Study of core-collapse supernovae: Post-bounce evolution and EOS effects

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• Supernova simulations beyond 300 msec after bounce:
  - General relativistic neutrino-transfer hydrodynamics
  - Relativistic EOS vs Lattimer-Swesty EOS
  - EOS effects on stalled-shock & supernova cores
Why supernova does not explode?, What is missing?

- **Microphysics issues:**
  - Equation of state
  - $\nu$-reaction rates
  - e-capture rates,…

- **Macrophysics issues:**
  - Hydrodynamics
  - $\nu$-transfer
  - Convection, rotation,…

Both issues should be examined to clarify the explosion mechanism.

- **We focus on microphysics by simulations of neutrino-transfer hydrodynamics**

- **In spherical symmetry, neutrino-transfer can be solved, we can examine microphysics**
Physics of unstable nuclei

• Recent advance of radioactive nuclear beam facilities provides us with data on n-rich nuclei: RIKEN, GSI, RIA, …
  ex. RIKEN-RI Beam Factory

  RI Beam Factory (RIBF):
  Upgrading project of RIKEN Accelerator Research Facility (RARF)

• Relativistic EOS table is based on data of unstable nuclei
  Shen et al. NPA, PTP (1998)

• We should examine supernova simulations in the light of physics of unstable nuclei
Efforts on examining microphysics

• No explosion (shock stalled) in 1D so far with:
  – Lattimer-Swesty EOS, Bruenn’s weak rates
  – Improved neutrino-rates, electron capture rates
  – Different sets of EOS
  – ~300 msec after bounce

• Post-bounce evolution for a long period of ~1 sec?
• EOS effects on shock dynamics and supernova cores?

• A new numerical code for general relativistic hydrodynamics with Boltzmann eq. for 
Purpose of our studies

- **Within the exact treatment of $\Delta$-transfer hydrodynamics in spherical symmetry**
  - Find out the fate of core collapse
    - Explosion or not?
  - Long-term evolution of supernova core
    - Up to $\sim 1$ sec after bounce
  - Examine the effect of new EOS
    - Some hints on the explosion mechanism

- **The first comparison of EOS effects beyond 300 msec**
  - Core bounce, shock dynamics and $\Delta$-heating mechanism
  - Evolution of proto-neutron star, supernova $\Delta$
Roles of Equation of state (EOS)

1. Pressure, stiffness,
   - structure, core bounce,..
2. Entropy, Temperature
   - $\mathcal{E}$-energy, spectrum,..
3. Composition ($n$, $p$, $\alpha$, nuclei)
   - e-capture, $\mathcal{E}$-interaction,..

• Systematic studies by parameterized EOS
  - Baron-Cooperstein, Takahara-Sato, Bruenn, Swesty,..

• Set of physical EOS
  - Wolff-Hillebrandt EOS (1985)
  - Lattimer-Swesty EOS (1991) Used so far
  - Relativistic (Shen) EOS (1998) NEW
Relativistic equation of state for supernovae

• Relativistic Mean Field + Local-Density Approx.
  Shen, Toki, Oyamatsu & Sumiyoshi, 1998, NPA, PTP
  – Based on relativistic Brückner Hartree-Fock
  – Checked by exp. data of unstable nuclei
    • Nuclear structure: mass, charge radius, neutron skin,
      • EOS data table (~60MB) covers
        – Density: $10^5 \sim 10^{15}$ g/cm$^3$
        – Proton fraction: $0 \sim 0.56$
        – Temperature: $0 \sim 100$ MeV
Relativistic Mean Field Theory - Effective Lagrangian
Serot, Walecka 1986

\[ L_{RMF} = \partial \mu \partial \nu \partial \rho \partial \sigma M g_{\mu \nu \rho \sigma} + g_{\mu \nu \rho} \eta^{\sigma} \eta^a \eta_{\mu \nu} A^a \frac{1}{2} \epsilon_{\mu \nu \rho \sigma} \]

\[ + \frac{1}{2} \partial \mu \partial \nu \partial \rho \partial \sigma \partial \tau \frac{1}{2} m^2 \partial^2 \partial^2 - \frac{1}{3} g_2 \partial^3 \partial^3 - \frac{1}{4} g_3 \partial^4 \]

\[ \frac{1}{4} \epsilon_{\mu \nu \rho \sigma} H_{\mu \nu} H^{\rho \sigma} + \frac{1}{2} m^2 \partial^2 \partial^2 + \frac{1}{4} c_3 \left( \partial^3 \eta^a \eta_{\mu \nu} \right)^2 \]

\[ \frac{1}{4} G^a \epsilon_{\mu \nu \rho \sigma} \eta_{\mu \nu} G^{\rho \sigma} + \frac{1}{2} m_2 \partial^a \partial^a - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]

Parameters determined by nuclear data (masses, radii)


Nuclear structure calculations \(\rightarrow\) EOS calculations
Neutron skins of Na isotopes

Symbols: Exp. Data
Lines: RMF

Local density approximation in cell

- Non-uniform
- Uniform

- Mix of: Neutron, Proton, Alpha, Nucleus

Shen et al. PTP (1998)
## Comparison of EOSs (1)

<table>
<thead>
<tr>
<th></th>
<th>LS-EOS</th>
<th>Shen-EOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>Compressible liquid drop model</td>
<td>Rel. Mean Field + Local-Density Approx.</td>
</tr>
<tr>
<td>Bulk EOS</td>
<td>“Skyrme”-like</td>
<td>RMF (RBHF)</td>
</tr>
<tr>
<td>Interaction</td>
<td>Saturation</td>
<td>Mass, $R_c$, $R_n$</td>
</tr>
<tr>
<td>Nucl. Data</td>
<td>---</td>
<td>Yes (incl. unstable)</td>
</tr>
<tr>
<td>n-skin</td>
<td>---</td>
<td>Yes</td>
</tr>
<tr>
<td>$M^*$</td>
<td>---</td>
<td>Yes</td>
</tr>
<tr>
<td>Info</td>
<td>Subroutine</td>
<td>Data table</td>
</tr>
</tbody>
</table>
## Comparison of EOSs (2)

<table>
<thead>
<tr>
<th></th>
<th>LS-EOS</th>
<th>Shen-EOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>K [MeV]</td>
<td>180, 220, 375</td>
<td>281</td>
</tr>
<tr>
<td>$A_{\text{sym}}$ [MeV]</td>
<td>29.3</td>
<td>36.9</td>
</tr>
<tr>
<td>Max. NS mass [$M_{\text{sol}}$]</td>
<td>1.8, 2.0, 2.7</td>
<td>2.2</td>
</tr>
</tbody>
</table>

- **Symmetry energy effect is large** cf. non-rel.
- Neutron Star: $Y_p = 0.1 \sim 0.2$ for rel-EOS [Sumiyoshi et al. NPA595 (1995)]
- **Difference of composition, chemical potential**
  - e-capture, $\beta$-scattering rates → supernova dynamics
Numerical code

• General relativistic hydrodynamics: lagrangian
  • Shock tubes, point explosion, collapse tests

• General relativistic Boltzmann eq. solver: $S_N$-method
  • Comparisons with Monte-Carlo calculations

• Fully implicit in time: advantageous to have long time steps
  • Comparisons with Liebendörfer et al. (astro-ph/0310662)
  • Consistent up to ~200 msec

• Other applications (hydrodynamics + heating-cooling)
Boltzmann equation for $n$ (GR, spherical)

Lindquist, Castor 1972, Bruenn 1985

$$
e^{-f} \frac{\partial}{\partial t} f + 4 e^{-f} \frac{\partial e^r}{\partial m}$$

$$+ \frac{\partial}{\partial m} \left(1 - m^2 \right)^2 \frac{\partial r^2}{\partial m} + e^{-f} \frac{\partial \ln(r_B^3)}{\partial m}$$

$$+ e^{-f} \frac{\partial \ln(r_B^3)}{\partial t} e^{-f} \frac{\partial \ln(r)}{\partial t}$$

$$= \frac{1}{r_B} e^{-f} \frac{\partial}{\partial t} f_{\text{collision}}$$

Finite differenced in $(t, m, \Box)$

Mezzacappa-Bruenn 1993
Neutrino processes


- **Emission and absorption:**
  \[
  \begin{align*}
  e^- + p & \rightarrow n + e^- \\
  e^+ + n & \rightarrow p + e^+
  \end{align*}
  \[
  \begin{align*}
  e^- + A & \rightarrow A^\prime + e^- \\
  e^+ + A & \rightarrow A^\prime + e^+
  \end{align*}
  \]

- **Scattering:**
  \[
  \begin{align*}
  \bar{n}_i + N & \rightarrow \bar{n}_i + N \\
  \bar{n}_i + e & \rightarrow \bar{n}_i + e
  \end{align*}
  \[
  \begin{align*}
  \bar{n}_i + A & \rightarrow \bar{n}_i + A \\
  \bar{n}_i + N & \rightarrow \bar{n}_i + N
  \end{align*}
  \]

- **Pair process:**
  \[
  \begin{align*}
  e^- + e^+ & \rightarrow n + n^* \\
  N + N & \rightarrow \bar{n}_i + \bar{n}_i
  \end{align*}
  \[
  \begin{align*}
  \bar{n}_i + \bar{n}_i & \rightarrow \bar{n}_i + \bar{n}_i \\
  \bar{n}_i + N & \rightarrow \bar{n}_i + N
  \end{align*}
  \]

- **Nucleon-Nucleon Bremsstrahlung**

i = e, \( \bar{n} \), \( n \)
Set up of numerical simulations

• **Initial model:** Fe core of $15M_{\text{solar}}$  
  – $M_{\text{Fe}} = 1.32M_{\text{solar}}$

• **Mesh size:** radial $N_r = 127$,  
  -energy $N_E = 14$, angle $N_q = 6$  
  – $\square$-distribution: $f(t, m, E_{\square}, \square)$

• **$\square$-species:** $\square_e, \square_e, \square_m, \square_m, (\square_t, \square_t)$ ($N_\square = 4$)  
  – ~120 hours on Fujitsu-vpp5000 (8PE)  
  – Block tridiagonal matrix: cyclic reduction in parallel  
  – $N_r * (N_E * N_\square * N_\square)^3$

*Woosley-Weaver ‘95*

*Sumiyoshi, Parallel Computing, 1997*
Set up of numerical simulations II

• Follow up to ~1 sec
  – More grids for outer layer to resolve accretions
  – Lower resolution for inner part
  – Will be improved by rezoning

• Using 2 EOS sets (Shen-EOS, LS-EOS)
  – Comparisons of collapse, bounce, shock propagation
  – Other settings (weak rates) are the same

• First comparison of post-bounce beyond 300 msec
  Janka et al. astro-ph/0405289
Collapse phase in Shen-EOS case

Difference from LS-EOS case

- **Stiffness**
  - *Less* compression
  - $\rho_c(\text{peak})=3.3 \times 10^{14} \text{ g/cm}^3$ (LS: $4.2 \times 10^{14} \text{ g/cm}^3$)

- **Composition**
  - *Less* n-rich nuclei: large symmetry energy
  - *Smaller* free proton fraction: $X_p \downarrow$, $\rho_p \downarrow$

- **Smaller e-capture rates**
  - Size of bounce core
    - e-capture on nuclei suppressed due to N>40 (Bruenn’s rate)
Composition of dense matter during collapse: $\rho_c = 10^{11}$ g/cm$^3$

Mass fraction

![Graph showing mass fraction as a function of $M_b [M_{solar}]$]

- Shen-EOS
- LS-EOS

Composition of dense matter during collapse: $\rho_c = 10^{11}, 10^{12}$ g/cm$^3$

Z, N of Nuclei

Profiles at bounce: $t_{pb} = 0$ ms

Lepton fraction

Velocity

Y\text{lepton}

Y\text{e}

Y\text{n}

Velocity [cm/s]

$M_b [M_{\text{solar}}]$

$Y_{\text{lepton}}$

$Y_{\text{e}}$

$Y_{\text{n}}$

$M_b [M_{\text{solar}}]$

preliminary

Shen-EOS

LS-EOS
Trajectories of fluid elements (Shen-EOS)

- surface
- Fe core
- collapse
- radius [km]
- time [sec]

- shock wave
- proto-neutron star
- bounce

preliminary

Trajectories of position of shock wave

\[ R_{\text{shock}} \text{ [km]} \]

\[ \text{time after bounce [sec]} \]

- Red line: Shen-EOS
- Blue line: LS-EOS

*preliminary*
Neutrino heating behind the shock after bounce

Heating:

```
\bar{e}_e + n \rightarrow e^- + p
\bar{\nu}_e + p \rightarrow e^+ + n
```
Post-bounce phase in Shen-EOS case

Difference from LS-EOS case

- **Heating rate**
  - *Smaller* due to *lower* luminosity & *smaller* $X_p$
    - Shock recession to be checked by higher resolution

- **Thermal evolution of central core**
  - *Lower* temperature, *lower* central density
  - Effect of effective mass $M^*$ & stiffness

- **Possible effects of EOS on**
  - Neutrino-heating mechanism
  - Supernova [], proto-neutron star evolution
    - Collapsar, black hole formation
Heating rate after bounce: $t_{pb}=100$ ms

$\sim [\text{MeV/s/N}]$

Heating region

Heating rate [erg/g/s]

radius [km]

Shen-EOS

LS-EOS

preliminary
Profiles after bounce: \( t_{pb} = 100 \text{ms} \)

**Luminosity**

- **Shen-EOS**
- **LS-EOS**

**Mass fraction**

- \( X_n \)
- \( X_p \)
- \( X_e \)

**Profiles**

- **nuclei**

**LS-EOS** vs. **Shen-EOS**

- Preliminary
Profiles after bounce: $t_{pb} = 100\text{ms}$

**Temperature**

- **Shen-EOS**
- **LS-EOS**

**Entropy**

*preliminary*
Comparison of numerical results

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</tr>
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<tbody>
<tr>
<td>$R_{\text{shock}}^{(\text{max})}$</td>
<td>164 km</td>
<td>160 km</td>
</tr>
<tr>
<td>$R_{\text{shock}}^{600\text{ms}}$</td>
<td>23 km</td>
<td>20 km</td>
</tr>
<tr>
<td>$\rho_c$ at 600ms</td>
<td>$6.0\times10^{14}$ g/cm$^3$</td>
<td>$3.9\times10^{14}$ g/cm$^3$</td>
</tr>
<tr>
<td>$M_\text{c}^*$ 600ms</td>
<td>938 MeV</td>
<td>440 MeV</td>
</tr>
<tr>
<td>$T_{\text{peak}}^{600\text{ms}}$</td>
<td>48 MeV</td>
<td>37 MeV</td>
</tr>
</tbody>
</table>

- Less compression
- $M^*(<M)$: increase of level density: $E \sim \frac{p^2}{2M^*}$
  → Lower T to get the same entropy
Profiles after bounce: $t_{pb}=20-600$ ms

Temperature

LS-EOS
$T_{peak} \sim 50$ MeV

Shen-EOS
$T_{peak} \sim 40$ MeV

preliminary
Profiles after bounce: $t_{pb}=20-600$ ms (Shen-EOS)

Entropy

Lepton fraction

preliminary
Properties of supernova neutrinos

Luminosity [erg/s]

$\langle E_n \rangle$ [MeV]

- Shen-EOS
- LS-EOS

preliminary
Summary

• Numerical simulations of supernovae
  – General relativistic $\square$-transport hydrodynamics in spherical symmetry

• New nuclear physics with unstable nuclei
  – Relativistic EOS table

• Long-term simulation beyond 300 msec
  – Stalled shock and supernova cores

• Comparisons with LS-EOS
  – No prompt/delayed explosion
  – Difference in composition & stiffness
  – Difference in thermal evolution of central core
To be done

- Follow up to $t_{\text{pb}} \sim 1$ sec and beyond
  - Continue to proto-neutron star cooling
    - Accretion induced collapse, Neutrino-driven winds
      - More grids for inner core
        » To check stalled shock
        » Adaptive rezoning

- Extension of relativistic EOS table
  - Include hyperons (Ishizuka-Ohinishi)

- Update electron capture, $\bar{n}$-rates
  - Consistent with EOS