Phase Transition in Asymmetrical Fermion Superfluids

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Motivation:
In the last few years there has been a great interest, both theoretically and experimentally, in asymmetrical fermionic systems!

Possible examples of systems exhibiting pairing between particles with different Fermi surfaces:

- Hyperfine States in a Trap
- Ultracold systems (atoms of different elements: $^{40}K$ or $^6Li$ - atomic traps)
- QCD (quarks of different species: up, down, etc): color superconductivity.
We have studied these asymmetrical fermion superfluids at zero T:

Since in real experiments the temperature is always non zero, some questions naturally arise:

• Does the particle’s masses and chemical potentials asymmetry alter the phase transition between the superfluid and normal states?

• How does the critical temperature of an asymmetric system depend on its asymmetry?
Phase Transitions at Fixed Chemical Potentials

The (non-relativistic) dilute system:

Particles a: $m_a$, $\mu_a$, $P_F^a = \sqrt{2m_a\mu_a}$ Spin $\uparrow$

Particles b: $m_b$, $\mu_b$, $P_F^b = \sqrt{2m_b\mu_b}$ Spin $\downarrow$
We investigate:

\[ P_F^a = P_F^b \text{ with } m_a \neq m_b \text{ and } \mu_a \neq \mu_b \]

\[ P_F^a \neq P_F^b \text{ pairing around } P_F \equiv \sqrt{2M\mu} = \sqrt{\frac{2m_a m_b}{m_a + m_b} (\mu_a + \mu_b)} \]
The model:
The system is described by the following Hamiltonian:

\[
H - \sum_{i=a,b} \mu_i n_i =\]

\[
\sum_k \varepsilon^a_k a_k^* a_k + \varepsilon^b_k b_k^* b_k - g \sum_{k,k'} a_k^* b_{-k}^* b_{-k} a_k
\]

\[
\varepsilon^a_i = \frac{k^2}{2m_i} - \mu_i
\]

\[- g < 0 \rightarrow attraction!
\]
From $H$ we obtain

$$\Omega = \frac{\Delta^2}{g} + \sum_{K<K_1} \left[ \varepsilon_k^+ - \sqrt{\varepsilon_k^+} + \Delta^2 - T \ln\left(e^{-\beta E_k^a} + 1\right) - T \ln\left(e^{-\beta E_k^b} + 1\right) \right]$$

$$+ \sum_{K_1} \left[ \varepsilon_k^b - T \ln\left(e^{-\beta |K_k^b|} + 1\right) \right]$$

$$\mathbf{E}_{\mathbf{a}, \mathbf{p}} = \mathbf{e}_- + \mathbf{e}_+ + \mathbf{e}_+ + \nabla \mathbf{3}$$

$$\varepsilon^- = \frac{\varepsilon_k^a - \varepsilon_k^b}{2}$$

$$\varepsilon^+ = \frac{\varepsilon_k^a + \varepsilon_k^b}{2}$$
Depending on the relative magnitudes of $P_F^{a,b}$ and $m_{a,b}$, $E_k^{a,b}$ (QPE) can be $< 0$. We take $P_F^b > P_F^a$ and $m_b > m_a$ such that only $E_k^b < 0$ for $k_1 < k < k_2$.
Roots of \( E_k^b \):

\[
k_{1,2}^2 = \frac{1}{2} \delta P_F^+ + \frac{1}{2} \left[ \delta P_F^- - 16m_a m_b \Delta^2 \right]^{1/2}
\]

\[
\delta P_F^+ \equiv P_F^{b2} + P_F^{a2} \quad \delta P_F^- \equiv P_F^{b2} - P_F^{a2}
\]

\( k_{1,2} \) is real if \( \Delta \leq \Delta_c = \frac{|\delta P_F^-|}{4\sqrt{m_a m_b}} \) or if \( P_F^b \neq P_F^a \)

\( \equiv \Delta_S \quad \text{Sarma phase} \)
The Sarma phase would, *supposedly*, have its energy lowered if the space between $k_1$ and $k_2$ is filled with $b$ particles.

The state which minimizes the energy can be written

$$|\Psi\rangle = \prod_{k<k_1}^{k>k_2} [u_k + v_k a_k^+ b_{-k}^+] \prod_{k_1}^{k_2} b_k^+ |0\rangle$$
For $\Delta \geq \Delta_c$ or $P_F^a = P_F^b \Omega$ turns out to be

$$\Omega_{\text{assym.}} = \frac{\Delta^2}{g} + \sum_k \left[ \epsilon_k^+ - \sqrt{\epsilon_k^+ + \Delta^2} - T \ln\left(e^{-\beta E_k^a} + 1\right) - T \ln\left(e^{-\beta E_k^b} + 1\right) \right]$$

From which we get the (thermal) gap equation

$$1 = g \sum_k \frac{1}{2 E_k} \left( 1 - \frac{1}{e^{E_k^a/T} + 1} - \frac{1}{e^{E_k^b/T} + 1} \right)$$

$E_k^{a,b} \xrightarrow{\Delta \to 0} \epsilon_k^{a,b}$
From the gap equation we obtain a non linear equation for $T_c$

$$T_c = \frac{\sigma \Delta_0}{2\pi} e^{-\frac{1}{2}F \left( \frac{1}{2T_c} \frac{m_b \mu_b - m_a \mu_a}{m_a + m_b} \right)}$$

$$\sigma = \sqrt{\frac{m_a m_b}{m_a + m_b}}$$

$$F(x) \equiv \Psi \left( \frac{1}{2} + \frac{ix}{\pi} \right) + \Psi \left( \frac{1}{2} \frac{-ix}{\pi} \right)$$
Since $P_F^a = P_F^b$ we get $F(0) = -2[\gamma - \ln(4)]$

\[
\frac{T_c}{\Delta_0} = 2\sigma \frac{e^\gamma}{\pi} *
\]

\[
\sigma = \frac{\sqrt{m_am_b}}{m_a + m_b} = \frac{\sqrt{\mu_a\mu_b}}{\mu_a + \mu_b} \rightarrow \frac{1}{2} \quad (m_a = m_b, \mu_a = \mu_b)
\]

* No cutoff dependence! Also, a la Sakita:
H. Caldas, A.L. Mota and C.W. Morais
PRD 72 045008 (2005).
\( \frac{T_c}{\Delta_0} \) behavior as a function of the mass asymmetry \( \frac{m_b}{m_a} \)
\[ \Omega \text{ for } m_b = 2m_a \text{ and } \mu_a = 2\mu_b \text{ such that } \]
\[ p_F^b = p_F^a \text{ at } \neq T \]
\( \Omega(T = 0) \) for \( P_F^b \neq P_F^a \) with increasing asymmetries, keeping \( \frac{P_F^2}{2M} = \frac{P_{F}^{a^2}}{2m_a} + \frac{P_{F}^{b^2}}{2m_b} \) fixed.
$\Omega(T)$ for fixed asymmetry and increasing $T$
Conclusions:

- There are no gapless excitations in this model at zero or finite temperature!

- If an asymmetrical fermion system is constrained to have both equal or different Fermi surfaces? 1st order with Increasing T!
Things to do:

- **Fixed chemical potentials:** To investigate phase transition when the gap has momentum structure, conjectured to bring stability to the Sarma phase (Forbes et al. 2004).

- **Fixed number of particles:** To investigate phase transition in the Mixed Phase (fixed densities). It would also be necessary to estimate the surface energy present when the system is not large.