Challenges in SN Ia Explosion Modelling

Wolfram Schmidt

- Subgrid Scale Models for Turbulent Deflagration
- Delayed Detonations
- Propagation of Detonation Waves
Challenging Questions: Turbulent Deflagration

- Initial conditions? (F. Röpke, MPA)
- What is the significance of the subgrid scale (SGS) model?
- Which SGS model is the optimal one?
- Does the optimal SGS model yield a “healthy explosion”? (W. Schmidt)
- How does rotation influence the explosion? (J. Pfannes)
Challenging Questions: Delayed Detonations

- How does a deflagration to detonation transition (DDT) come about?
- If there were DDTs, what would be the consequences for SN Ia explosions? (I. Golombek)
- Can detonation waves cross ash regions? (A. Maier)
Turbulent Deflagration in SNe Ia

- Thermonuclear burning of $^{12}\text{C}$ and $^{16}\text{O}$
- Runaway due to degenerate electron gas
- Subsonic propagation of thin flame fronts
- Low-density burning products are rising in convective plumes (RT instability)
- Shear instabilities produce turbulence
- Turbulence stretches and folds the flames
Turbulent Deflagration: Numerical Resolution

- High resolution
  - Large surface area
  - Fast burning

- Low resolution
  - Small surface area
  - Slow burning
Turbulent Deflagration: Subgrid Scale Model

- Turbulent flame propagation speed $s_t$
- $s_t \sim$ turbulent velocity fluctuations $v'$ on length scales $l \leq \Delta$
- $v'$ given by SGS turbulence energy $k_{sgs}$

\[ s_t = \max (s_{lam}, \sqrt{2k_{sgs}}) \quad \text{Niemeyer & Hillebrandt, 1995} \]

\[
\frac{D k_{sgs}}{Dt} - \frac{1}{\rho} \nabla \cdot \left( \rho C_v \Delta_{eff} k_{sgs}^{1/2} \nabla k_{sgs} \right) = C_v \Delta_{eff} k_{sgs}^{1/2} \left| S^* \right|^2 - \frac{2}{3} k_{sgs} d - C_\varepsilon \frac{k_{sgs}^{3/2}}{\Delta_{eff}}
\]
Turbulent Deflagration: Closure Parameters

Turbulence production and dissipation:
Similarity parameters $C_v$ and $C_\varepsilon$

Statistical values $C_v \approx 0.05$, $C_\varepsilon \approx 0.5$ (theory, DNS)

Ad hoc wall proximity functions (WPF): Compressibility
Clement, 1993

Localised model: Computation from resolved flow properties
Kim & Menon, 1999

Not applicable to SNe Ia

Used in MPA simulations so far

New implementation

August 5, 2004
3D Simulation of SNe Ia

- Reference model c3_3d
- Moderate resolution $256^3$
- Axisymmetric initial conditions
- Homologous grid expansion (F. Röpke)
- Influence of SGS model on energy generation, total Ni mass, flame morphology
3D Simulation (2D Sections): Energy and Mass Density

total energy

mass density
3D Simulation (2D Sections): Kinetic Energy and Strain

kinetic energy

rate of strain
Comparison of SGS Models: Flame Morphology

WPF model

localised model
Variants of the Localised SGS Model

- **Backscattering** of kinetic energy from subgrid to resolved scales: $C_v < 0$
- Complete **coupling**: SGS stresses in momentum equation
- **Asymptotic** flame speed relation for the fully turbulent regime:

  $$s_t \approx \beta \sqrt{2k_{sgs}}$$

- Optimal (with PPM): Suppress BS, $\beta \approx 1$
Comparison of SGS models: Statistics

- Total energy
- Turbulence energy
- Nickel mass
- Rate of turbulence production
Delayed Detonations

- **Hypothetical** deflagration to detonation transition *(Khokhlov, 1991)*

- **Supersonic** detonation wave burns most of remaining C+O

- Ignition of a detonation in the **distributed burning regime**: \( \delta_F \geq l_G \) if \( \rho \leq 10^7 \text{ g/cm}^3 \)

  flame thickness  Gibson length
Simulation of Artificial Delayed Detonations

- 2D simulations with PROMETHEUS
- Primary level set for deflagration front
- Ignition of a detonation once \( l_G = \delta_F \) in some numerical cell
- Secondary level set for detonation wave
- Wave propagation speed computed via ZND theory (cf. Khokhlov, 1989)
Delayed Detonation: Central Ignition

- Model Z3a
- 2D simulation
- Central ignition of deflagration at $t = 0$
- Single-point ignition of a detonation at $t = 0.85 \text{ s}$
Delayed Detonation: Non-Central Ignition (Scenario I)

- Model B1a
- Single off-centre initial bubble
- Detonation wave crosses ash regions
Delayed Detonation: Non-Central Ignition (Scenario II)

- Model B1a
- Single initial bubble
- Detonation wave cannot penetrate ash regions
Delayed Detonations: Energy Production

Total energy $\geq 0.9 \cdot 10^{51}$ erg
Delayed Detonations: Nucleosynthesis

<table>
<thead>
<tr>
<th>Model</th>
<th>$E_{\text{nuc}} \times 10^{51}$ erg</th>
<th>$E_{\text{kin}} \times 10^{51}$ erg</th>
<th>$M_{\text{Mg}} [M_{\text{sol}}]$</th>
<th>$M_{\text{Ni}} [M_{\text{sol}}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z3a</td>
<td>1.44</td>
<td>0.87</td>
<td>0.58</td>
<td>0.65</td>
</tr>
<tr>
<td>B1a</td>
<td>1.75</td>
<td>1.19</td>
<td>0.87</td>
<td>0.42</td>
</tr>
<tr>
<td>B5a</td>
<td>1.51</td>
<td>0.95</td>
<td>0.68</td>
<td>0.53</td>
</tr>
</tbody>
</table>
Propagating of Detonation Waves Through Ash

- Presently 1D simulations with FLASH
- AMR treatment of a detonation wave propagating through C+O
- Obstacle consisting of a layer of burned material ($^{56}\text{Ni}$)
Snapshots of Wave Propagation

maximum refinement level = 6

maximum refinement level = 8
Wave Propagation Speed

Insufficient numerical resolution may cause spurious detonation crossings!

![Graph showing velocity of detonation front vs. time]
Conclusions

- Flame morphology and evolution of burning is sensitive to the SGS model:
  Initially slow burning, up to 40% variation of final energy, anisotropic turbulence

- Delayed detonations produce phenomenologically realistic SNe Ia:
  Plenty of energy, nickel and intermediate mass elements

- Detonation waves possibly stall in ash