Low-momentum interactions for nuclei

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Nuclear Forces and the Quantum Many-Body Problem
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Outline

1) Introduction and motivation

2) Low-momentum nucleon-nucleon interaction

3) Cutoff dependence as a tool to assess missing forces
   Perturbative low-momentum 3N interactions
   Nogga, Bogner, AS, nucl-th/0405016.

4) Selected applications:
   perturbative $^4$He, $^{16}$O and $^{40}$Ca, perturbative nuclear matter?
   Bogner, Furnstahl, AS, in prep.
   pairing in neutron matter

5) Summary and priorities
1) Introduction and motivation

Nuclear forces and the quantum many-body problem

Choice of nuclear force starting point:

If system is probed at low energies, short-distance details are not resolved.

Improvement of many-body methods:
Bloch-Horowitz, NCSM, CCM, DFT + effective actions, RG techniques,…
1) Introduction and motivation

Nuclear forces and the quantum many-body problem

Choice of nuclear force starting point:

Use low-momentum degrees-of-freedom and replace short-dist. structure by something simpler w/o distorting low-energy observables.

Infinite number of low-energy potentials (diff. resolutions), use this freedom to pick a convenient one.
What depends on this resolution?

- strength of 3N force relative to NN interaction
- strength of spin-orbit splitting obtained from NN force
- size of exchange-correlations, Hartree-Fock with NN interaction bound or unbound
- convergence properties in harmonic oscillator basis

Change of resolution scale corresponds to changing the cutoff in nuclear forces. [This freedom is lost, if one uses the cutoff as a fit parameter (or cannot vary it substantially).]

Observables are independent of the cutoff, but strength of NN, 3N, 4N,… interactions depend on it! Explore…
2) Low-momentum nucleon-nucleon interaction

Many different NN interactions, fit to scattering data below $E_{\text{lab}} \lesssim 350 \text{MeV}$ (with $\chi^2/\text{dof} \approx 1$)

Details not resolved for relative momenta larger than $\Lambda \sim 2.1 \text{ fm}^{-1}$ or for distances $r \lesssim 0.5 \text{ fm}$

Strong high-momentum components, model dependence
Separation of low-momentum physics + renormalization

Integrate out high-momentum modes and require that the effective potential $V_{\text{low } k}$ reproduces the low-momentum scattering amplitude calculated from potential model $V_{\text{NN}}$

Cutoff $\Lambda$ is boundary of unresolved physics

[NB: cutoff only on potential]

$$
T(k', k; k^2) = V_{\text{NN}}(k', k) + \frac{2}{\pi} \mathcal{P} \int_0^\infty \frac{V_{\text{NN}}(k', p) T(p, k; k^2)}{k^2 - p^2} \, p^2 \, dp
$$

$$
T(k', k; k^2) = V_{\text{low } k}(k', k) + \frac{2}{\pi} \mathcal{P} \int_0^\Lambda \frac{V_{\text{low } k}(k', p) T(p, k; k^2)}{k^2 - p^2} \, p^2 \, dp
$$

$V_{\text{low } k}$ sums high-momentum modes (according to RG eqn)
RG evolution also very useful for $\chi$EFT interactions

- choose cutoff range in $\chi$EFT to include maximum known long-distance physics $\Lambda_{\chi} \sim 500\text{-}700$ MeV for N3LO

- run cutoff down lower for application to nuclear structure (e.g., to $\Lambda \approx 400$ MeV)
  - observables (phase shifts, …) preserved
  - higher-order operators induced by RG

comp. fitting a $\chi$EFT truncation at lower $\Lambda$ (less accurate)
Details on $V_{\text{low } k}$ construction:

1. Resummation of high-momentum modes in energy-dep. effective interaction (BH equation) [largest effect]

$$\hat{Q}(k', k; \omega) = V_{\text{NN }}(k', k) + \frac{2}{\pi} \mathcal{P} \int_\Lambda^\infty \frac{V_{\text{NN }}(k', p) \hat{Q}(p, k; \omega)}{\omega - p^2} p^2 dp$$

2. Converting energy to momentum dependence through equations of motion; below to second order (to all orders by iteration, 1+2 = Lee-Suzuki transformation)

$$V_{\text{low } k}(k', k) = \hat{Q}(k', k; k^2) + \int_p \frac{\hat{Q}(k'; p, k^2) - \hat{Q}(k'; p, p^2)}{k^2 - p^2} \hat{Q}(p, k; k^2) + \mathcal{O}(\hat{Q}^3)$$

[small changes, $Q(\omega=0)$ good for $E_{\text{lab}}$ below $\sim 150$ MeV]

Both steps equivalent to RG equation Bogner et al., nucl-th/0111042.

$$\frac{d}{d\Lambda} V_{\text{low } k}(k', k) = \frac{2}{\pi} \frac{V_{\text{low } k}(k', \Lambda) T(\Lambda, k; \Lambda^2)}{1 - (k/\Lambda)^2}$$

[Hermitize $V_{\text{low } k}$ or symmetrized RG equation (very small changes)]
Exact RG evolution of all NN models below $\Lambda \sim 2.1 \text{ fm}^{-1}$ leads to model-independent low-mom. interaction $V_{\text{low } k}$ (all channels)
Collapse of off-shell matrix elements as well

\begin{align*}
\text{N}^2\text{LO} \\
\text{N}^3\text{LO}
\end{align*}
Collapse due to same long-distance ($\pi$) physics + phase shift equivalence
$V_{\text{low } k}$ is much softer, without strong core at short-distance, see relative HO matrix elements $< 0 \mid V_{\text{low } k} \mid n >$

convergence will not require basis states up to $\sim 50$ shells
Poor convergence in SM calculations (due to high-mom. components \( \sim 1 \) GeV in NN potentials)

Unsatisfactory starting-energy dependence (due to loss of BH self-consistency in 2-body subsystem)

\( V_{\text{low } k} \) will lead to improvements

from P. Navratil, talk @ INT-03-2003

from T. Papenbrock, talk @ MANSC 2004
Comparison of $V_{\text{low } k}$ to $G$ matrix elements up to $N=4$

**BUT:** $V_{\text{low } k}$ is a bare interaction!
Comments:

1. Renormalization of high-momentum modes in free-space easier (before going to a many-body system)

2. Soft interaction avoids need for G matrix resummation (which was introduced because of strong high-momentum components in nuclear forces)

3. $V_{\text{low } k}$ is energy-independent, no starting-energy dep.

4. Cutoff is not a parameter, no “magic” value, use cutoff as a tool…
3) Cutoff dependence as tool to assess missing forces
Perturbative low-momentum 3N interactions

All low-energy NN observables unchanged and cutoff-indep.

Nijmegen partial wave analysis
△ CD Bonn
○ $V_{\text{low } k}$
All NN potentials have a cutoff ("P-space of QCD") and therefore have corresponding 3N, 4N, … forces.

If one omits the many-body forces, calculations of low-energy 3N, 4N, … observables will be cutoff dependent.

By varying the cutoff, one can assess the effects of the omitted 3N, 4N, … forces. Nogga, Bogner, AS, nucl-th/0405016.
Potential model dependence in $A=3,4$ systems (Tjon line)

Cutoff dependence due to missing three-body forces along Tjon line
Nogga, Bogner, AS, nucl-th/0405016.

Results for reasonable cutoffs seem closer to experiment
Renormalization: three-body forces inevitable!
Faddeev, $V_{low \ k}$ only

Cutoff dep. of low-energy 3N observables due to missing three-body forces (see $\pi$ EFT)

$A=3$ details
Adjust **low-momentum three-nucleon interaction** to remove cutoff dependence of A=3,4 binding energies

Use leading-order effective field theory 3N force given by van Kolck, PR C49 (1994) 2932; Epelbaum et al., PR C66 (2002) 064001.

Motivation: At low energies, all phenomenological 3N forces (from $\omega$, $\rho$,… exchange, high-mom. N, $\Delta$,… intermed. states) collapse to this form; cutoffs in $V_{low k}$ and $\chi$ EFT similar.
Constraint on D- and E-term couplings from fit to $^3$H

$E(3H) = E(V_{\text{low } k} + 2\pi \ 3NF) + c_D < \text{D-term} > + c_E < \text{E-term} >$

Second constraint from fit to $^4$He [E-term fixed by left Fig.]

$\eta=1$: $^4$He fitted exactly

$\eta=1.01$: deviation from exp.

$\approx 600$ keV

non-linearities at larger cutoffs
2 couplings for D- and E-terms fitted to $^3$H and $^4$He

We find all 3N parts perturbative for cutoffs $\Lambda \lesssim 2$ fm$^{-1}$

| $\Lambda$ [fm$^{-1}$] | $T$ | $V_{low\ k}$ | $c$-terms | $D$-term | $E$-term | $^3$H | $^4$He
|----------------------|-----|-------------|-----------|----------|----------|------|------
| 1.0                  | 21.06 | -28.62 | 0.02 | 0.11 | -1.06 | 38.11 | -62.18 | 0.10 | 0.54 | -4.87 |
| 1.3                  | 25.71 | -34.14 | 0.01 | 1.39 | -1.46 | 50.14 | -78.86 | 0.19 | 8.08 | -7.83 |
| 1.6                  | 28.45 | -37.04 | -0.11 | 0.55 | -0.32 | 57.01 | -86.82 | -0.14 | 3.61 | -1.94 |
| 1.9                  | 30.25 | -38.66 | -0.48 | -0.50 | 0.90 | 60.84 | -89.50 | -1.83 | -3.48 | 5.68 |
| 2.5(a)               | 33.30 | -40.94 | -2.22 | -0.11 | 1.49 | 67.56 | -90.97 | -11.06 | -0.41 | 6.62 |
| 2.5(b)               | 33.51 | -41.29 | -2.26 | -1.42 | 2.97 | 68.03 | -92.86 | -11.22 | -8.67 | 16.45 |
| 3.0(*)               | 36.98 | -43.91 | -4.49 | -0.73 | 3.67 | 78.77 | -99.03 | -22.82 | -2.63 | 16.95 |

- $<k^2> \approx (A-1) m <T> \approx m_\pi^2 \ll \Lambda^2$
- $c$- and $E$-terms increase and cancel
- 3N force parts increase by factor $\sim 5$ from $A=3$ to $A=4$
- Larger cutoffs: contributions become nonperturbative, fit to $^4$He non-linear (a,b) and approximate solution (*)

20% is beyond $(Q/\Lambda)^3 \sim (m_\pi/\Lambda)^3$
5) Selected applications

Perturbative calculations for $^4\text{He}$, $^{16}\text{O}$ and $^{40}\text{C}$


Results for $V_{\text{low } k}$

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$V_{\text{low } k}$ from N3LO</th>
<th>HF</th>
<th>HF+2nd</th>
<th>HF+2nd +3rd</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}\text{O}$</td>
<td></td>
<td>$B/A$</td>
<td>3.23</td>
<td>7.22</td>
<td>7.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\langle r_c \rangle$</td>
<td>2.30</td>
<td>2.52</td>
<td>2.65</td>
</tr>
<tr>
<td>$^{40}\text{Ca}$</td>
<td></td>
<td>$B/A$</td>
<td>6.19</td>
<td>9.10</td>
<td>9.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\langle r_c \rangle$</td>
<td>2.610</td>
<td>3.302</td>
<td>3.444</td>
</tr>
</tbody>
</table>

$V_{\text{low } k}$ binds nuclei on HF level (contrary to all other microscopic NN interactions)

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$V_{\text{low } k}$ from (all for $\Lambda=2.1 \text{ fm}^{-1}$)</th>
<th>Nijmegen II</th>
<th>AV18</th>
<th>CD-Bonn</th>
<th>N3LO</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4\text{He}$</td>
<td>$BE/A$</td>
<td>6.88</td>
<td>6.85</td>
<td>6.95</td>
<td>6.61</td>
<td>7.07</td>
</tr>
<tr>
<td></td>
<td>$\langle r^2 \rangle^{1/2}$</td>
<td>1.68</td>
<td>1.69</td>
<td>1.63</td>
<td>1.75</td>
<td>1.67</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>$BE/A$</td>
<td>8.26</td>
<td>8.26</td>
<td>8.30</td>
<td>8.11</td>
<td>7.98</td>
</tr>
<tr>
<td></td>
<td>$\langle r^2 \rangle^{1/2}$</td>
<td>2.58</td>
<td>2.59</td>
<td>2.49</td>
<td>2.66</td>
<td>2.73</td>
</tr>
<tr>
<td>$^{40}\text{Ca}$</td>
<td>$BE/A$</td>
<td>9.66</td>
<td>9.53</td>
<td>9.93</td>
<td>9.50</td>
<td>8.55</td>
</tr>
<tr>
<td></td>
<td>$\langle r^2 \rangle^{1/2}$</td>
<td>3.20</td>
<td>3.22</td>
<td>3.10</td>
<td>3.29</td>
<td>3.485</td>
</tr>
</tbody>
</table>

FIG. 1. Behavior of $E_{\text{HF}}$ with $\hbar \omega$ and $N$ for $^{16}\text{O}$. 

**exact 7.30**
Pairing in neutron matter

\[1S_0 \text{ pairing} \]
\[3P_2 \text{ pairing} \]

\[V_{\text{low } k} \]

Suppression due to polarization effects (spin-/LS fluct.)
No strong core → simpler many-body starting point

- **BHF** Bao et al. [NP A575 (1994) 707.]
- **Simple \( V_{\text{low } k} \) Hartree-Fock**
- **FHNC** Akmal et al. [PR C58 (1998) 1804.]

**Neutron matter EoS**


nuclear matter from low-mom. NN + 3N int. Bogner, Furnstahl, AS, in prep.
5) Summary and priorities

+ Model-independent low-momentum interaction $V_{\text{low }} k$
+ Cutoff independence as a tool, not fit parameter
+ Three-body forces required by renormalization, perturbative low-momentum 3N interaction

* Cutoff dependence and convergence properties of NCSM and CCM results obtained from $V_{\text{low }} k + 3\text{NF}$, isospin dependence of 3N force sufficient? $^{10}\text{B}$?


* Derivation of $V_{\text{low }} k$ from phase shifts + pion exchange
  Bogner, Birse, AS, in prep.

* Calculations of SM effective interactions from $V_{\text{low }} k + 3\text{N}$ force in regions where two-body G matrix fails