MHD in stars

Topics:

1. Fossil magnetic fields vs. internally generated
   - nature of the Ap, WD fields

2. Instabilities in toroidal fields in stars

3. Dynamo process in radiative zones
   - angular momentum transport

Applications

1. Magnetic stars:

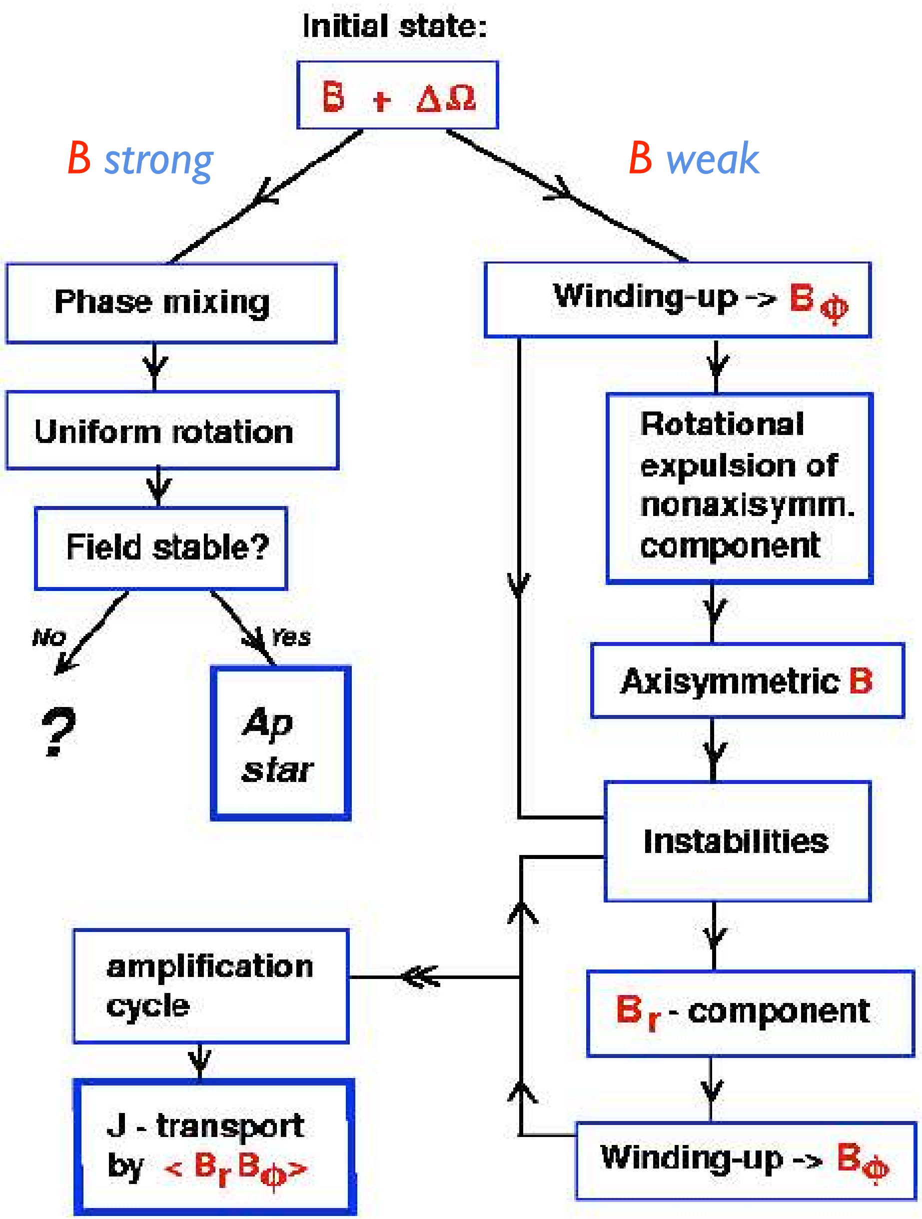
- Ap-Bp,
- magnetic WDs
- pulsars, magnetars

Main question: structure/stability of the field

2. Internal rotation of stars

- pulsar rotation rates
- core rotation in collapsar scenario

Evolution of B in a star



HCS 1999, 2002, A&A

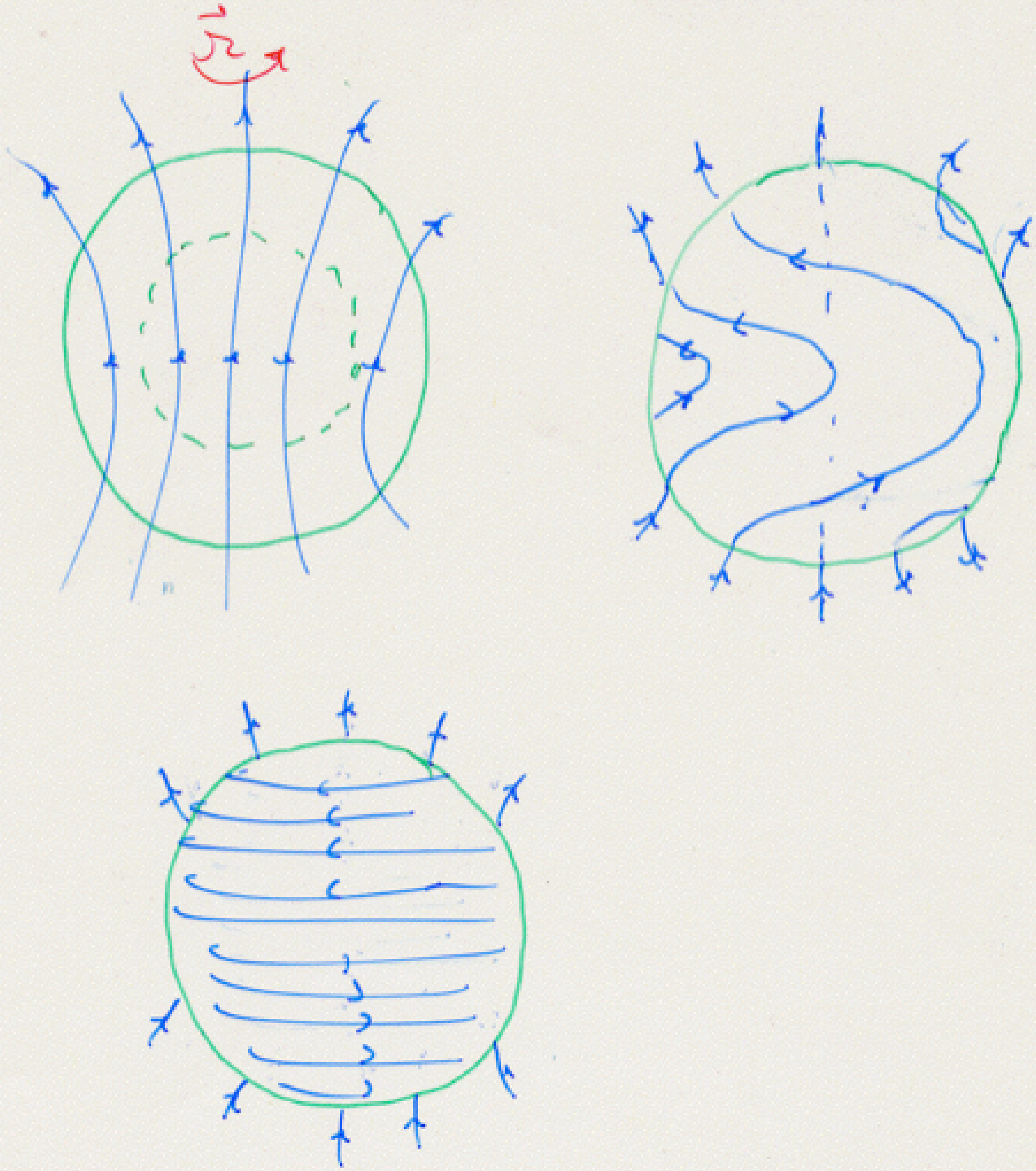
- *Bifurcation: type of outcome depends on initial field strength:*

Strong initial field: ⟶ uniformly rotating 'Ap star'

Weak initial field: ⟶ small scale self-sustained field, initial conditions forgotten

Field lines wound-up by differential rotation

Assume $\Omega = \Omega(r)$
("shellular" rotation)



Strong initial field case

Poloidal: $\mathbf{B}_\mathrm{p} = (B_r, B_\theta)$

Toroidal: $\mathbf{B}_\varphi = B_\varphi \mathrm{e}_\varphi$

initial: B_0 $\tau_\mathrm{A} \sim r/V_{\mathrm{A}0} = \dfrac{r(4\pi\rho)^{1/2}}{B_0}$

- Differential rotation winds field in azimuthal direction →
- Opposing Lorentz force →
- Oscillation at τ_A , (Mestel 1953)
- Damped by ‘phase mixing’ →

End result: *Uniform rotation*

‘Fossil field’ hypothesis (Ap stars, magnetic WDs)

Stability problem: $\tau_{\rm A} \ll t_0$

Stable configurations ‘rare’:

all purely poloidal fields unstable Mestel, Wright, Flowers & Ruderman

all purely toroidal fields unstable R.J. Tayler

Need ‘linked poloidal-toroidal field’ ? Prendergast 1958

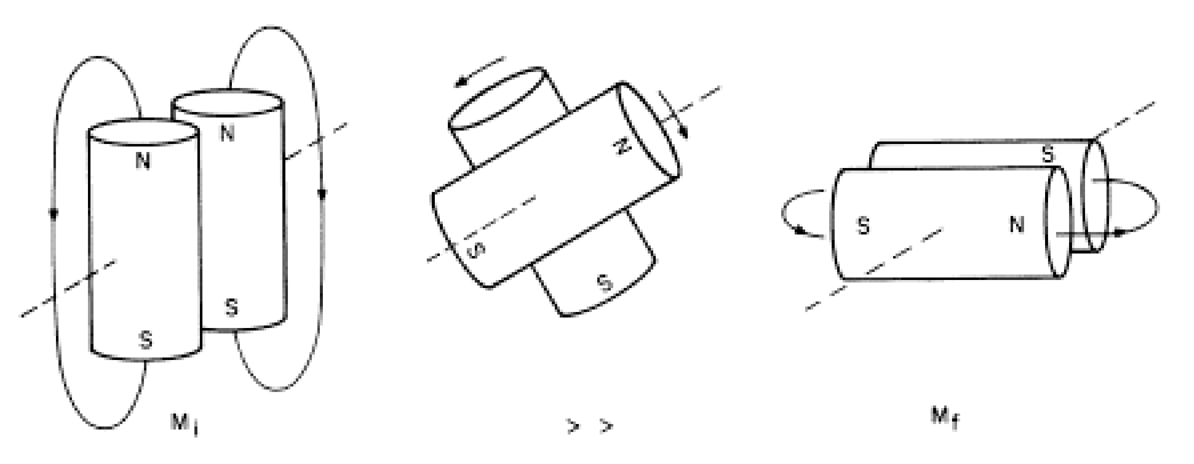
Instability of poloidal fields:

Flowers and Ruderman 1977

Initial field:

- uniform inside the star,
- matched to dipole outside

(magnetic helicity: 0)



Test of the fossil field hypothesis

Numerical results

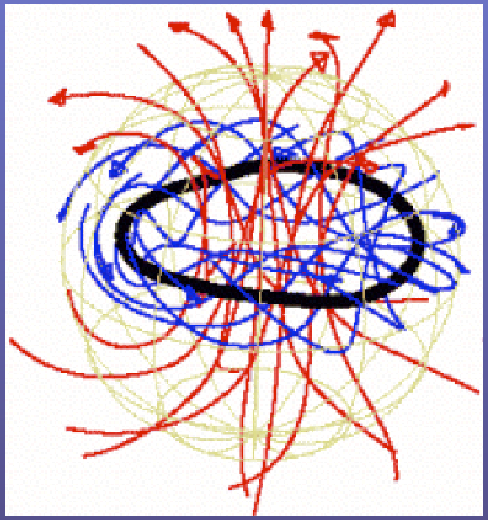
- 3D MHD simulations of global B-field in a polytropic star

- Initial random field

- Evolved over ~ 100 $\tau_{\rm A}$

(J. Braithwaite and HS, 2004, *Nature*, to appear)

Evolution of a random initial magnetic field

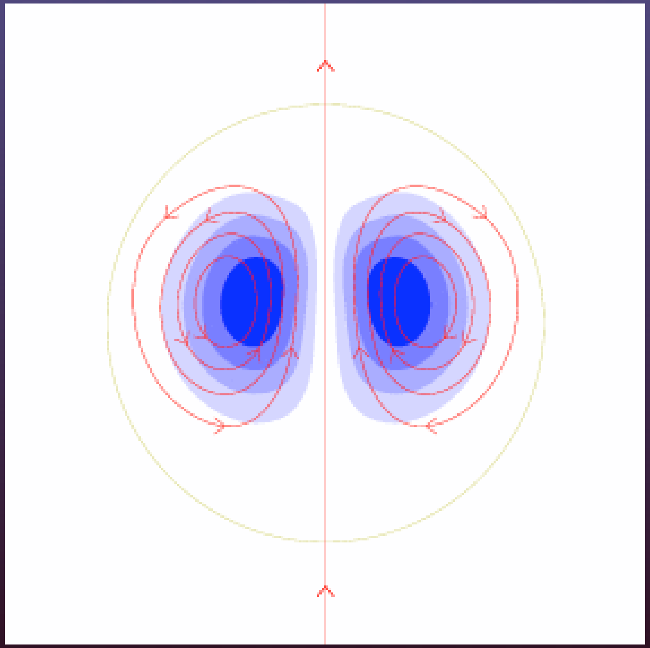


Two stages:

1. rapid evolution on τ_A

→ *Linked poloidal-toroidal torus*

- confined by gravity,
- stabilized by *stable stratification* and *twist of field*



2. Slow evolution on (Ohmic) diffusion time

Properties of the stable field

1. torus of twisted field below the surface, at the surface an approximate dipole.

2. torus somewhat decentered and deformed.

3. all initial states develop to similar configuration.

4. final strength depends on initial *magnetic helicity.*

Magnetic helicity

$$H = \int_V \mathrm{B} \cdot \mathrm{A} \qquad\qquad (\mathrm{B} = \nabla \times \mathbf{A})$$

- measures degree of global 'twist',
- tends to be conserved
  - 'Taylor states'
  - 'spheromaks'
- configuration with finite helicity can not decay completely

Interim summary: Ap, mWD fields

- fields of the magnetic Ap-Bp stars probably fossil.

- they may survive stellar evolution to produce the fields of magnetic WD.

- perhaps also relevant for cores of highly evolved stars, pulsars.

Weak initial field case

- winding-up of field lines by differential rotation.

- azimuthal field ~ t , exceeds radial component after one differential turn.

- stress $\langle B_r B_\varphi / 4\pi \rangle$ eventually reacts back.

- but if B_r weak, need to worry about *stability of* B_φ .

Instabilities in a toroidal field

1. Buoyant (Parker-) instability

2. Pinch-type instabilities ⬅

3. Magnetorotational instability (MRI)

Magnetic buoyancy instability

Newcomb 1961, Parker 1963

- happens when B decreases outward

- wavy: seaserpent displacements

- growth rate $\sim H/V_{\mathrm{A}}$

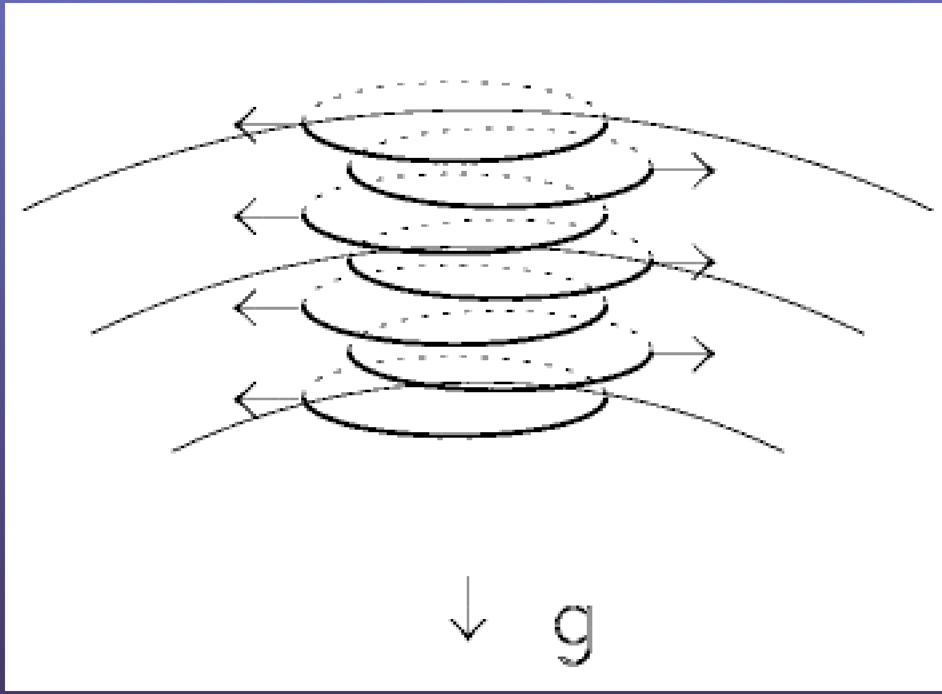
- requires strong field

$$\frac{B}{H(4\pi\rho)^{1/2}} = v_{\mathrm{A}}/H > N\left(\frac{\eta}{\kappa_{\mathrm{t}}}\right)^{1/2}$$

η : magnetic diffusivity, κ_{t} : thermal diffusivity

Tayler-instability (pinch-type in stable stratification)

- small radial length scale
- any* field strength
- growth on Alfvén time
- always present near the pole: →



R.J. Tayler 1973

First instability to set in, in a toroidal field

* without diffusion, viscosity

Note on MRI

(magnetorotational instability)

MRI is a 'magnetically enabled shear instability'

- growth rate independent of **B**,
- affected by stratification in same way,
- transport properties like shear turbulence.
- effective viscosity ~ thermal diffusivity

→ $$\nu_{\rm eff} \sim \kappa_{\rm t} = \frac{3}{16} \frac{T^3}{\kappa_{\rm R} \rho^2 c_{\rm p}}$$ (Zahn 1974)

magnetic instabilities in a stable stratification

(toroidal B-field)

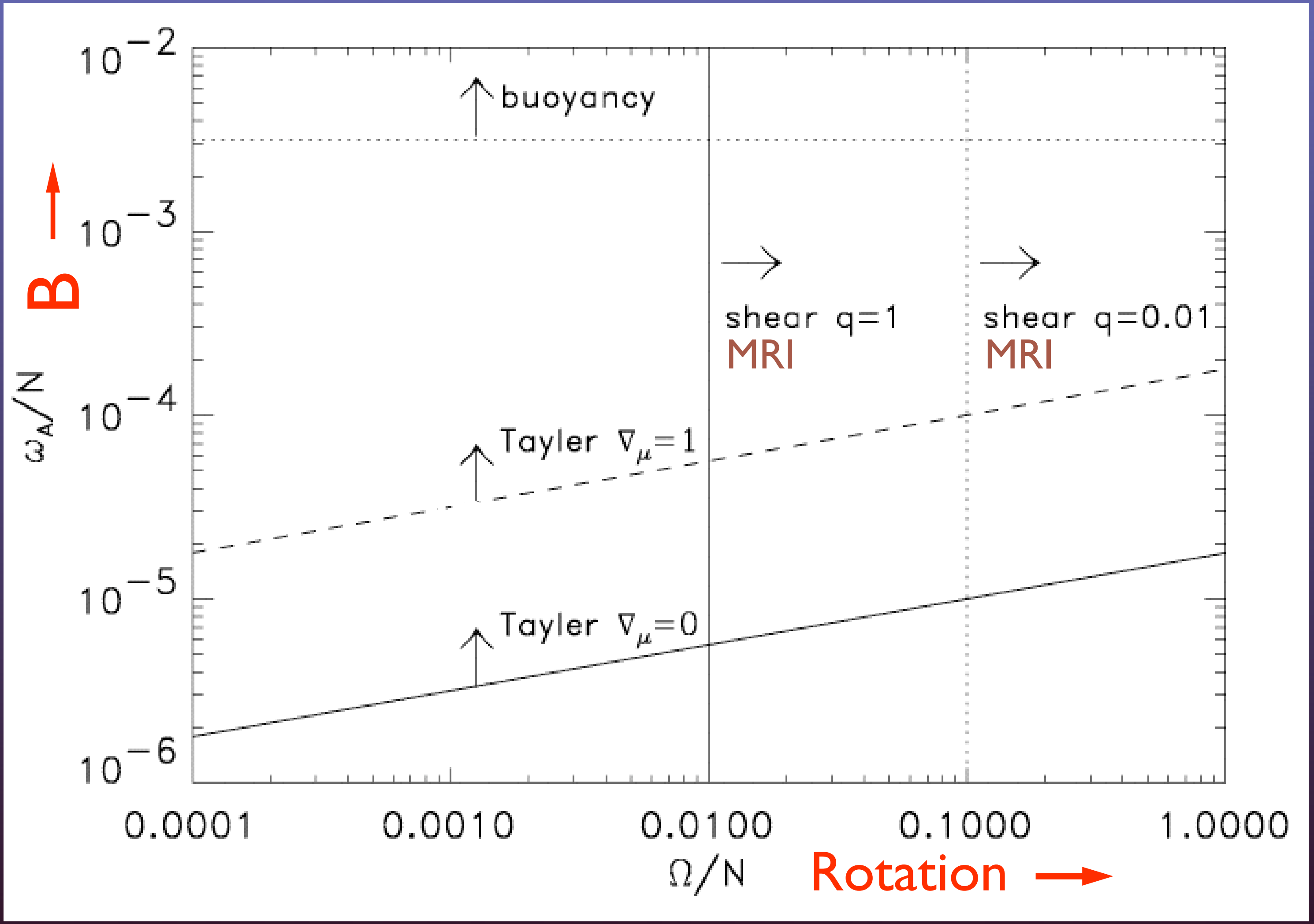
$$\mathbf{B} = B\mathbf{e}_\varphi$$

$$\omega_A = V_A/r$$

N: Brunt-V

$$q = \frac{\mathrm{d}\ln\Omega}{\mathrm{d}\ln r}$$

$$\nabla_\mu = g\frac{\mathrm{d}\ln\mu}{\mathrm{d}r}$$



A ‘dynamo’ process by (only) differential rotation

- weak field wound up into toroidal:

  $B_\varphi = t\, B_r\, r \mathrm{d}\Omega / \mathrm{d}r$

- B_φ becomes unstable to Tayler inst.

- this starts destroying B_φ (by reconnection at pole) and creates new B_r

- equilibrium when winding up (linear in time) balances instability (exponential)

  which wavenumbers most are unstable determines B_r / B_φ

‘dynamo’ (ctd)

- take into account:
  - magnetic (Ohmic) diffusion η,
  - thermal diffusion $\kappa_{\rm t}$,
  - composition gradient $\mathrm{d}\mu/\mathrm{d}r$.

- result: a field with $B_\varphi \gg B_r$
  global scale in φ, θ,
  small scale in r

- acts like a (nonlinear) viscosity

  $\nu_{\rm eff}(\Omega, \mathrm{d}\Omega/\mathrm{d}r, N, \mathrm{d}\mu/\mathrm{d}r, ...)$

- Proof of concept with 3D numerical MHD simulations by Jon Braithwaite (Thesis 2004)

consequences

- much more effective J-transport than hydro processes
- Sun: sufficient to explain uniform rotation in the core
- little mixing
- shear accumulates in the μ - gradients
- predicted intial pulsar spin (Heger et al 2004)

  ~ 10 ms

Summary

1. magnetic evolution of a star follows 1 of 2 distinct paths.

2. a strong initial field → uniformly rotating magnetic star with stable twisted torus field. explains stability of Ap, WD fields.

   importance of magnetic helicity

3. weak initial field → selfsustained (mostly toroidal) field independent of initial condition.

   importance of μ - gradients