Weak coupling approach to dense quark matter: condensed matter physics of QCD

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INT program, 2004 Seattle
QCD and dense matter:
From lattices to stars

Outlines

- Quark Matter and Color Superconductivity
- Pattern of Symmetry Breaking
- The gap equation in CSC
- Meissner and Debye mass of gluon and photon in CSC
- Guage parameter independence of the gap
- Comparison of Color-SC with normal SC and superfluidity

This talk is based on

1. A. Schmitt, Q. Wang, D. H. Rischke, Phys. Rev. D66, 114010(2002).
2. A. Schmitt, Q. Wang, D. H. Rischke, Phys. Rev. Lett. 91, 242301(2003).
3. A. Schmitt, Q. Wang, D. H. Rischke, Phys. Rev. D69, 094017(2004)
4. D.-F. Hou, Q. Wang, D. H. Rischke, Phys. Rev. D69, 071501(2004),
5. A. Mishra, Q. Wang, D. H. Rischke, in preparation.

Quark Matter and Color Superconductivity

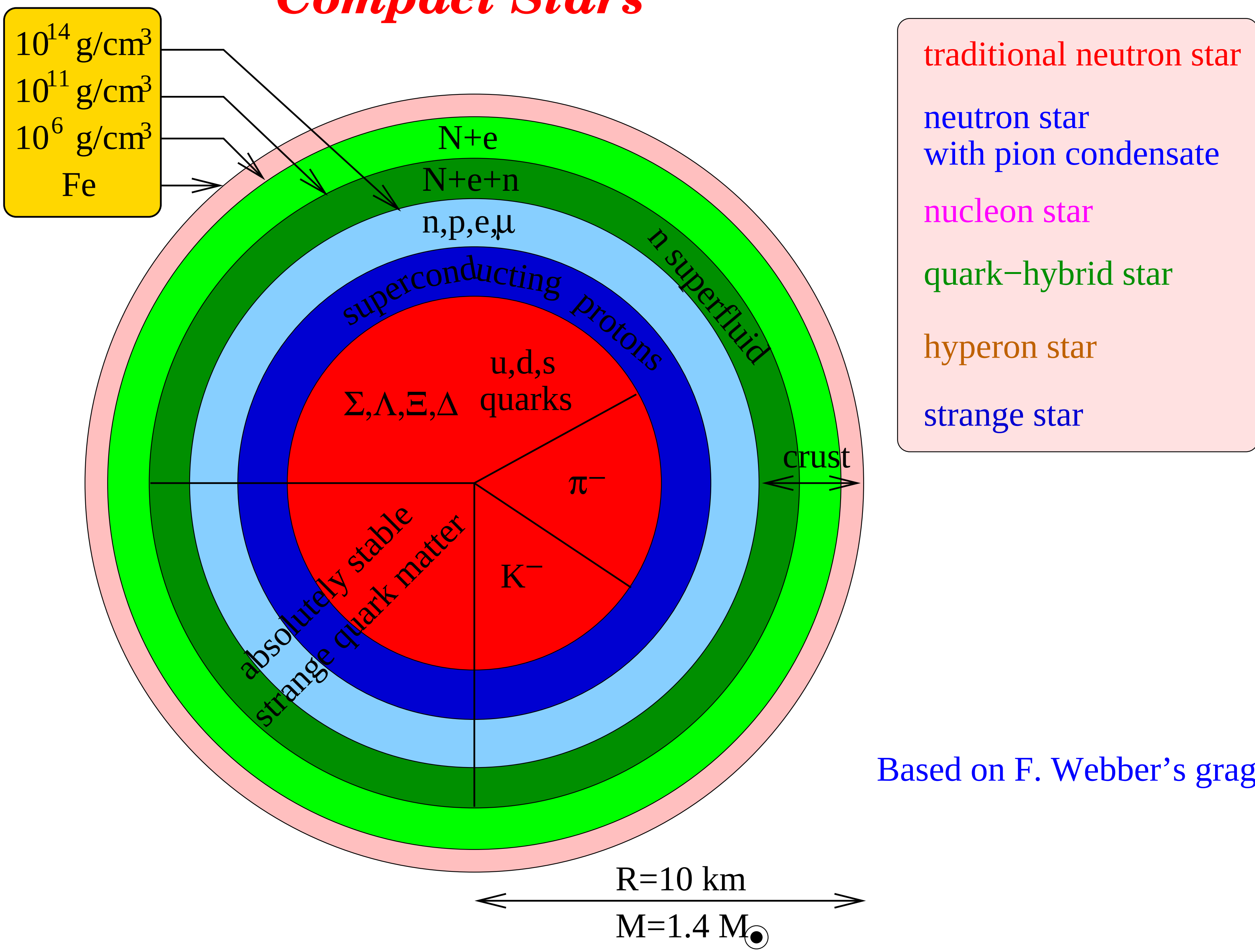
Matter at High Density

We study this because we want to know Properties of

▶ dense matter that exists in our universe
(density in compact stars: $\sim$ 200 million ton/cm^3)

▶ Condensed matter physics of Quantum Chromodynamics, the theory of strong interaction

Compact Stars

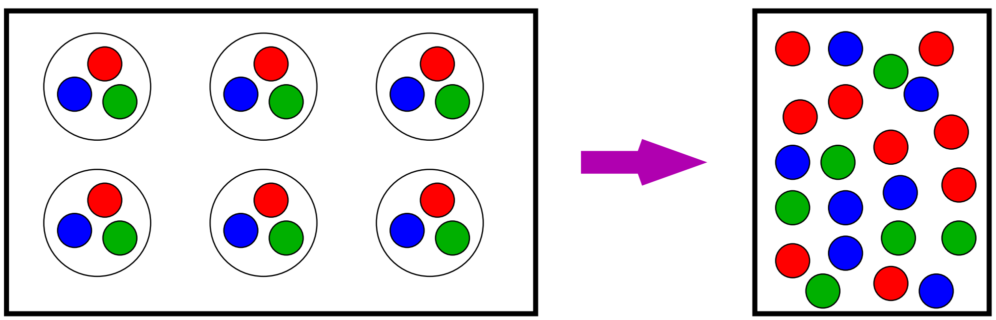


Based on F. Webber's gragh

Quark Matter and Color Superconductivity

Dense Quark Matter

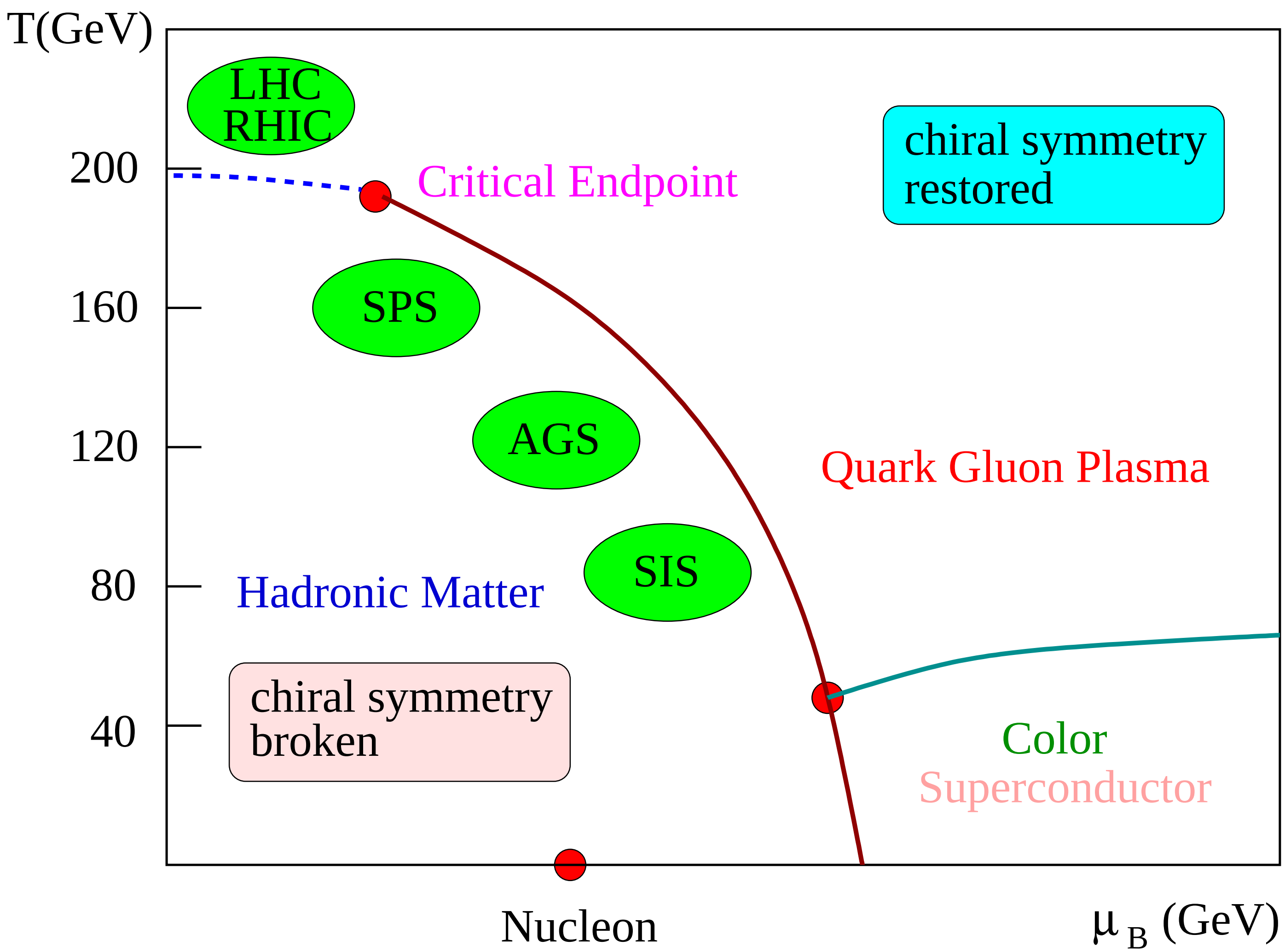
- Squeezing baryonic matter to produce quark matter



- Very dense quark matter is weakly interacting [Collins & Perry, 1975].

- Asymptotic freedom: $\alpha_s(\mu) \ll 1$ for $\mu \gg \Lambda_{QCD}$ [Gross, Wilczek and Politzer, 1973].

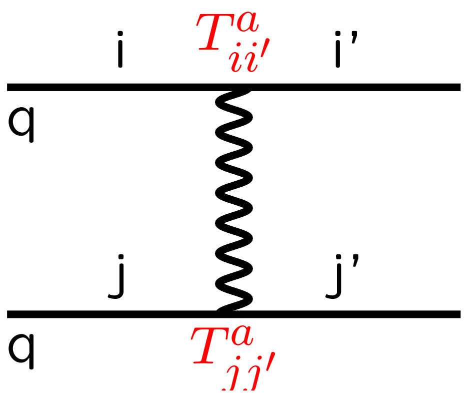
QCD phase diagram



Ruester,Shovkovy,Rischke (2004)
Kryjevsky,Kaplan,Schafer (2004)
Iida, et al. (2004)
Alford (2001)
Rajagopal (1999)

Quark Matter and Color superconductivity

Ground State of Quark Matter: Color superconductor



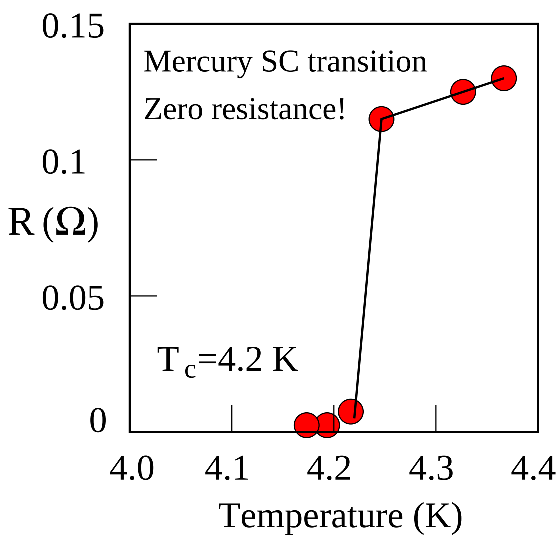
$$T^a_{ii'}T^a_{jj'} = \frac{1}{2}\delta_{ij'}\delta_{i'j} - \frac{1}{2N_c}\delta_{ii'}\delta_{jj'}$$
$$= \frac{1}{6}(\delta_{ii'}\delta_{jj'} + \delta_{ij'}\delta_{i'j}) - \frac{1}{3}(\delta_{ii'}\delta_{jj'} - \delta_{ij'}\delta_{i'j})$$

★ CSC: Robust and Simpler

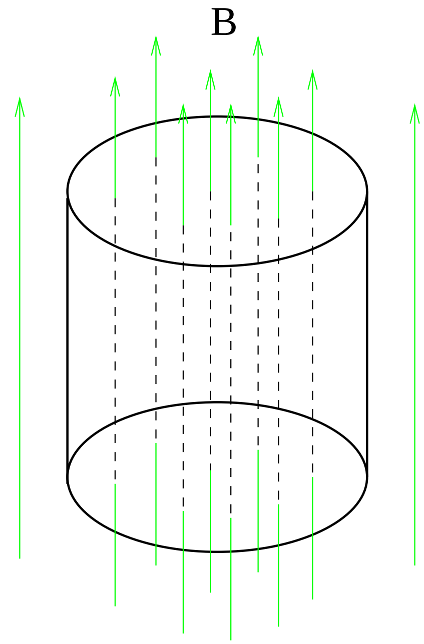
- Any attractive interaction will lead to Cooper instablity.
- The attractive interaction in QCD is directly between two quarks.

Basics of color superconductivity

Normal superconductivity



H. K. Onnes
Commun. Phys. Lab. 12, 120(1911)

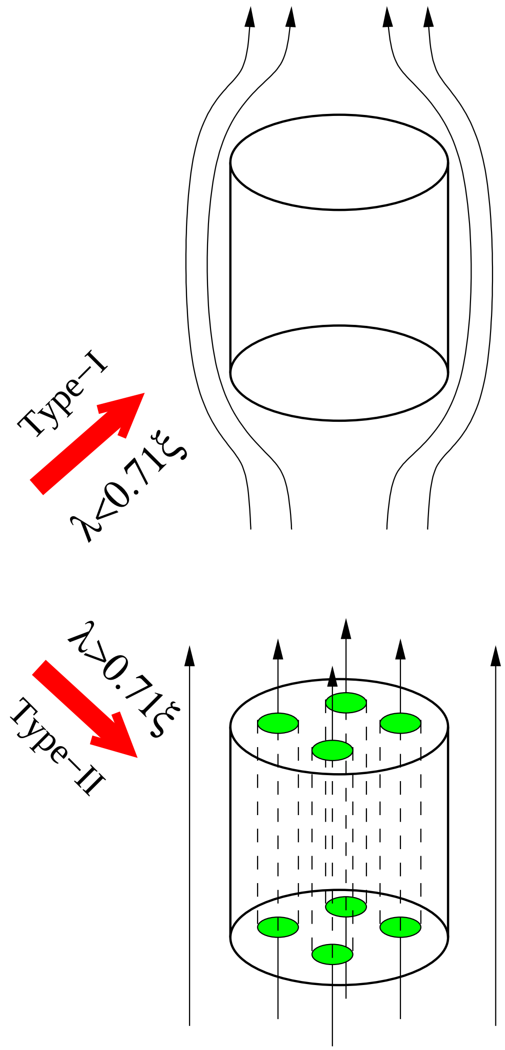


two scales:

penetration length λ

coherence length ξ

Meissner−Ochsenfeld Effect



Type II SC and magnetic vortex

Landau & Ginzburg, 1950

Abrikosov, 1957

Meissner and Debye mass of gluon and photon in CSC

Two scales

- The penetration length λ is the inverse of gluon mass of $O(g\mu)$: $\lambda \sim 1/m_g \sim 1/(g\mu)$. Assume gluon acquires a mass, EOM becomes

$$
\begin{aligned}
& \partial_\mu \partial_\mu A_\nu + m^2 A_\nu = 0 \\
\rightarrow \ & (-\nabla^2 + m^2) A_\nu = 0 \ \ (static \ \ limit) \\
\rightarrow \ & (\frac{\partial}{\partial x^2} - m^2) A_\nu = 0 \ \ (1 - dimension) \\
\rightarrow \ & A_\nu \sim \exp(-mx) \equiv \exp(-x/\lambda) \ \ (solution)
\end{aligned}
$$

- Coherence length ξ is inverse of the gap of $O(\phi)$: $\xi \sim 1/\phi$

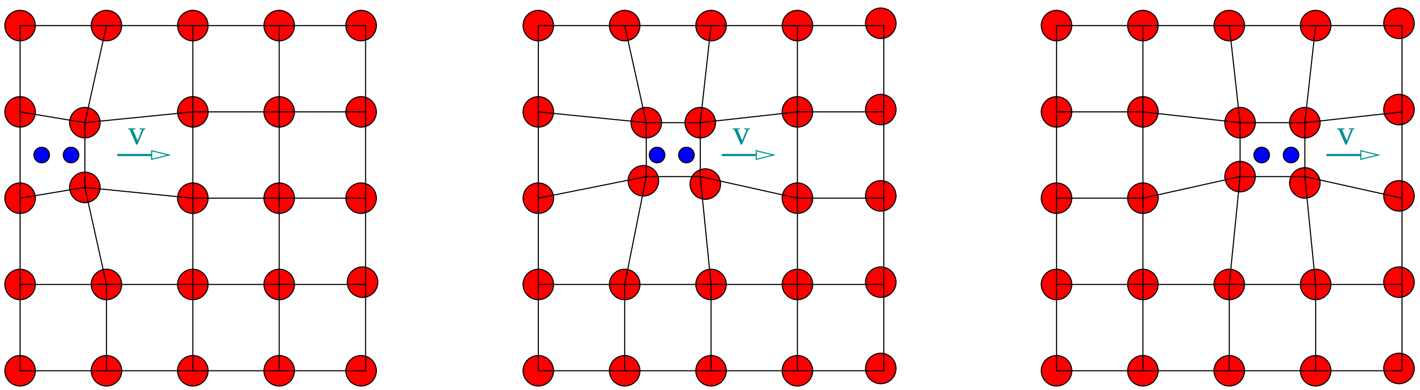
- $\lambda \ll \xi \rightarrow$ type-I; $\lambda \gg \xi \rightarrow$ type-II.

Quark Matter and Color Superconductivity

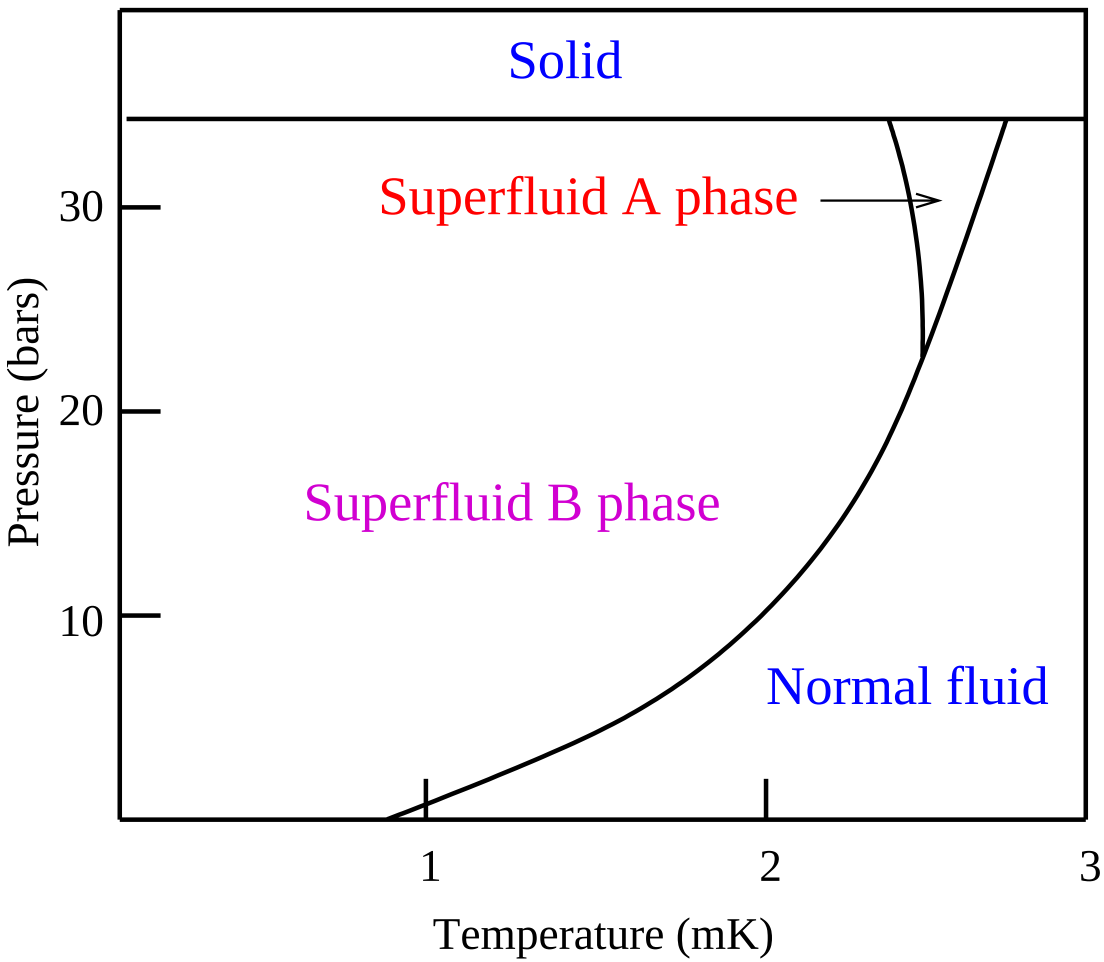
Normal superconductivity

Theory of Bardeen, Cooper & Schriefer (BCS)

The concept of Cooper pair: Two electrons work coherently to travel in the potential valley without feeling the resistance of the lattice, where the effective interaction between them is attractive though they are Coulomb-repulsive.



Superfluid He3

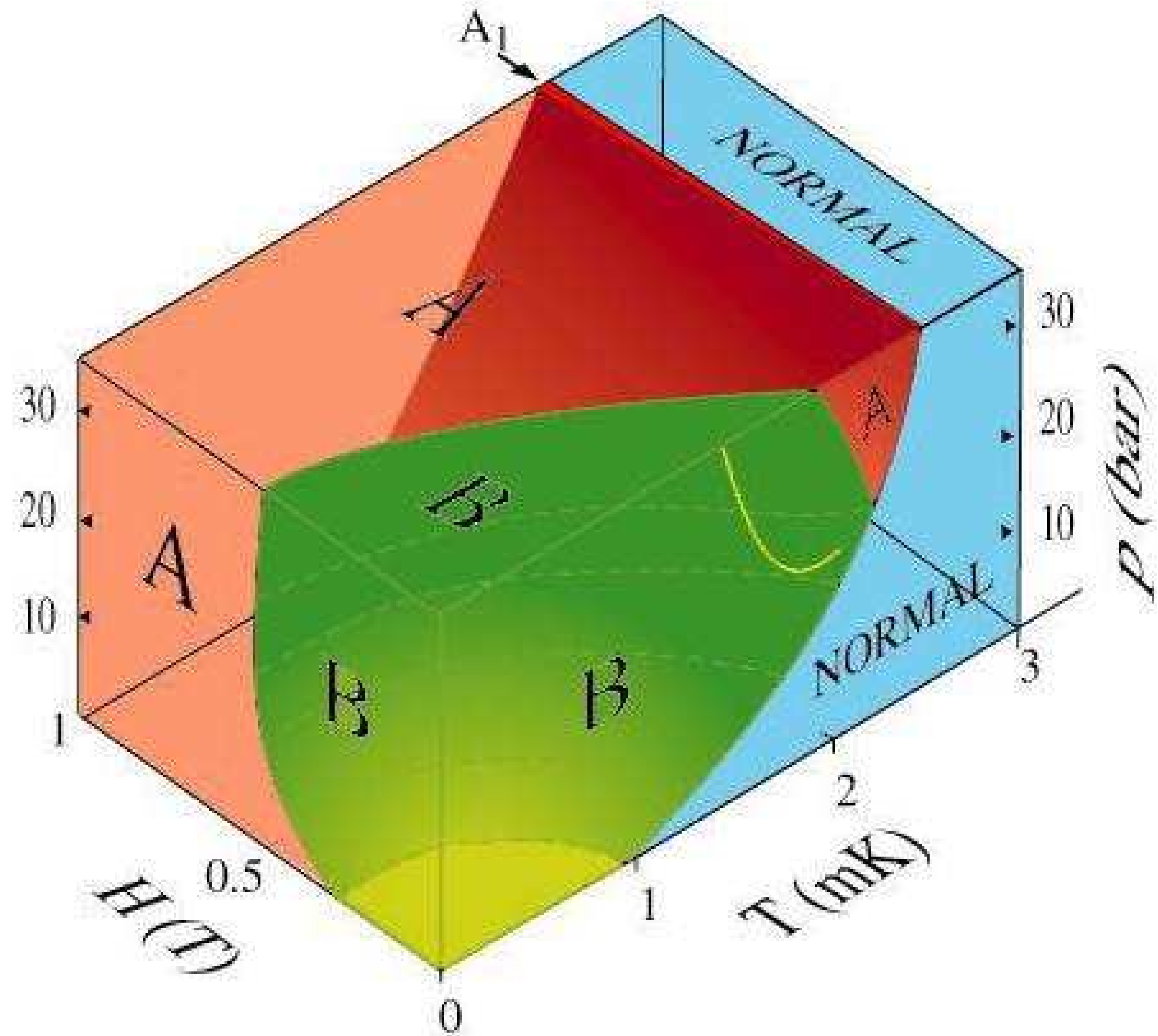


Osheroff, Richardson, Lee, 1973

Legget 1975

Cooper pair of two helium atom

due to Van−der−Waals force



A1
NORMAL
A
A
A
B
B
B
B
NORMAL
30
20
10
1
0.5
0
H (T)
1
2
3
T (mK)
30
20
10
p (bar)

Quark Matter and Color Superconductivity

Observation of a fermionic condensate

C. A. Regal, M. Greiner and D. S. Jin

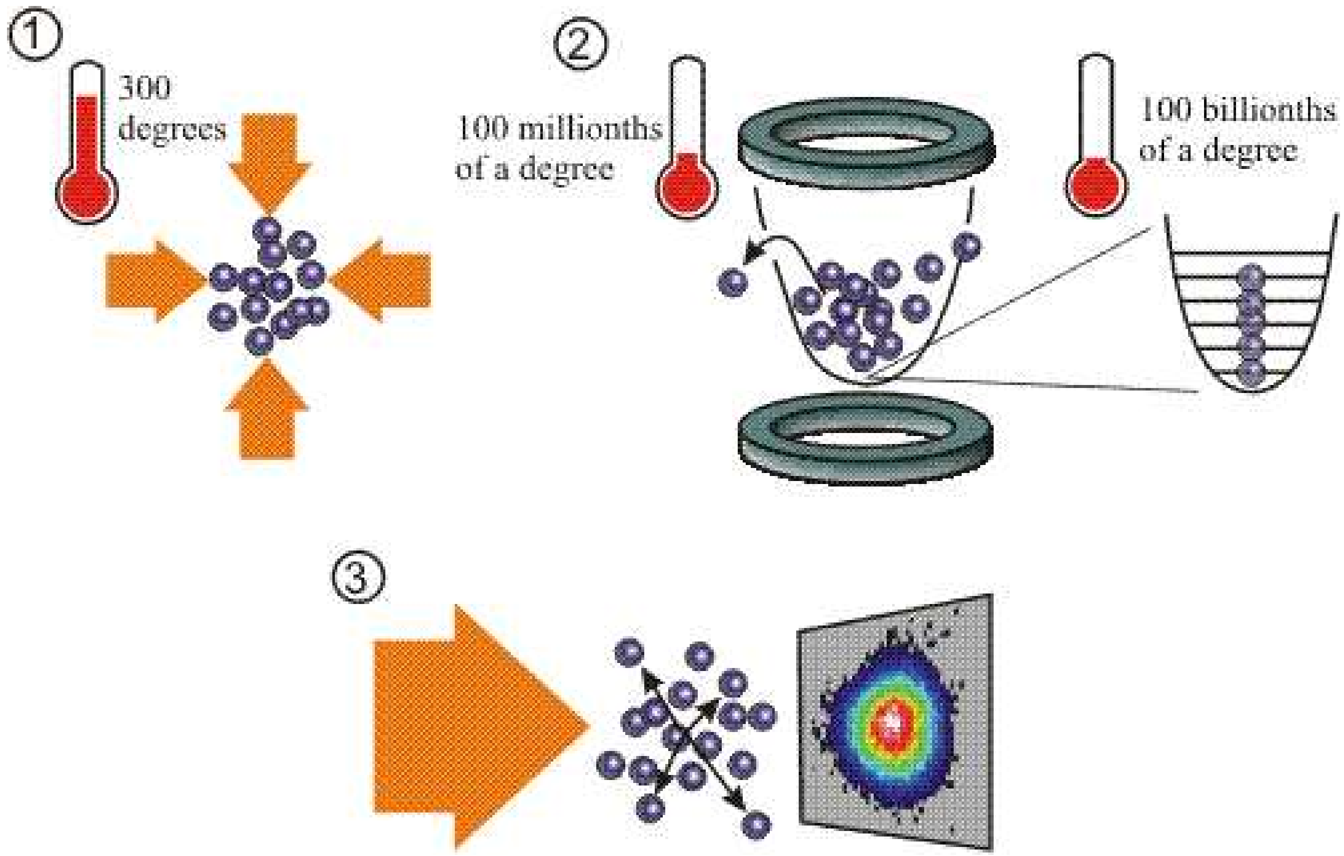
Observation of resonance condensation of fermionic atom pairs

Physical Review Letters 92, 040403(2004).

- ^{40}K at 150 nK in magnetic field
- attraction between pairs of atoms
- the gas collapsed into the BEC

"Making atoms in a gas pair like the electrons in a superconductor might lay bare the inner workings of high-temperature superconductors, neutron stars, and primordial matter–and perhaps win the creator of this much-anticipated atomic soup a Nobel Prize..."

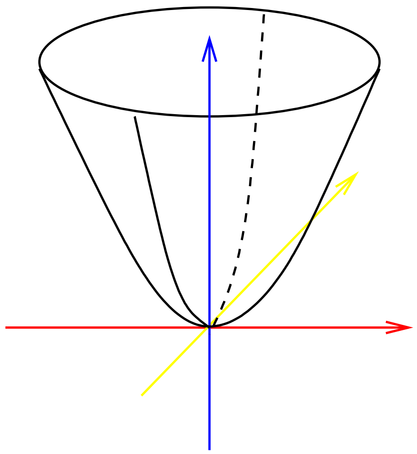
[Science 301, 750(2003)]

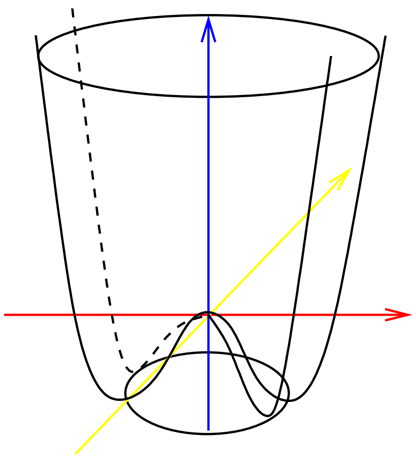


1
300
degrees
2
100 millionths
of a degree
100 billionths
of a degree
3

Pattern of symmetry breaking

Anderson-Higgs mechanism





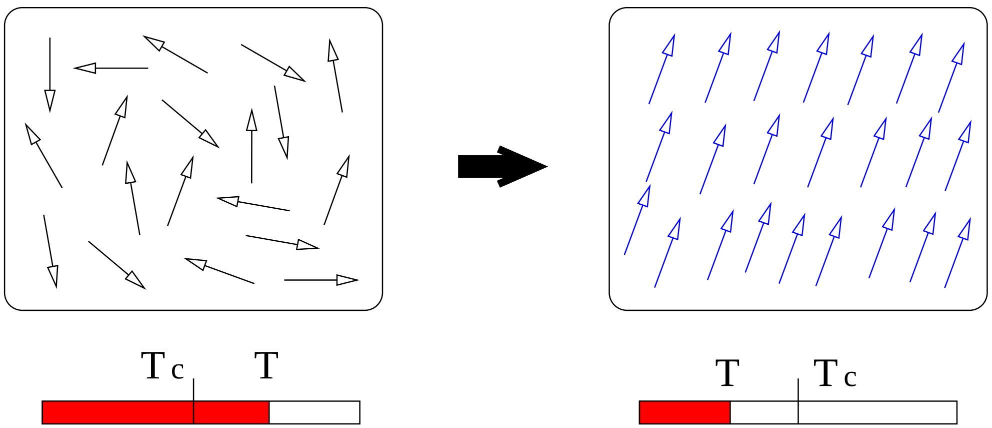
- $V_{eff} = a\phi^2 + b\phi^4$
  $a = a'(T - T_c), \;\; a', b > 0$
- $T < T_c \rightarrow$ Goldstone modes
- Goldstone modes can be eaten by gauge boson
  $\rightarrow$ massive gauge boson
  $\rightarrow$ Meissner effect

Goldstone Theorem: Suppose $\mathcal{L}(\phi) \in G$ and $\mathcal{L}(\phi_0) \in H$, where $H \subset G$, then there are $n = dim(G) - dim(H)$ Nambu-Goldstone bosons and massive gauge bosons.

Pattern of symmetry breaking

Example I: Ferromagnetism

Order parameter: magnetization $\langle M \rangle$

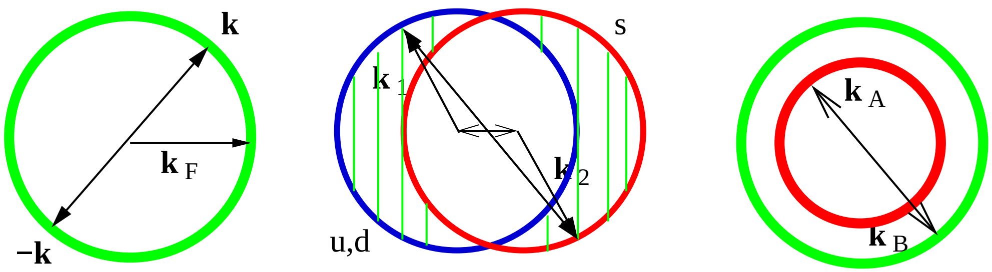


$$\begin{array}{rcl} SO(3) & \longrightarrow & U(1) \\ \langle M \rangle = 0 & \longrightarrow & \langle M \rangle \neq 0 \\ T > T_c & \longrightarrow & T < T_c \end{array}$$

Pattern of symmetry breaking

Example II: (Normal) Superconductivity

Order parameter: pair density $\langle \psi^\dagger \psi^\dagger \rangle$; Symmetry breaking pattern: particle number and electric charge conservation is broken.



q=k−k=0 BCS pairing | 2q= k 1 + k 2 LOFF pairing | q= k A + k B Interior paring

$$U(1)_N, U(1)_{em} \longrightarrow 1$$
$$\langle \psi^\dagger \psi^\dagger \rangle = 0 \longrightarrow \langle \psi^\dagger \psi^\dagger \rangle \neq 0$$
$$T > T_c \longrightarrow T < T_c$$

Pattern of symmetry breaking

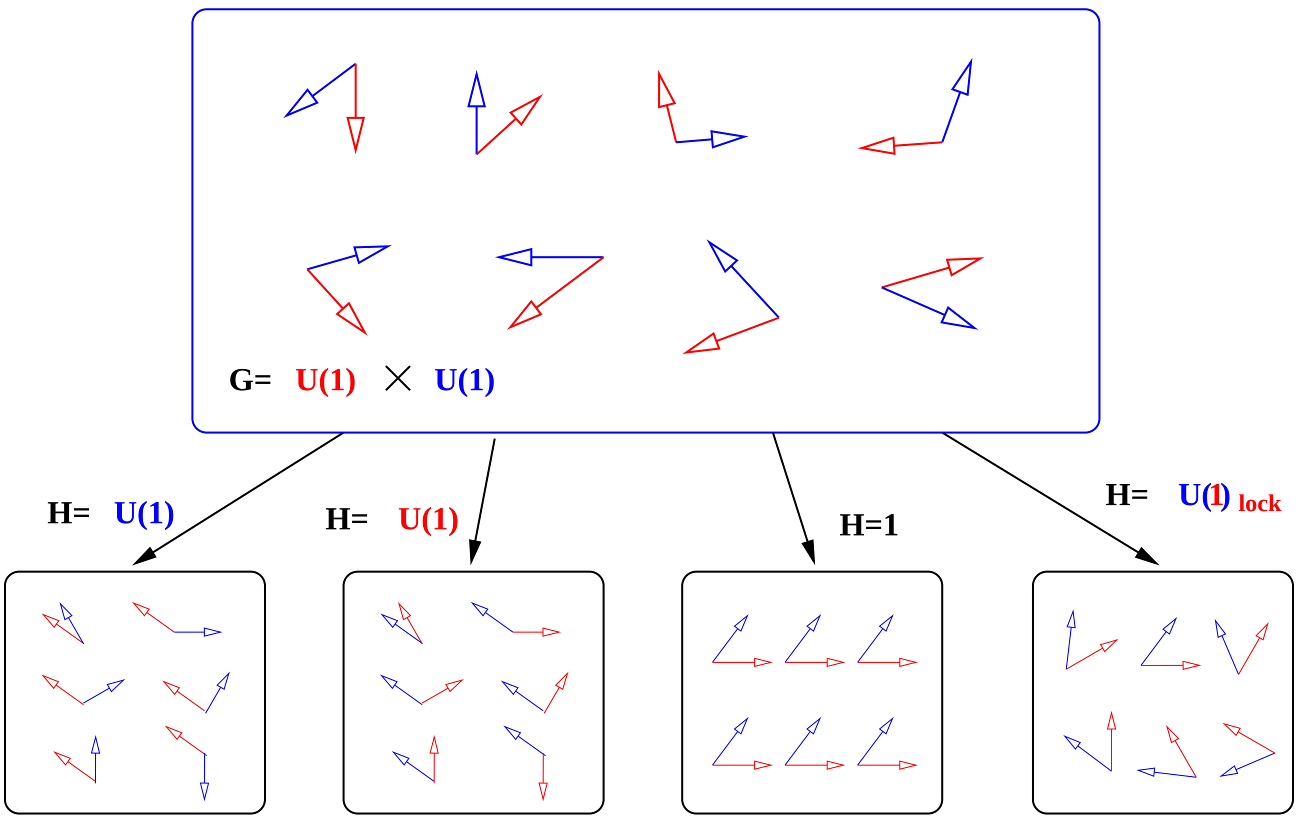
Example III: ^3He Superfluidity

Because the atomic interaction potential becomes repulsive for short distance, then the Cooper pair in ^3He must be in $L = 1$ (anti-symmetric), $S = 1$ (symmetric) state.

Symmetry group: $G = SO(3)_L \times SO(3)_S \times U(1)_N$. Three typical symmetry breaking pattern:

- $G \to H_B = SO(3)_{L+S}$ (B phase)
- $G \to H_A = U(1)_S \times U(1)_{L+N}$ (A phase)
- $G \to H_{A_1} = U(1)_{S+N} \times U(1)_{L+N}$ (A_1 phase)

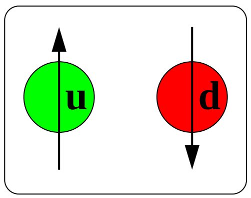
Pattern of symmetry breaking



An example of symmetry breaking and locked state: $G = U(1) \times U(1) \rightarrow H$

Pattern of symmetry breaking

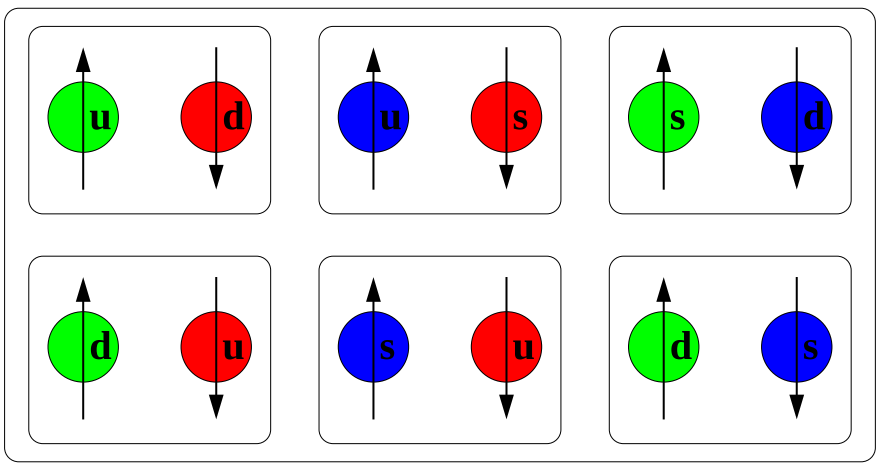
Some well-known color super-phases



- 2SC, $N_f = 2$, spin-0 gap

$$SU(3)_c \times SU(2)_f \rightarrow SU(2)_c \times 1_f$$

[D. Bailin, A. Love, Phys. Rep. 107, 325(1984)]



- Color Flavor Locking, CFL

$N_f = 3$, Spin-0 gap

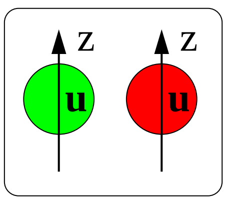
$$SU(3)_c \times SU(3)_f \rightarrow SU(3)_{c+f}$$

[M. Alford, K. Rajagopal, F. Wilczek, Nucl. Phys. B537, 443(1999)]

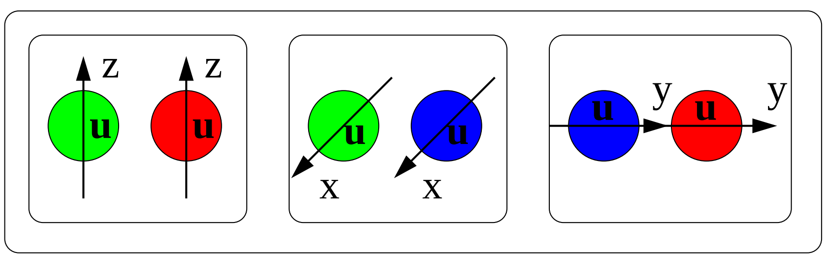
Pattern of symmetry breaking

Some well-known color super-phases

• If $\mu_u \neq \mu_d \neq \mu_s$ and their differences are large enough, 2SC and CFL are not favored. Pairing occurs between quarks of the same flavor $\rightarrow$ CSL or polar phase.



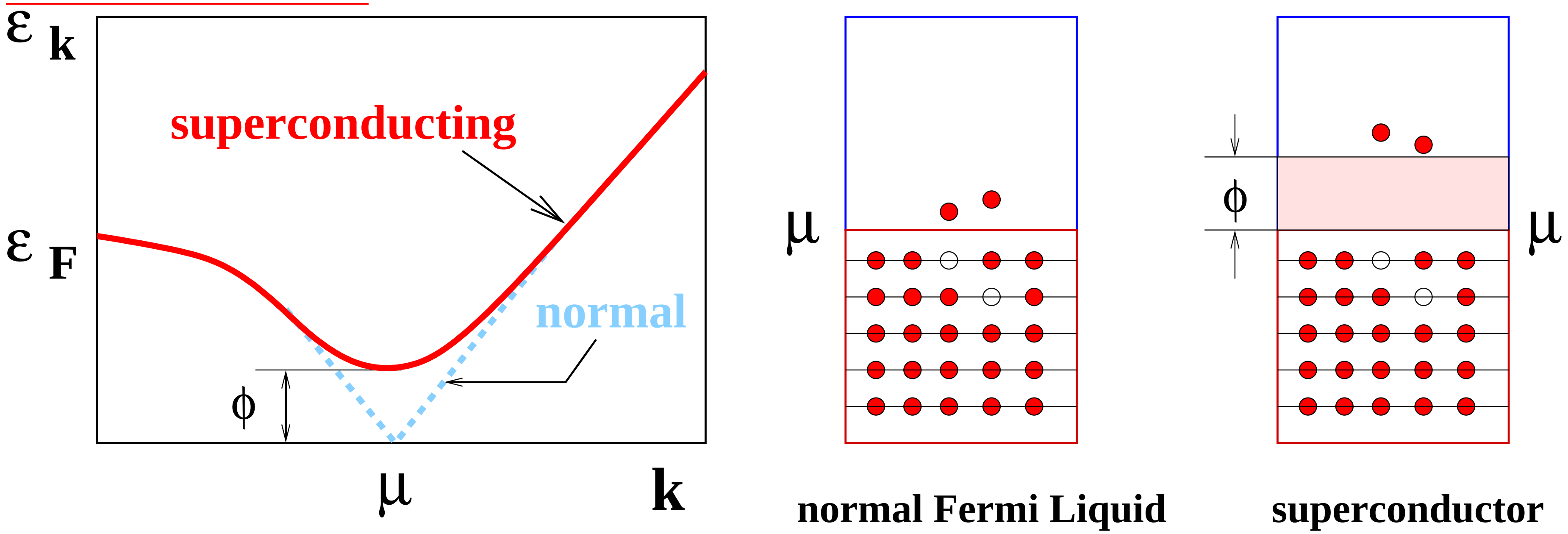
• Polar phase, $N_f = 1$, spin-1 gap
$SU(3)_c \times SO(3)_J \rightarrow SU(2)_c \times U(1)_J$
[T. Schafer, Phys. Rev. D62, 094007(2000)]



• Color spin locking, CSL
$N_f = 1$, spin-1 gap
$SU(3)_c \times SO(3)_J \rightarrow SO(3)_{c+J}$
[D. Bailin, A. Love, Phys. Rep. 107, 325(1984),
T. Schafer, Phys. Rev. D62, 094007(2000)]

The gap equation in CSC

The energy gap



Excitation energy of quasi-particle:

- No gap: $\epsilon_{\mathbf{k}} = |k - \mu|$;
- With gap: $\epsilon_{\mathbf{k}} = \sqrt{(k-\mu)^2 + \phi^2}$

Gap equation and its solution to subleading order

- Nambu-Gorkov basis:

$$\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix} \quad \overline{\Psi} = (\overline{\psi}, \overline{\psi}_C)$$

$\psi_C = C\overline{\psi}^T$, $\psi = C\overline{\psi}_C^T$
$\overline{\psi}_C = \psi^T C$, $\overline{\psi} = \psi_C^T C$
$C = i\gamma^2\gamma_0$

- The action is:

$$I(\overline{\Psi}, \Psi) = \frac{1}{2}\int_{x,y} \overline{\Psi}(x) S^{-1}(x,y) \Psi(y)$$
$$= \frac{1}{2}\sum_k \overline{\Psi}(k) \frac{S^{-1}(k)}{T} \Psi(k)$$

$\int_x \equiv \int_0^{1/T} d\tau \int_V d^3\mathbf{x}$

$k_0 = -i(2n+1)\pi T$ is Matsubara frequency
$\sum_k \equiv V \sum_n \int d^3\mathbf{k}/(2\pi)^3$

Gap equation and its solution

Propagator

Propagator for free fermions
$S^0{}_{11} = (\gamma^\mu Q_\mu + \mu\gamma_0)^{-1}$

Standard 1-L SE

$\Sigma_{12} = \gamma_0 \Sigma_{21}^\dagger \gamma_0$

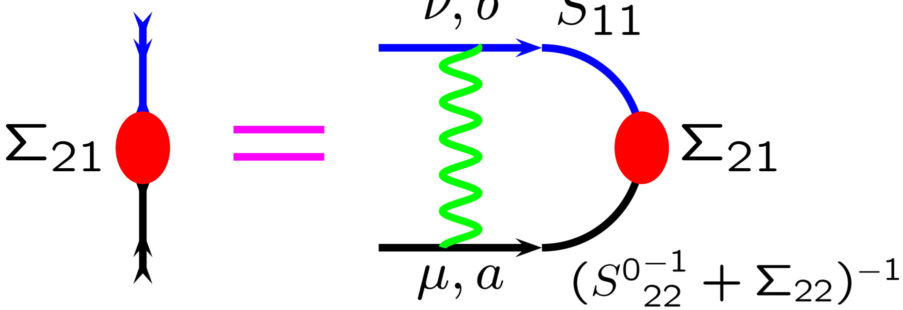
$$S^{-1} = \begin{pmatrix} {S^0}_{11}^{-1} + \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & {S^0}_{22}^{-1} + \Sigma_{22} \end{pmatrix}$$

$\Phi^+ = \langle \psi_C \overline{\psi} \rangle$
gap matrix for CS

$S^0{}_{22}$ is for charge-conjugate free fermions
$S^0{}_{22} = (\gamma^\mu Q_\mu - \mu\gamma_0)^{-1}$

Gap equation and its solution

Mean field approximation



Nambu-Gorkov matrix
$\hat{\Gamma}_a^\mu = \mathrm{diag}(\gamma^\mu T_a, -\gamma^\mu T_a^T)$

$$\Sigma_{21}(K) = -g^2 \frac{T}{V} \sum_Q \Delta^{ab}_{\mu\nu}(K-Q) \left[\hat{\Gamma}^\mu_{a22} S_{21}(Q) \hat{\Gamma}^\nu_{b11} \right]$$

Gap equation and its solution

Weak-coupling solution

The solution to the gap equation: [Son 1999; Schafer & Wilczek 2000; Hong et al. 2000; Pisarski & Rischke 2000]

$$\phi_0 = 2\,\tilde{b}\,b_0\,b_1\,\mu\,\exp\left(-\frac{\pi}{2\bar{g}}\right)$$

$\tilde{b} \equiv 256\pi^4[2/(N_f g^2)]^{5/2}$

b_0: $\exp\left(-\frac{4+\pi^2}{8}\right)$

b_1: Depends on phases

Gap equation and its solution

Summary of results for weak-coupling solution

Gap magnitude at $T = 0$ and transition temperature T_c:

	2SC	CFL	CSL	polar
spin	0	0	1	1
ϕ_0/ϕ_0^{2SC}	1	$2^{-1/3}$	$2^{-2/3}e^{-5}$	$e^{-d(\theta)}$
T_c/ϕ_0	0.57	$2^{1/3} \times 0.57$	$2^{2/3} \times 0.57$	0.57

• Two gap structure (non-zero gap) leads to violation of BCS relation about the ratio of T_c to ϕ_0.

• The exponential factor is from angular structure of color-spin-locking state.

Meissner and Debye mass of gluon and photon in CSC

Residual symmetry

Question: whether all eight gluons and and the photon become massive in super phase?
Answer: this depends on pattern of local symmetry breaking. If there is a residual symmetry, the corresponding gauge bosons remain massless. The residual symmetry group of $SU(3)_c \times U(1)_{em}$ leaves the gap matrix Δ invariant:

$$(g_c \times g_{em})\Delta(g_c^T \times g_{em}^T) = \Delta$$

where $g_c \in SU(3)_c$ and $g_{em} \in U(1)_{em}$.

	representation	order parameter	generators of residual group	η
2SC	$\bar{\mathbf{3}}_c \otimes \mathbf{1}_f \otimes \mathbf{1}_J$	δ_{i3}	$T_1, T_2, T_3, Q + \eta T_8$	$-1/\sqrt{3}$
CFL	$\bar{\mathbf{3}}_c \otimes \bar{\mathbf{3}}_f \otimes \mathbf{1}_J$	δ_{ij}	$Q + \eta T_8$	$2/\sqrt{3}$
CSL	$\bar{\mathbf{3}}_c \otimes \mathbf{3}_J$	δ_{ik}	$-$	$-$
polar	$\bar{\mathbf{3}}_c \otimes \mathbf{3}_J$	$\delta_{i3}\delta_{k3}$	$T_1, T_2, T_3, Q + \eta T_8$	$-2\sqrt{3}\,q$

Meissner and Debye mass of gluon and photon in CSC

Mixing of photon and gluon

- New charge $\tilde{Q}$ and $\tilde{T}_8$:

$$\begin{pmatrix} \tilde{Q} \\ \tilde{T}_8 \end{pmatrix} = \begin{pmatrix} 1 & \eta \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} Q \\ T_8 \end{pmatrix}$$

- New photon and new gluon:

$$\begin{pmatrix} \tilde{A} \\ \tilde{A}_8 \end{pmatrix} = \begin{pmatrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{pmatrix} \begin{pmatrix} A \\ A_8 \end{pmatrix}$$

where $\cos\theta = g^2/(g^2 + \eta^2 e^2)$.

- New coupling constants:

$$\tilde{g} = g\cos\theta \; , \; \tilde{e} = e\cos\theta \; , \; \lambda = -\eta \frac{e^2}{g^2}$$

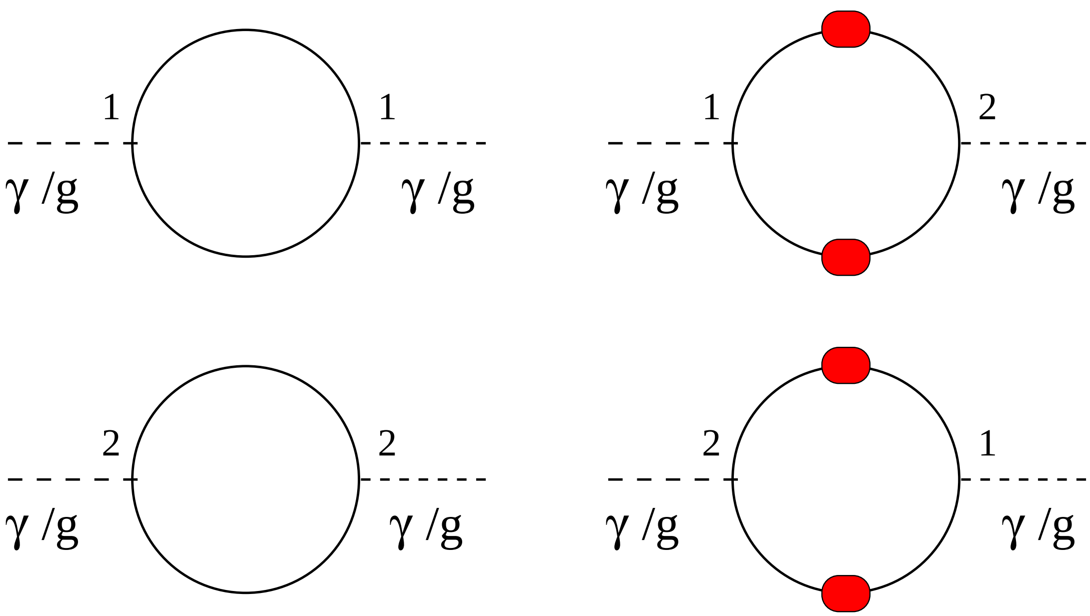
Analogy to Standard Model

	Weinberg-Salam	$\mathbf{N_f = 2, 3}$ color superconductivity
gauge group	$SU(2) \times U(1)$ isospin, hypercharge	$SU(3)_c \times U(1)_{em}$ color, electromagnetism
gauge fields	W_1, W_2, W_3, W_0	$A_1, \ldots, A_8, A$ gluons & photon
coupling constants	G, G'	g, e
symmetry breaking	$SU(2) \times U(1) \to U(1)_{em}$	$SU(3)_c \times U(1)_{em} \to \tilde{U}(1)_{em}$
new fields	W^+, W^- $Z = \cos\theta_W W_3 + \sin\theta_W W_0$ $A = -\sin\theta_W W_3 + \cos\theta_W W_0$	$A_1, \ldots, A_7$ $\tilde{A}_8 = \cos\theta A_8 + \sin\theta A$ $\tilde{A} = -\sin\theta A_8 + \cos\theta A$
new coupling constants	$e = G' \cos\theta_W$	$\tilde{e} = e\cos\theta$
massless fields	A	$\tilde{A}$ (= no Meissner mass!)

Meissner and Debye mass of gluon and photon in CSC

Quark loops with condensate

To get masses, we compute these diagrams in NG basis



diagonalize (9,9) matrix Π^{ab} for a,b=0,1,...,8

Meissner and Debye mass of gluon and photon in CSC

Summary of results

- Zero-temperature rotated Debye masses

	$\tilde{m}^2_{D,88}$	$\tilde{m}^2_{D,\gamma\gamma}$	$\cos^2\theta_D$
2SC	$3\,g^2$	$2\,e^2$	1
CFL	$(4\,e^2 + 3\,g^2)\,\zeta$	0	$3g^2/(3g^2+4e^2)$
polar	$3\,g^2$	$18\,q^2e^2$	1
CSL	$3\,\beta\,g^2$	$18\,q^2e^2$	1

$$\begin{aligned} \text{Unit} &= N_f\mu^2/6\pi \\ \zeta &\equiv (21-8\ln 2)/54 \\ \alpha &\equiv (3+4\ln 2)/27 \\ \beta &\equiv (6-4\ln 2)/9 \end{aligned}$$

- Zero-temperature rotated Meissner masses

	$\tilde{m}^2_{M,88}$	$\tilde{m}^2_{M,\gamma\gamma}$	$\cos^2\theta_M$
2SC	$\frac{1}{3}g^2+\frac{1}{9}e^2$	0	$3g^2/(3g^2+e^2)$
CFL	$\left(\frac{4}{3}e^2+g^2\right)\,\zeta$	0	$3g^2/(3g^2+4e^2)$
polar	$\frac{1}{3}g^2+4\,q^2e^2$	0	$g^2/(g^2+12\,q^2e^2)$
CSL	$\beta\,g^2$	$6\,q^2e^2$	1

No Meissner effect for 2SC and CFL: electromagnetic wave can propagate in 2SC and CFL phase in the form of rotated photon

Meissner and Debye mass of gluon and photon in CSC

Summary of results

• Rotated photon can penetrate into 2SC and CFL phases $\longrightarrow$ no electromagnetic Meissner effect.

• Rotated photon in CSL phase has a non-vanishing mass $\longrightarrow$ Electromagnetic Meissner effect.

• Although rotated photon in polar phase has a zero mass but a system with 2 or 3 flavors still exhibits Electromagnetic Meissner effect because of different chemical potential or no single mixing angle for all flavors.

• Since $\lambda \sim 1/(g\mu) \ll \xi \sim 1/\phi$, spin-1 color-superconductor (CSL or polar phase) is expected to be a type-I superconductor.

Gauge parameter independence of the gap

Some concepts

- Gauge parameter ξ in covariant gauge:

$$D_{\mu\nu}(P) = \frac{P^L_{\mu\nu}}{P^2 + \Pi_l} + \frac{P^T_{\mu\nu}}{P^2 + \Pi_t} - \xi \frac{P_\mu P_\nu}{P^4}$$

- Gauge parameter independence: $\phi(\xi) = \phi(\xi')$

[Gerhold & Rebhan 2003]

- Approximated gauge independence in the mean field approach:

[Rajagopal & Shuster 2000; Pisarski & Rischke 2002]

$$\phi(\xi) = \phi(\xi') + \Delta\phi$$
$$\Delta\phi \sim O(g^2\phi)$$

Gap equation and its solution

Weak-coupling solution and schematical gap equation

The solution to the gap equation: [Son 1999; Schafer & Wilczek 2000; Hong et al. 2000; Pisarski & Rischke 2000]

$$\phi_0 \;=\; 2\,\tilde{b}\,b_0\,b_1\,\mu\,\exp\left(-\frac{\pi}{2\bar{g}}\right)$$

$\tilde{b} \equiv 256\pi^4[2/(N_f g^2)]^{5/2}$

b_0: $\exp\left(-\frac{4+\pi^2}{8}\right)$

b_1: Depends on phases

Schematical gap equation

$$\phi \;=\; g^2\phi_0\left[\xi\,\ln^2\left(\frac{\mu}{\phi_0}\right) + \beta\,\ln\left(\frac{\mu}{\phi_0}\right) + \alpha\right]$$

ξ: $O(1/g^2)$ leading static magnetic

β: $O(1/g)$–subleading non-static magnetic static electric quark-selfenergy

α: $O(g^2)$–subsubleading

Gauge parameter independence

Problem in covariant gauge

- The gap is NOT invariant to subleading order in covariant gauge,

[D.K. Hong, V.A. Miransky, I.A. Shovkovy, L.C.R. Wijewardhana, Phys. Rev. D61, 056001(2000)]

$$\phi = e^{3/2\xi}\phi_0$$

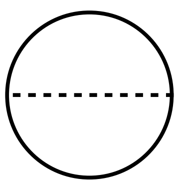
From 1-loop contribution to the gap eqn. at subleading order

- Hint to solution of the problem: Impose on-shell condition!

$$\begin{aligned} K_{on} = (\epsilon_{\mathbf{k}}, \mathbf{k}) &\longrightarrow (\phi, \mu\hat{\mathbf{k}}) \\ \mathbf{S}^{-1}(K_{on})\Psi_{on}(K_{on}) &= 0 \\ \overline{\Psi}_{on}(K_{on})\mathbf{S}^{-1}(K_{on}) &= 0 \end{aligned}$$

Gauge independence in covariant gauge

2PI Diagrams



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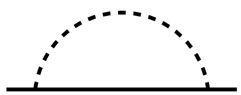
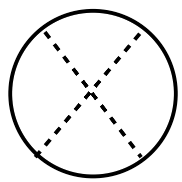


(a1)



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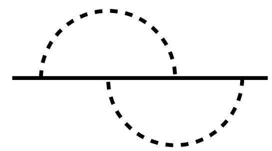

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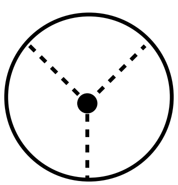

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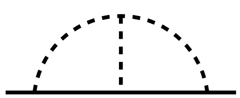

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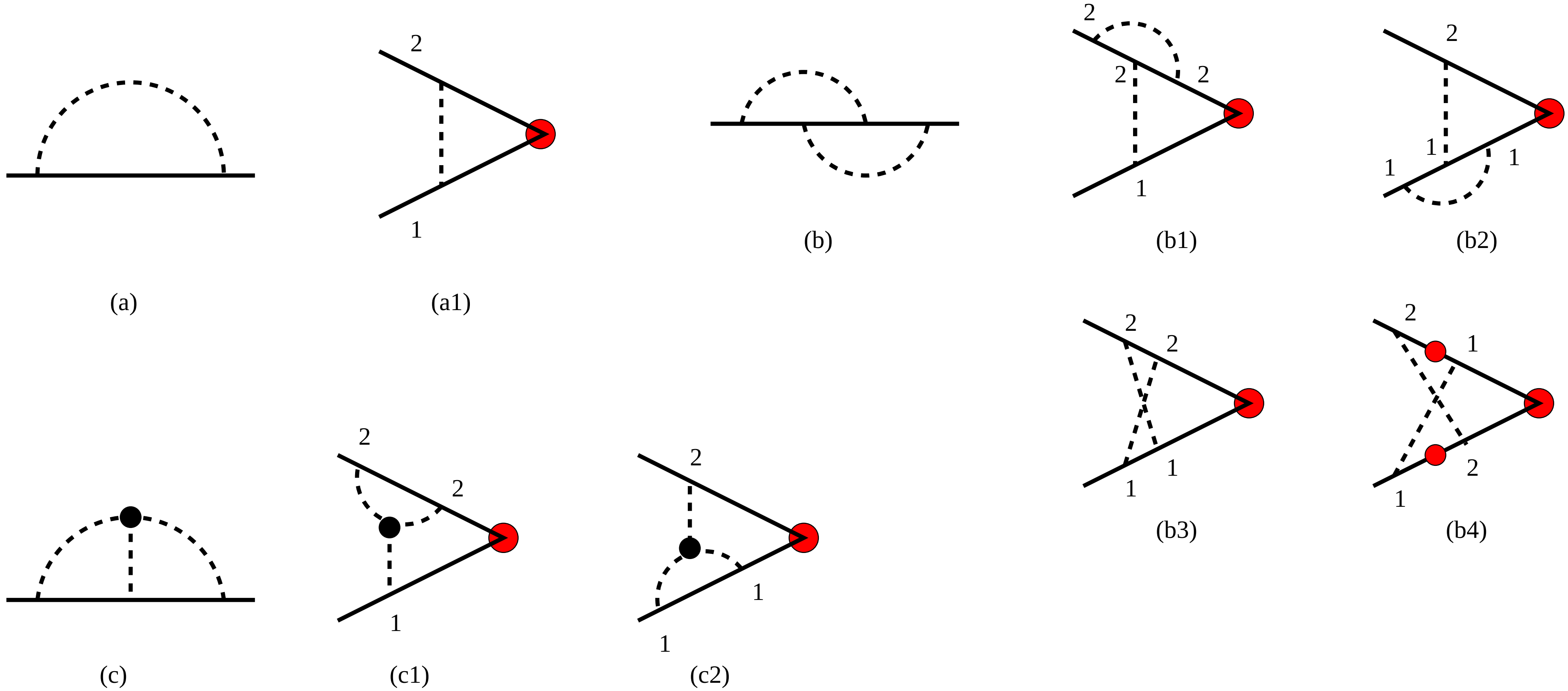
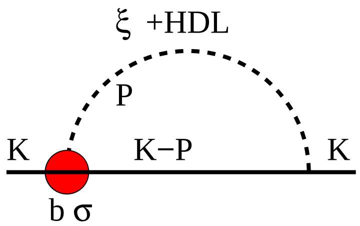
Gauge independence in covariant gauge

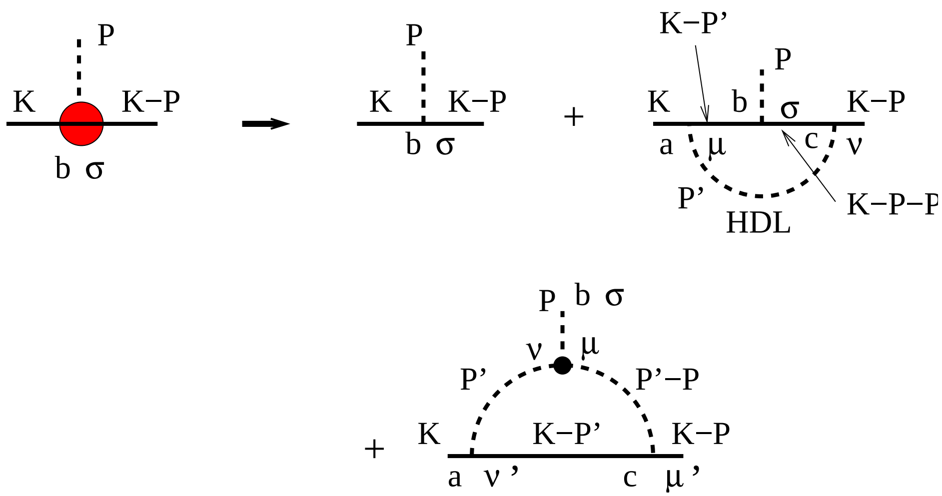
Diagrams upto 2-loop in NG basis



Gauge independence in covariant gauge

Full vertex





Gauge independence in covariant gauge

1-loop correction involving Abelian vertex

$$iP_\sigma \Lambda^b_{1\sigma}(P,K,K-P)$$
$$= \frac{g^3}{(2\pi)^4}\int d^4P'\; iD^{HDL}_{\mu\nu}(P')i\mathbf{T}^a\gamma_\mu i\mathbf{S}(K-P')$$
$$\times i\mathbf{T}^b \not{P} i\mathbf{S}(K-P-P')i\mathbf{T}^a\gamma_\nu$$

$$\mathbf{S}^{-1}(K-P') - \mathbf{S}^{-1}(K-P-P')$$

$$
\begin{aligned}
& iP_\sigma \Lambda^b_{1\sigma}(P,K,K-P) \\
&= g[i\mathbf{\Sigma}(K)\mathbf{T}^b - \mathbf{T}^b i\mathbf{\Sigma}(K-P)] \\
&+ g f^{abc}\mathbf{T}^a[\mathbf{\Sigma}_{nc}(K) - \mathbf{\Sigma}_{nc}(K-P)]\mathbf{T}^c \\
&- g^3 \int \frac{d^4P'}{(2\pi)^4}\, D^{HDL}_{\mu\nu}(P')\mathbf{T}^a\gamma_\mu \mathbf{S}(K-P') \\
&\times [\mathbf{T}^b, \mathbf{S}^{-1}]\mathbf{S}(K-P-P')\mathbf{T}^a\gamma_\nu
\end{aligned}
$$

$$
\begin{pmatrix}
0 & (T^a J_3 + J_3 T^{aT})\tau_2\gamma_5\phi^{e*}\Lambda^{-e} \\
(T^{aT} J_3 + J_3 T^a)\tau_2\gamma_5\phi^e\Lambda^e & 0
\end{pmatrix}
$$

Gauge independence in covariant gauge

1-loop correction involving 3g vertex

$$P_\sigma i\Lambda^b_{2\sigma}$$
$$= g^2 \int \frac{d^4P'}{(2\pi)^4} iD^{HDL}_{\nu'\nu}(P') iD^{HDL}_{\mu'\mu}(P'-P)$$
$$\times iV^{abc}_{\nu\sigma\mu}(P',P,P'-P) P_\sigma \mathbf{T}^a i\gamma_{\nu'} i\mathbf{S}(K-P')\mathbf{T}^c i\gamma_{\mu'}$$

$$iV = iV^{(0)} + iV^{HDL}$$

$$
\begin{aligned}
iV^{(0)} &= iV^{(0)P} + iV^{(0)F} \\
&= -gf^{abc}\Big\{\Big[P'_{\nu} g_{\sigma\mu} + (P' - P)_{\mu} g_{\sigma\nu}\Big] \\
&\quad + \Big[(-2P' + P)_{\sigma} g_{\mu\nu} - 2P_{\nu} g_{\sigma\mu} + 2P_{\mu} g_{\sigma\nu}\Big]\Big\}
\end{aligned}
$$

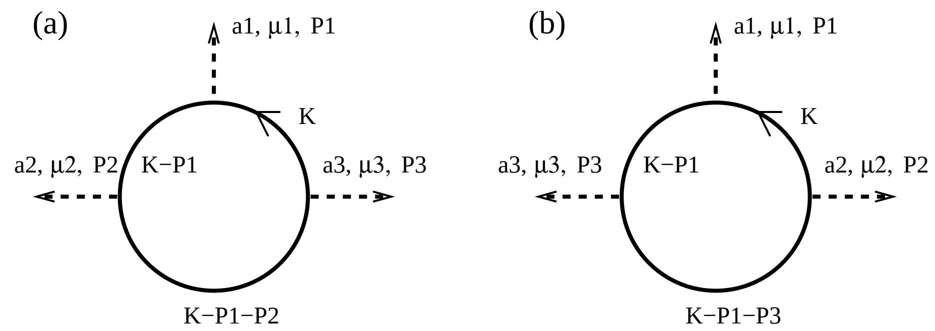
Pinch technique

[J.M. Cornwall, PRD 26,1453(1982); J.M. Cornwall, J. Papavassiliou, PRD 40,3474(1989);]

Gauge independence in covariant gauge

Ward identity for $3g$ vertex

$$P_\sigma i V^{(0)F}_{\nu\sigma\mu} = -g f^{abc}(P^2 - 2P\cdot P')g_{\mu\nu}$$
$$= -g f^{abc}\left[(P'-P)^2 - P'^2\right] g_{\mu\nu}$$



$$i P_{1\mu_1} V^{HDL;a_1a_2a_3}_{\mu_1\mu_2\mu_3} = ig\mathrm{Tr}[T^{a_1}(T^{a_2}T^{a_3} - T^{a_3}T^{a_2})]$$
$$\times\left[\Pi^{nc}_{\mu_2\mu_3}(P_3) - \Pi^{nc}_{\mu_2\mu_3}(P_2)\right]$$
$$= -g f^{a_1a_2a_3}\left[\Pi^{\mu_2\mu_3}(P_3) - \Pi^{\mu_2\mu_3}(P_2)\right]$$

Gauge independence in covariant gauge

Ward identity for $3g$ vertex

$$
\begin{aligned}
P_\sigma i V^{abc}_{\nu\sigma\mu} &= P_\sigma [i V^{(0)F;abc}_{\nu\sigma\mu} + i V^{HDL;abc}_{\nu\sigma\mu}] \\
&= -g f^{abc} \left[D^{-1}_{HDL\,\mu\nu}(P') - D^{-1}_{HDL\,\mu\nu}(P'-P) \right]
\end{aligned}
$$

Substituting it back into $iP_\sigma \Lambda^b_{2\sigma}$, we obtain

$$
\begin{aligned}
& P_\sigma i \Lambda^b_{2\sigma}(K, K-P) \\
= \; & -g f^{abc} \mathbf{T}^a \left[\mathbf{\Sigma}_{nc}(K) - \mathbf{\Sigma}_{nc}(K-P) \right] \mathbf{T}^a
\end{aligned}
$$

One can see that the above cancels the second term in $iP_\sigma \Lambda^b_{1\sigma}$.

Gauge independence in covariant gauge

Generalised Ward identity with condensate

$$P_\sigma i\Lambda^b_\sigma = g[i\mathbf{\Sigma}(K)\mathbf{T}^b - \mathbf{T}^b i\mathbf{\Sigma}(K-P)] + I^b_X$$

$$I^b_X = -g^3 \int \frac{d^4P'}{(2\pi)^4}\, D^{HDL}_{\mu\nu}(P')\mathbf{T}^a\gamma_\mu \mathbf{S}(K-P')$$
$$\times[\mathbf{T}^b, \mathbf{S}^{-1}]\mathbf{S}(K-P-P')\mathbf{T}^a\gamma_\nu$$
$$\sim g^3\phi \ln^2\phi$$

When inserted into the gap equation, we can show that I^b_X contributes to subsubleading order.

Generalised Ward identity with condensate

$$P_\sigma i\Gamma^b_\sigma = ig[\mathbf{S}^{-1}(K)\mathbf{T}^b - \mathbf{T}^b\mathbf{S}^{-1}(K-P)] + I^b_X$$

This generalised Ward identity can also be derived from more formal path integral (or functional) approach. [Hou, Wang & Rischke, 2004]

[Similar form for normal superconductor was derived by Nambu 1964].

Gauge independence in covariant gauge

Vertex correction to the gap

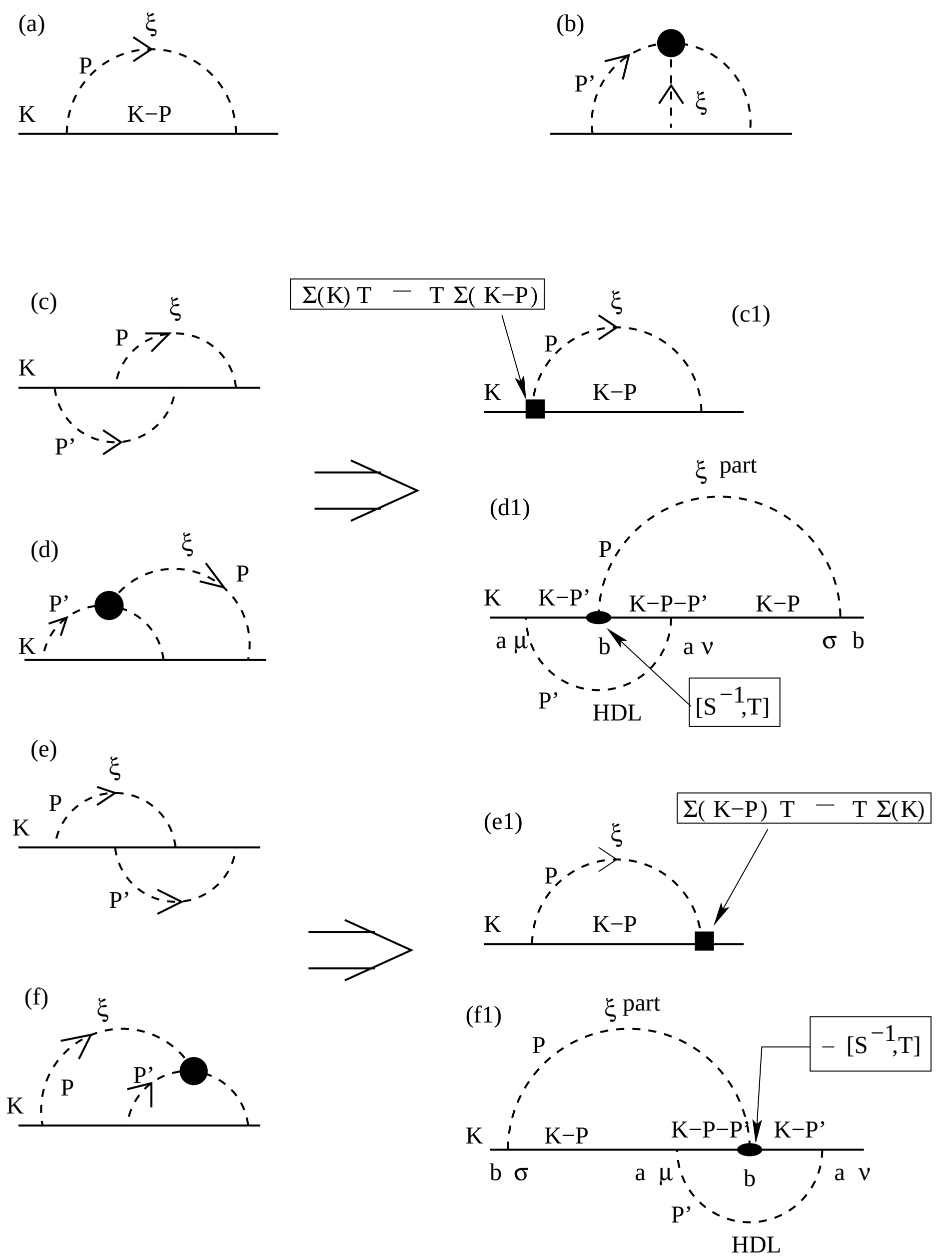
$$\begin{aligned} i\Lambda_i^b &= \lim_{\mathbf{p}\to 0} \lim_{p_0 \to 0} i\Lambda_i^b(K, K-P) \\ &\sim \frac{\partial}{\partial \mathbf{p}_i}(r.h.s.) \sim 0 \end{aligned}$$

$$\begin{aligned} i\Lambda_0^b &= \lim_{p_0\to 0} \lim_{\mathbf{p} \to 0} i\Lambda_0^b(K, K-P) \\ &\sim \frac{\partial}{\partial p_0}(r.h.s.) \sim g\gamma_0 \ln\phi \end{aligned}$$

Although non-vanishing, $i\Lambda_0^b$ corresponds to electric gluon which is Deybe screened and therefore subsubleading. It can be proved that I_X^b does not contribute to $i\Lambda_\mu^b$ to subleading order.

No vertex correction at subleading order!!

ξ term


(a)
ξ
P
K
K−P
(b)
P'
ξ
(c)
ξ
P
K
P'
(d)
ξ
P
P'
K
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Σ(K) T − T Σ(K−P)
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(e1)
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K
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ξ part
P
− [S^{−1},T]
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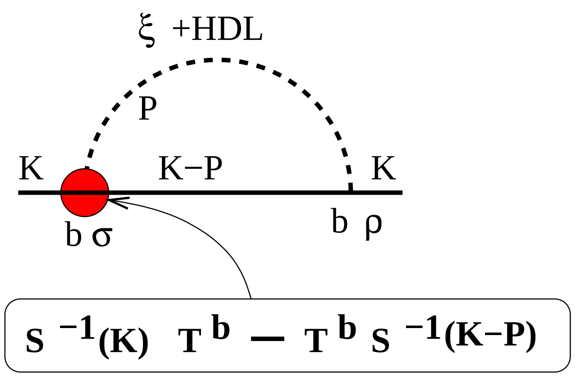
HDL

Gauge independence in covariant gauge

Where does $\exp(3/2\xi)$ go



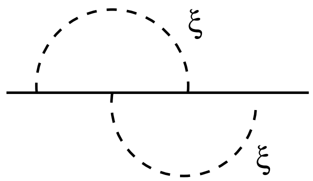
$$I^{\xi}[a,c,d] \sim -\xi g^2 \int \frac{d^4P}{(2\pi)^4}\frac{1}{P^4} \times \left[\mathbf{S}^{-1}(K)\mathbf{T}^b - \mathbf{T}^b\mathbf{S}^{-1}(K-P)\right] \times \mathbf{S}(K-P)\mathbf{T}^b\gamma_\rho P_\rho$$

$$\overline{\Psi}_{on}(K_{on})I^{\xi}[a,c,d]\Psi(K_{on}) \to \overline{\Psi}_{on}(K_{on})\mathbf{S}^{-1}(K_{on}) = 0$$

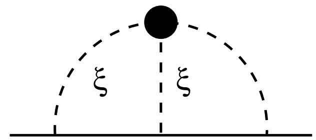
$$\int d^4P P_\rho / P^4 = 0$$

The exponential factor $\exp(3/2\xi)$ is removed by quasi-particle on-shell condition.

ξ^2 and ξ^3 term



(a)



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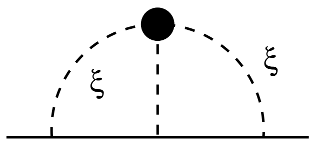

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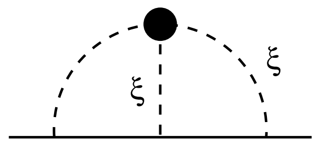

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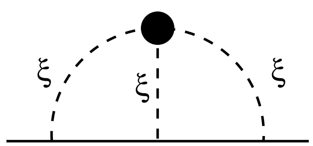

(e)

Gauge independence in covariant gauge

Summary of the proof

- Generalized Ward Identity
- quasi-particle on-shell condition

⇓

2SC gap is gauge parameter independent at subleading order in covariant gauge!

Comparison of Color-SC with normal SC

	color-superconductor	superconductor	superfluid (^{3}He)
Degree of freedom	quarks, gluons	electrons	atoms
Theory	QCD	QED	QED
phase structure	rich $\leftarrow$ C,F	simple(low-T_c), complex(high-T_c)	rich $\leftarrow$ L, S
T_c	$\sim$ MeV $\sim 10^{10}$ K	a few K to 100 K	$\sim$ mK
locking state	CFL, CSL	no (?)	B, A phases
Meissner effect	gluonic, EM	EM	no
mixing	yes, like SM $(A + Z^0)$	no	no
gauge invariance	color	EM	no

A colorful world in color superconductivity in perspective of condensed matter physics!

Acknowledgement:

Thank Dr. Igor Shovkovy for a lot of critical and helpful comments and discussions.

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