The Modeling of Surface TNR’s on Compact Stars

• Nova – review
• X-Ray bursts on NS – first steps
Multidimensional Nova Simulations

- Why?
- Who?
- How?
- What’s Now?
1D TNR Model for Classical Novae

- Hydrogen accretion on to degenerate core
- Heating by compression.
- Unstable to convection when base temp. $\approx 2 : 3 \times 10^7 \, ^0 K$
- Ignition of runaway:
  Luminosity $\approx 10^5 \, L_0$
  peak temperatures $\approx 2 : 3 \times 10^8 \, ^0 K$
Is the 1D approach justified?

- Deviation from spherical symmetry:
  (i) Convective flows.
  (ii) Early local temperature perturbations.
  (ii) Ignition; spreading mechanism.

Multi Dimensional effects can be related to:

- Source of Z enrichment?

- $\beta$ Abundances and burning rates: decay time vs. convective turnover time.
1D Temperature history

Local perturbation

H rich (solar)

C-O
(O-Ne-Mg)

\( R \sim 5.0 \times 10^8 \)

d \sim 1 \times 10^8
Time Scales

- Dynamic ~ 1 sec (sound crossing time)
- Thermal diffusion ~ 100-1000 years
- Convection (1D MLT):
  \[ V \sim 10^6 \text{ cm/sec}, \text{ scale height} \sim 5 \times 10^7 \text{ cm} \]
  Turnover time ~ 50 sec
- Burning:
  Depends on temperature and composition
  \[ 0.1 \text{ sec at } 10^8 \text{ K} \]
  \[ 10^4 \text{ sec at } 7 \times 10^7 \text{ K} \]
Mixing

- CNO (ONeMg) enrichment of the ejecta - observational fact.
- Accreted matter is solar.
- **Dredge up Mechanism?**
  
  (i) Element diffusion.
  
  
  (ii) Interaction of the convective flow with the core. Mixing increases with intensity of the flow.
  
  
  (iii) Shear flow instability (Rotation and/or Convective flow).
  
Reaction network CNO cycle

half life
O(15)=176 sec
O(14)=102 sec
N(13)=863 sec

turn over time
~ 50 sec
1D
Who?
Shankar, Arnett & Fryxell

How?
Kercek, Hillebrandt & Truran

Glasner, Livne & Truran

Alexakis, Young, Rosner, Truran, Hillebrandt, and the Flash Code Team
Semianalytical models & Dimensional considerations

  “Volcanic” localized eruptions.

  Dimensional analysis of multidimensional effects for TNRs that occur on thin stellar shells.
  Claim that for the Nova case there is initiation at a point and a flame that spreads by small scale turbulence with velocity:

\[ v = \left( \frac{h_p v_c}{\tau_b} \right)^{1/2} \]

- Eulerian - PPM - PROMETHEUS
  (Fryxell, Muller & Arnett 1989)
- Gravity, Nuclear Burning, without diffusive heat conduction.
- Computational domain:
  angular $\rightarrow$ 1 deg.
  radial $\rightarrow$ envelope (CO enriched) + few core zones.
- Very big temperature perturbations:
  100%-600%.
- Main results:
  (i) perturbations rise and cool dynamically.

- ALE – VULCAN – Livne 1993
- Explicit – Implicit
- Computational domain:
  - angular $\rightarrow$ 20 deg
  - radial $\rightarrow$ envelope (solar) + few core zones. Size $\sim$ 5x5 Km.
- Relax the convective flow; no pertur.
- Main results:
  - (i) Mixing by dredge up.
  - (ii) TNR takes place with very irregular local flames.
  - (iii) No trace for flame propagation.
  - (iv) Convective cells and convective velocities are bigger than those predicted by 1D MLT.
Kercek, Hillebrandt & Truran (A&A 337,379; 1998)

- Eulerian - PPM – improved PROMETHEUS (Fryxell, Muller & Arnett 1989)
- Computational domain: same as GLT 1997.
- Cell Size ~ 5x5 Km. (same as GLT) + HIGHER RESOLUTION ~ 1x1 Km.
- Remap the 1D zones, Relax hydro, Ignition in one zone.

- Main results:
  (i) Direct numerical 2D simulations of TNRs are feasible.
  (ii) Signs for convergence for higher resolution.
  (iii) General outcome resembles GLT 1997
  (iv) Mixing by dredge up but at a later stage.
  (iv) Somewhat less violent.
3D

- Eulerian - PPM – PROMETHEUS (Fryxell, Muller & Arnett 1989).
- Cell Size ~ 8x8 Km. (~same as 2D)
- Remap the 1D zones, Relax hydro, Ignition in one zone.

- Main results:
  (i) Flow patterns in 3D differ from those in 2D.
  (ii) Self enrichment with CO during the outburst is very slow.
  (iii) They don’t find fast Nova.
  (iv) Conclude that mixing should be prior to runaway.
A few of the things we learned up to now
Ignition process

• TNR takes place with very irregular local flames no trace for flame propagation.

• We can follow the ignition process almost from the set on of convection.
Fate of Early Perturbations

- Early perturbations are quenched.
- The pressure profile is restored on a few sound crossing times.
- Some mixing and a bit enhanced burning is induced by each perturbation.
Temp. at base $7 \times 10^7$; Perturbe at $T=50$

\[
\begin{align*}
T=50.5 & \quad \text{CNO} \\
T=53.5 & \quad \text{Q}
\end{align*}
\]
Burning rates – time history

All runs with $T_{\text{base}} = 7 \times 10^7$ Deg

Burning Rate vs. Time

- Eulerian - NO PERTURBATION
- PERTURBATION TEMPERATURE $7 \rightarrow 10$
- PERTURBATION TEMPERATURE $7 \rightarrow 8.5$
- EXTENDED ANGULAR GRID; PERTURBATION $7 \rightarrow 10$

Log($Q_{\text{out}}$) [erg/sec] vs. Time [sec]
Mixing

• Macroscopic overshot (undershoot) mixing exists from the moment the flow becomes unstable to convection. The amount of mixing increases (not linearly) with the approach towards runaway conditions.

Eulerian=>advection=>
Numerical Mixing !!!
Burning rates

- Without mixing, burning rates are limited by proton capture on C12:

\[ q_{\text{max}} = 5.8 \times 10^{13} \times \left( Z_{\text{cno}} / 0.01 \right) \]

Dredge up of fresh C12 on times shorter than beta decay time can lead to extreme burning rates that are temperature dependent.
What are the differences?

- Can we explain fast Nova?
- What is the amount of mixing during each stage of the runaway?
- What is the topology of the convective cells?
Why is it hard?
&
What are the reasons for the differences?

- Mapping the 1D initial model to 2D.
- Discontinuities in the initial model.
- Sensitivity of the burning rates to the exact amount of mixing.
- Sensitivity to the outer boundary conditions.
- The role of hydrodynamic instabilities (KH).
Initial Model

Steep gradients at the base of the envelope
Dependence of $Q$ on the amount of mixing of cold C12 with hot Hydrogen
Demonstration of the sensitivity to the outer boundary conditions

Time=130

Time=400
and indeed...

Outer Boundary Condition

Sensitivity

\[ \log(Q_{\text{out}}) [\text{erg/sec}] \]

- Eulerian-Free Flow
- Euler \rightarrow Lagrange
- Eulerian-Fix

Time [sec]
The future is…

INSTABILITIES !!!
Kelvin – Helmholtz instability
lines of equal C(12) abundance

\[ \lambda_{\text{min}} \propto U^2 / (g \cdot \Delta \rho) \]

Glasner Livne & Truran 1997 (fig.2)
What should be done?
&
Who started to do homework?
Theoretical analysis and simulations of Mixing through shear instabilities

- Distinct between:
  - (i) **KH Instability** due to a velocity jump.
  - (ii) **Critical Layer Instability** - Shear flow drive gravity waves by resonant interaction when both have the same velocity → Non linear “breaking waves” induce mixing.

- Linear analysis of the instabilities in order to find the relevant control parameters and the relevant unstable modes.

- Use the acquired knowledge in order to perform ‘wise’ simulations putting efforts to resolve the relevant modes.

- Work in those directions is done by the Flash code team in Chicago and at the Max Planck Institute.
Induced mixing by shear wind

The Flash code team; Chicago
NOVA Temp=4e7 t=10
T=20  v_scale=2e7
T = 38
T=38
Flashes on NS
H rich (solar)
Or He

\[ R \sim 1.0 \times 10^6 \]

\[ d \sim 1 \times 10^3 \]
**Time Scales**

- Dynamic ~ 1 micro sec (sound crossing time)
- Thermal diffusion ~ 10-100 Sec
- Convection (1D MLT):
  \[ V \sim 10^6 \text{ cm/sec} \quad \text{scale height} \sim 10-100 \text{ cm} \]
  \[ \text{Turnover time} \sim 10^{-5} \text{ sec} \]
- Burning: The whole fuel is consumed
- Rotation ~ 10^-3 Sec.
1D Models of the accretion
They give three different phenomenological estimates for the speed of the front:

- The width of the front is equal to the length scale of a convective cell, taken to be the scale height.
  \[ v = \frac{h\text{-scale}}{t\text{-nuc}} \sim 10^4 \text{ cm/sec} \]

- Heat is transported by turbulent convective diffusion.
  \[ v = \sqrt{D/t\text{-nuc}} \sim 2 \times 10^5 \text{ cm/sec} \quad D = h\text{-scale} \times v\text{\_conv} \]

- The turbulent scale is much larger than the front width.
  \[ v = v\text{\_conv} \sim \text{a few } 10^6 \text{ cm/sec} \]
PROPAGATION OF THERMONUCLEAR FLAMES ON RAPIDLY ROTATING NEUTRON STARS: EXTREME WEATHER DURING TYPE I X-RAY BURSTS

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ABSTRACT

We analyze the global hydrodynamic flow in the ocean of an accreting, rapidly rotating, non-magnetic neutron star in a low-mass X-ray binary during a type I X-ray burst. We use both analytical arguments and numerical simulations of simplified models for ocean burning. Our analysis extends previous work by taking into account the rapid rotation of the star and the lift-up of the burning ocean during the burst. We find a new regime for the spreading of a nuclear burning front, where the flame is carried along a coherent shear flow across the front. If turbulent viscosity is weak, the speed of flame propagation is \( v_{\text{flame}} \sim (gh)^{1/2}/f_{\nu} \sim 20 \text{ km s}^{-1} \), where \( g \) is the scale height of the burning ocean, \( h \) is the local gravitational acceleration, \( f_{\nu} \) is the timescale for fast nuclear burning during the burst, and \( f \) is the Coriolis parameter, i.e., twice the local vertical component of the spin vector. If turbulent viscosity is dynamically important, the flame speed increases, and reaches the maximum value, \( v_{\text{flame}}^{\text{max}} \sim (gh/f_{\nu})^{1/2} \sim 300 \text{ km s}^{-1} \), when the eddy overturn frequency is comparable to the Coriolis parameter \( f \).

We show that, due to rotationally reduced gravity, the thermonuclear runaway which ignites the ocean is likely to begin on the equator. The equatorial belt is ignited at the beginning of the burst, and the flame then propagates from the equator to the poles. Inhomogeneous cooling (equator first, poles second) of the hot ashes drives strong zonal currents which may be unstable to the formation of Jupiter-type vortices; we conjecture that these vortices are responsible for coherent modulation of X-ray flux in the tails of some bursts. We consider the effect of strong zonal currents on the frequency of modulation of the X-ray flux and show that the large values of the frequency drifts observed in some bursts can be accounted for within our model combined with the model of homogeneous radial expansion. Additionally, if vortices or other inhomogeneities are trapped in the forward zonal flows around the propagating burning front, fast chirps with large frequency ranges (\( \sim 25-500 \text{ Hz} \)) may be detectable during the burst rise. Finally, we argue that an MHD dynamo within the burning front can generate a small-scale magnetic field, which may enforce vertically rigid flow in the front's wake and can explain the coherence of oscillations in the burst tail.

Illustration of a burning front moving from the hot to the cold region in the atmosphere/ocean.
Global energy production rate for 2D model
Without any perturbation Temp=6e8
Radial Temp. profile at various times up to 0.002 Sec

![Temperature vs. Mass Fraction Graph](image)
Perturbation Temp $6\times 10^8 \Rightarrow 2\times 10^9$

At Time=$0$

Non existing initial velocity field
T=3e-7
Pressure
$T=1 \times 10^{-6}$
Pressure 1.0e-6
T=2e-6
Pressure
$T=5e^{-6}$
Pressure $t=5e-6$
$T = 6 \times 10^{-6}$
$T=7e^{-6}$
$T=8\times10^{-6}$
$T = 9 \times 10^{-6}$
T=10e-6
Pressure $t=10\times10^{-6}$
$T = 16 \times 10^{-6}$
Perturbation Temp $6e8 => 2e9$
At Time=$2.7e-3$ sec

existing initial
covective velocity field
Temp=2e9 t0=2.701e-3
$V_{\text{scale}}=2\times10^8$
Pressure
$T = 2.707 \times 10^{-3}$
T=2.712e-3 new Temp scale
$V_{\text{scale}}=4\times10^8$ $t=2.715\times10^{-3}$
T=2.725e-3
Perturbation Temp $6e8=>8e8$
At Time=$2.7e-3$ sec
existing initial convective velocity field
Convective Temp_pertube=8e8
t=2.701e-3  dt=1.0e-6  v_scale=5e7
Press $t=2.701e-3$
He abundance $t=2.701\times10^{-3}$
2.704e-3
\[ T = 2.707 \times 10^{-3} \]
T=2.709  v_scale=1e8
$T = 2.713 \times 10^{-3}$, $v$-scale $= 2 \times 10^8$
Change temp scale t=2.719e-3
T = 2.735 \times 10^{-3}
$T = 2.745 \times 10^{-3}$
$T = 2.755 \times 10^{-3}$
END