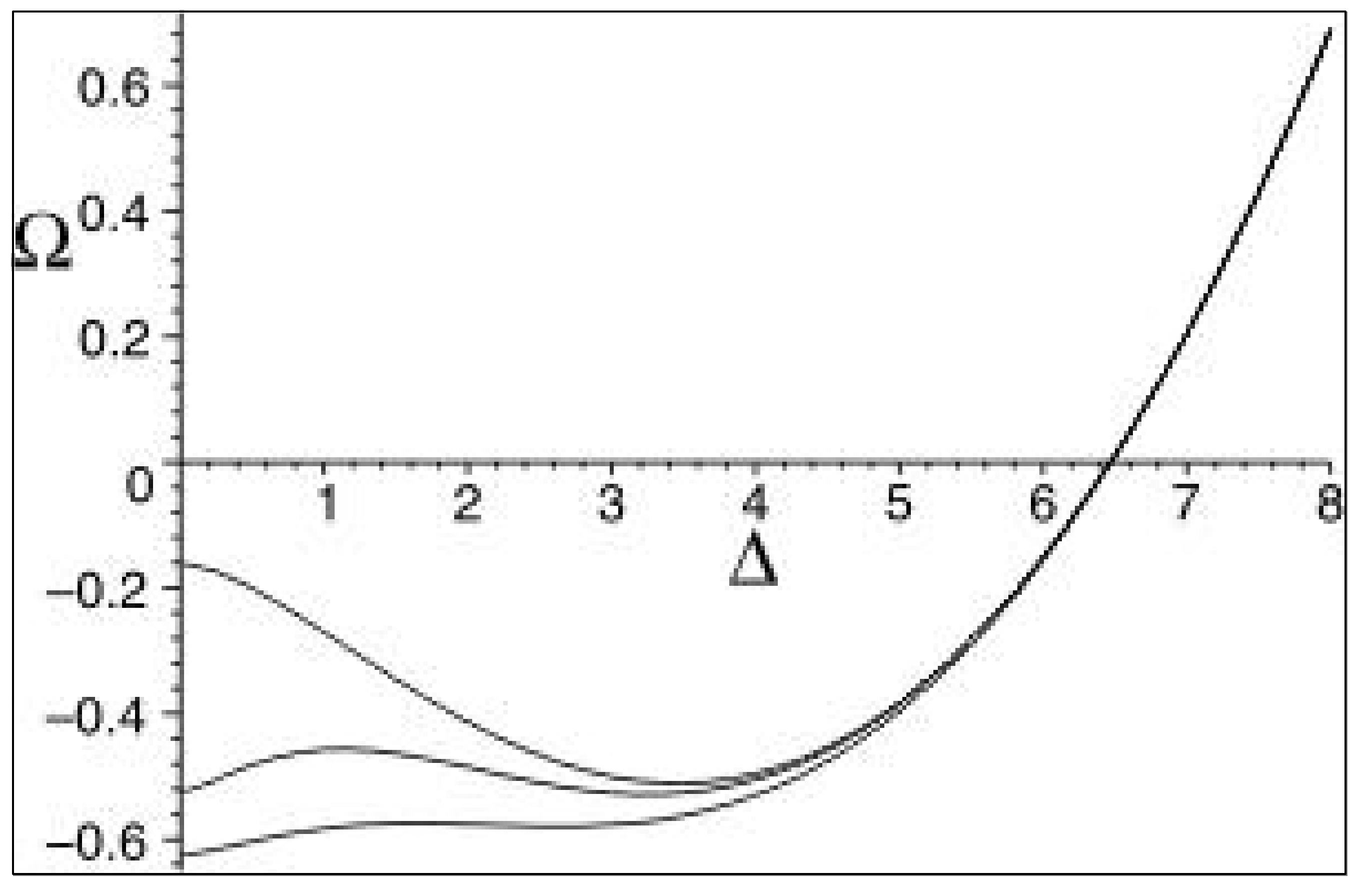
H. Caldas, A.L. Mota and C.W. Morais

PRD 72 045008 (2005).

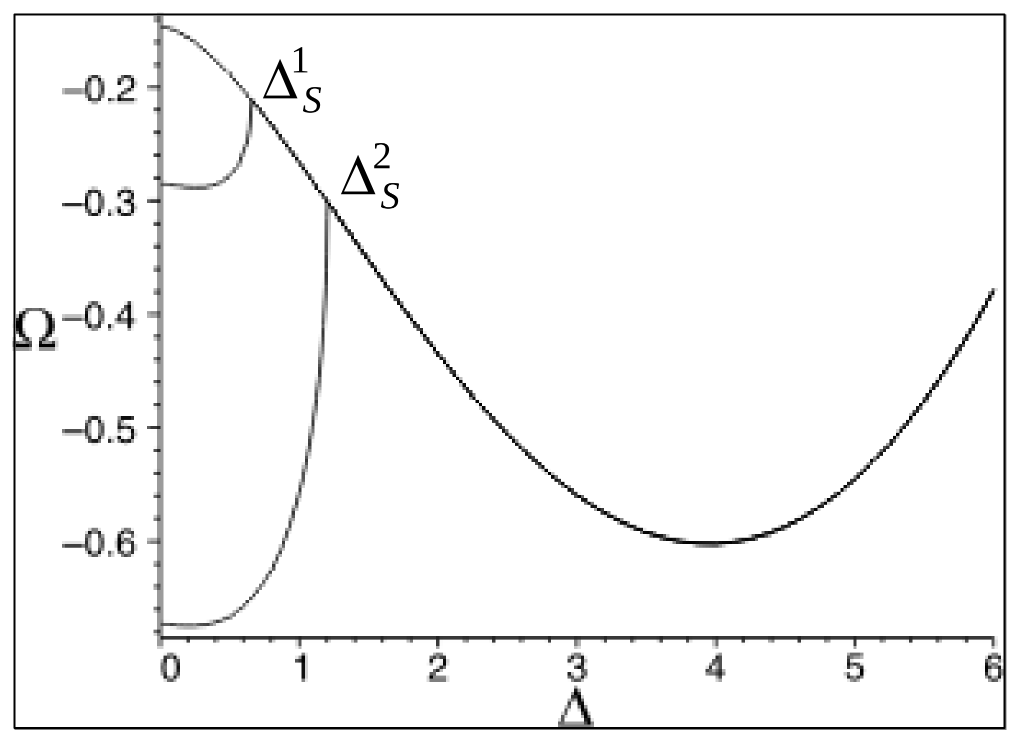
$\frac{T_c}{\Delta_0}$ behavior as a function of the mass asymmetry $\frac{m_b}{m_a}$



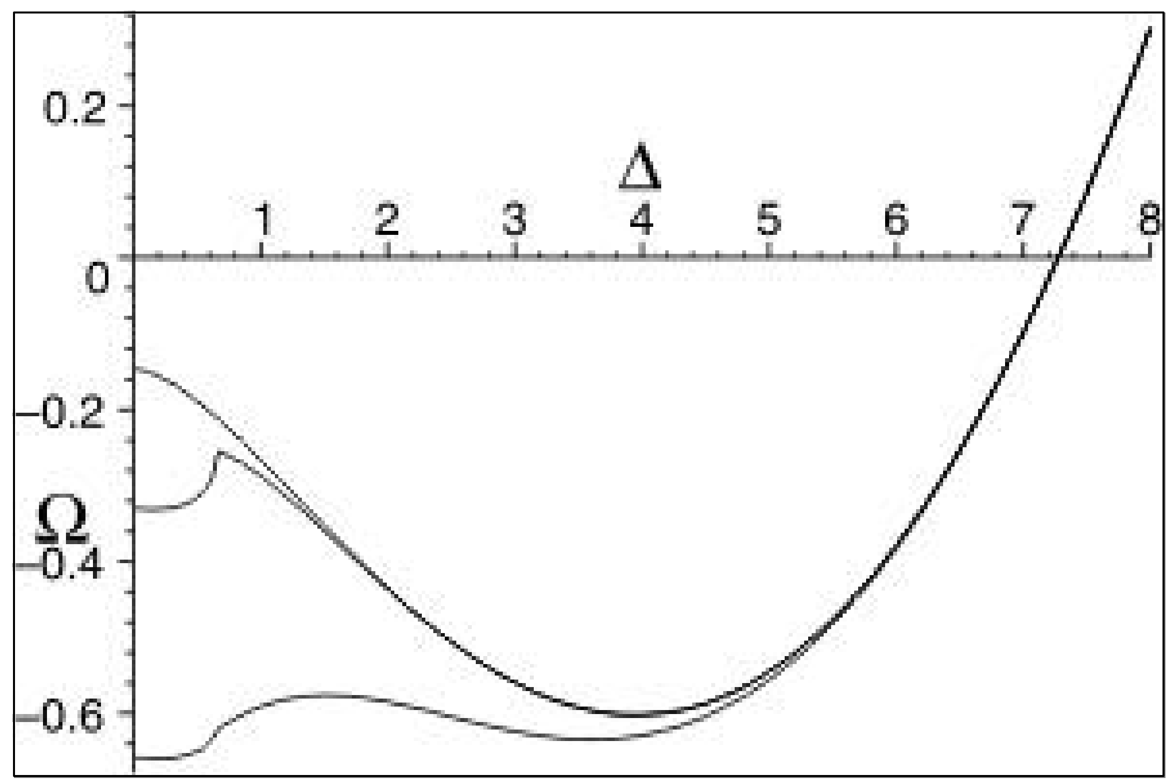
Ω for $m_b = 2m_a$ and $m_a = 2m_b$ such that $P_F^b = P_F^a$ at $\neq T$



$\Omega(T=0)$ for $P_F^b \neq P_F^a$ with increasing asymmetries, keeping $\frac{P_F^2}{2M} = \frac{P_F^{a\,2}}{2m_a} + \frac{P_F^{b\,2}}{2m_b}$ fixed.



$\Omega(T)$ for fixed asymmetry and increasing T



Conclusions:

- There are no gapless excitations in this model at zero or finite temperature!

- If an asymmetrical fermion system is constrained to have both equal or different Fermi surfaces ? 1st order with Increasing T!

Things to do:

- **Fixed chemical potentials:** To investigate phase transition when the gap has momentum structure, conjectured to bring stability to the Sarma phase (Forbes et al. 2004).

- **Fixed number of particles:** To investigate phase transition in the Mixed Phase (fixed densities). It would also be necessary to estimate the surface energy present when the system is not large.