The QCD Phase Diagram: What About Isospin?

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QCD and Dense Matter, Seattle, May 2004

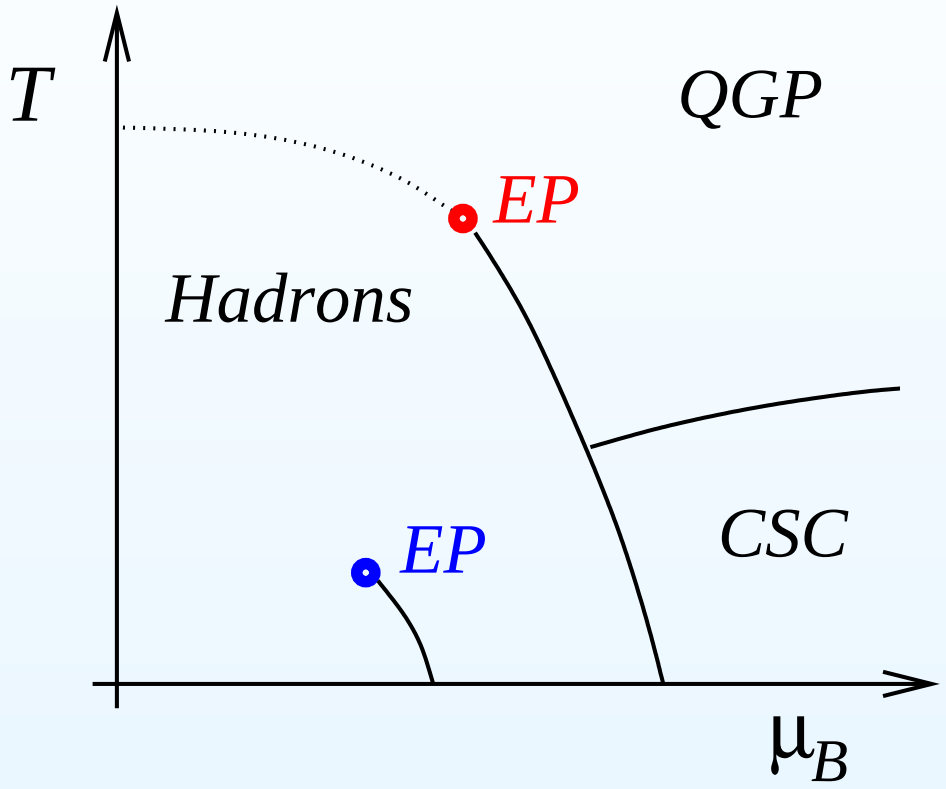
Introduction

- Experiments: baryon AND isospin density
  - ▷ Neutron stars (high μ_B, low T)
  - ▷ RHIC (low μ_B, high T)
- Most studies: $\mu_I = 0$, except low T
  - ▷ Neutron stars
  - ▷ LOFF phase
  - $\longrightarrow$ What about higher T?
- QCD phase diagram with μ_I, μ_B, $T \neq 0$
- Random Matrix model

Outline

- Quick overview of QCD phase diagram
  - ▷ $\mu_I = 0,\ \mu_B \neq 0,\ T \neq 0$
  - ▷ $\mu_B = 0,\ \mu_I \neq 0,\ T \neq 0$
- Random Matrix Theory
  - ▷ Description of the model
  - ▷ Range of validity at $\mu_I,\ \mu_B,\ T = 0$
- Random Matrix model with $\mu_I,\ \mu_B,\ T \neq 0$
- QCD phase diagram ($N_f = 2,\ m_q \neq 0$)
- How to test these results?

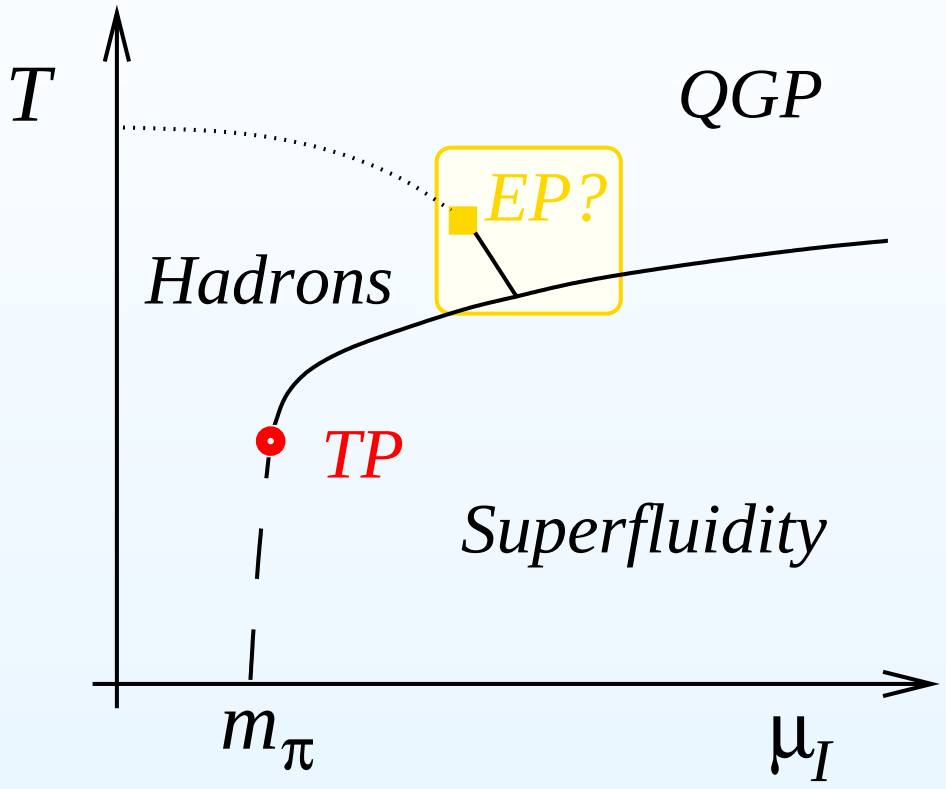
QCD: $\mu_I = 0$, $\mu_B \neq 0$, $T \neq 0$



▷ Color Superconductivity, Critical endpoint

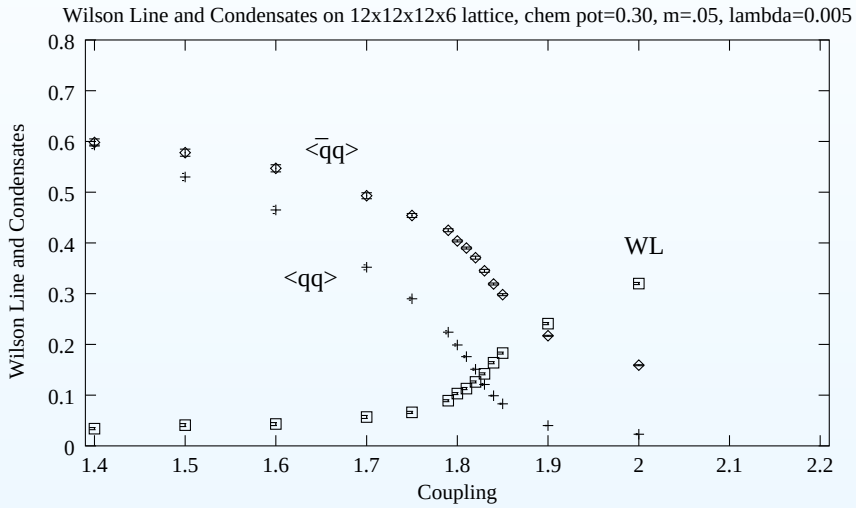
- Random Matrix, NJL, Ladder QCD
- Lattice $\longrightarrow$ low μ_B only

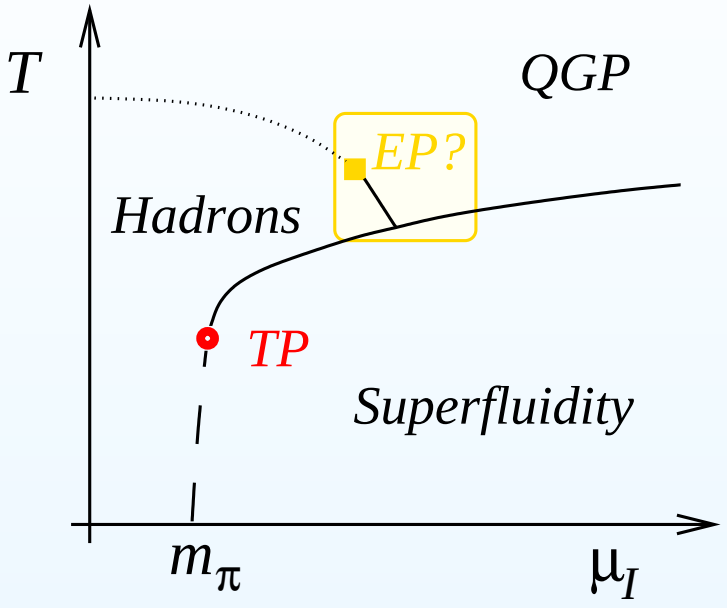
QCD: $\mu_B = 0$, $\mu_I \neq 0$, $T \neq 0$

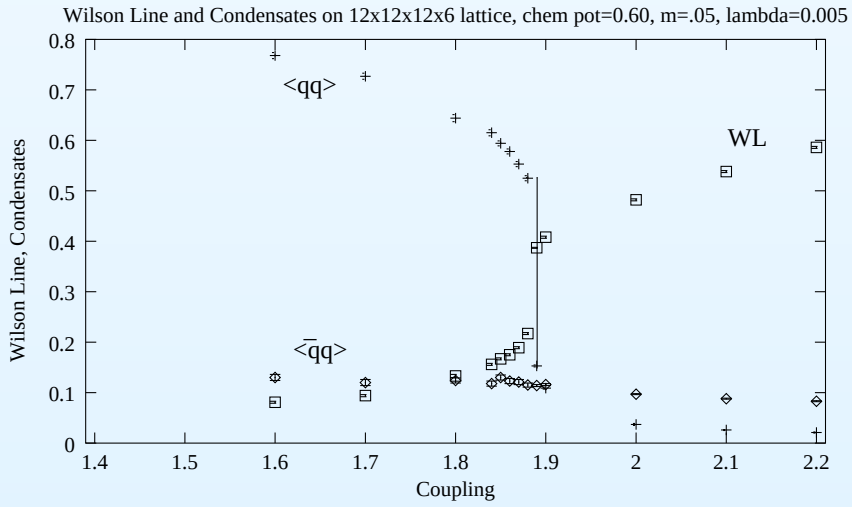


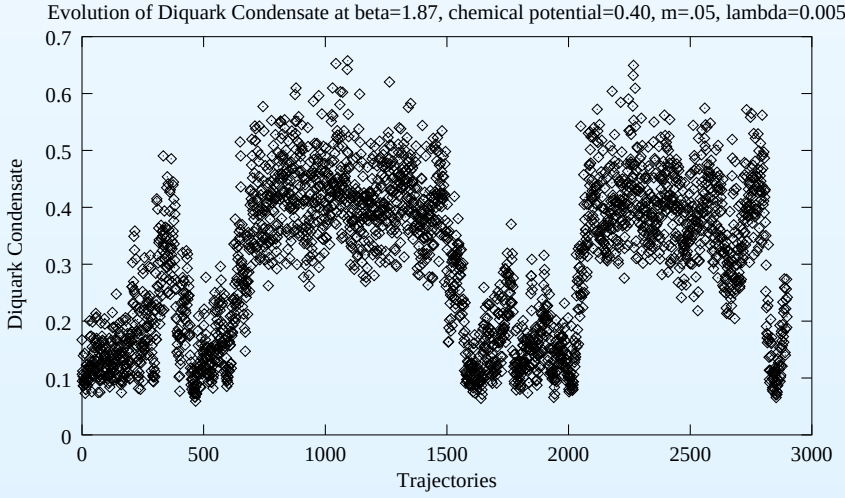
- ▷ Superfluidity, Tricritical point, Critical endpoint?
- Chiral Perturbation Theory, Lattice
- Phase diagram similar to $N_c = 2$, $\mu_B \neq 0$

Lattice: $N_c = 2,\ \mu_B \neq 0,\ T \neq 0$









Random Matrix Theory: μ_B, μ_I, $T=0$

- QCD partition function

$$Z_{\rm QCD} = \int [dA] \prod_f \det(i\not{D} + m_f)\ {\rm e}^{-S_{\rm YM}}$$

- Random Matrix Theory partition function

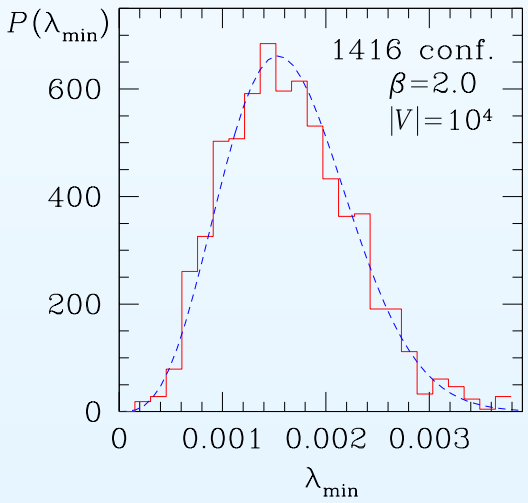
$$Z_{\rm RMT} = \int [dW] \prod_f \det(iD + m_f)\ {\rm e}^{-nG^2\ {\rm Tr} WW^\dagger}$$

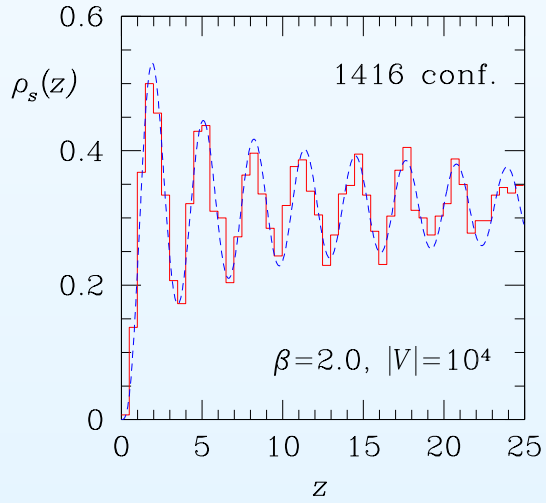
$$\triangleright \begin{vmatrix} m_1 & 0 & W & 0 \\ 0 & m_2 & 0 & W \\ -W^\dagger & 0 & m_1 & 0 \\ 0 & -W^\dagger & 0 & m_2 \end{vmatrix}, \quad W = n \times n$$

Random Matrix Theory: μ_B, μ_I, $T=0$

- Random Matrix Theory partition function
  - ▷ Same symmetry as QCD partition function
  - ▷ No dynamics
- Spectrum of Dirac operator

Berbenni-Bitsch et al., PRL 80 (1998) 1146





  - ▷ Lattice, Chiral Perturbation Theory
- Valid if $L \ll 1/m_\pi$

Random Matrix model: $\mu_B, \mu_I, T \neq 0$

- Partition function

$$Z_{\mathrm{RMT}} = \int [dW] \prod_f \det(iD + m_f + \mu_f \gamma_0)\ \mathrm{e}^{-nG^2\ \mathrm{Tr} WW^\dagger}$$

$$\triangleright \begin{vmatrix} m_1 & 0 & W+\Omega+\mu_1 & 0 \\ 0 & m_2 & 0 & W+\Omega+\mu_2 \\ -W^\dagger-\Omega^\dagger+\mu_1 & 0 & m_1 & 0 \\ 0 & -W^\dagger-\Omega^\dagger+\mu_2 & 0 & m_2 \end{vmatrix}$$

$$\triangleright \quad \Omega = \begin{pmatrix} iT & 0 \\ 0 & -iT \end{pmatrix}$$

Random Matrix model: μ_B, μ_I, $T \neq 0$

- Random Matrix model partition function

$$Z_{\mathrm{RMT}} = \int [dW] \prod_f \det(iD + m_f + \mu_f \gamma_0)\ \mathrm{e}^{-nG^2\ \mathrm{Tr} WW^\dagger}$$

1) Determinant as integral over fermions

2) Integration over W (Gaussian)

$\Rightarrow$ Four fermion term

3) Hubbard-Stratonovich transformation

$\Rightarrow$ Mesons

4) Integration over fermions

$\Rightarrow$ Effective action with mesons (exact map.)

Random Matrix model: μ_B, μ_I, $T \neq 0$

- Effective action identical to zero-momentum part of Chiral Perturbation Theory at μ_B, $T = 0$

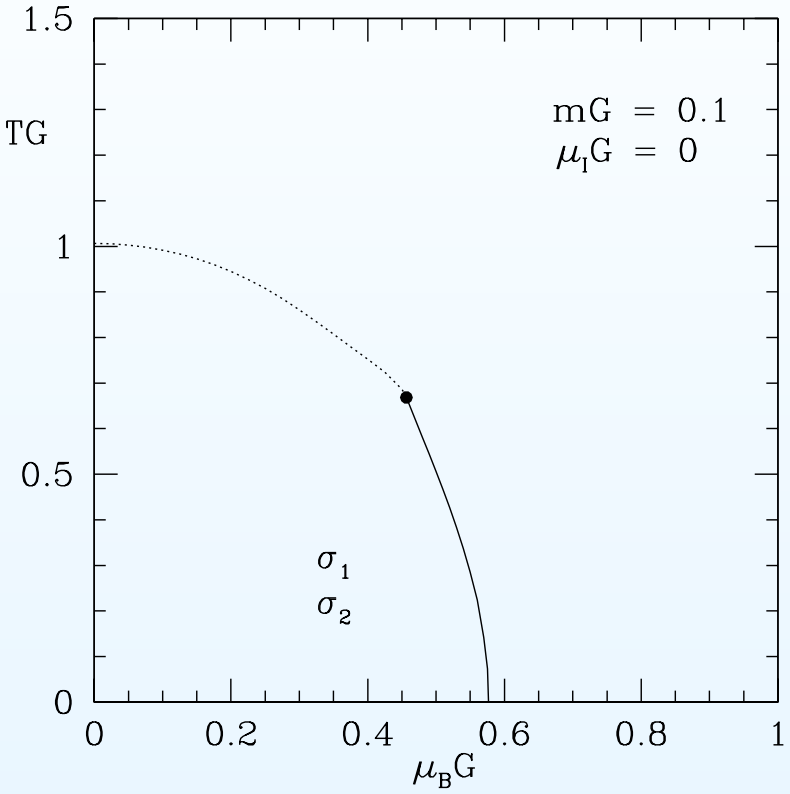
- Saddle point approximation of effective action

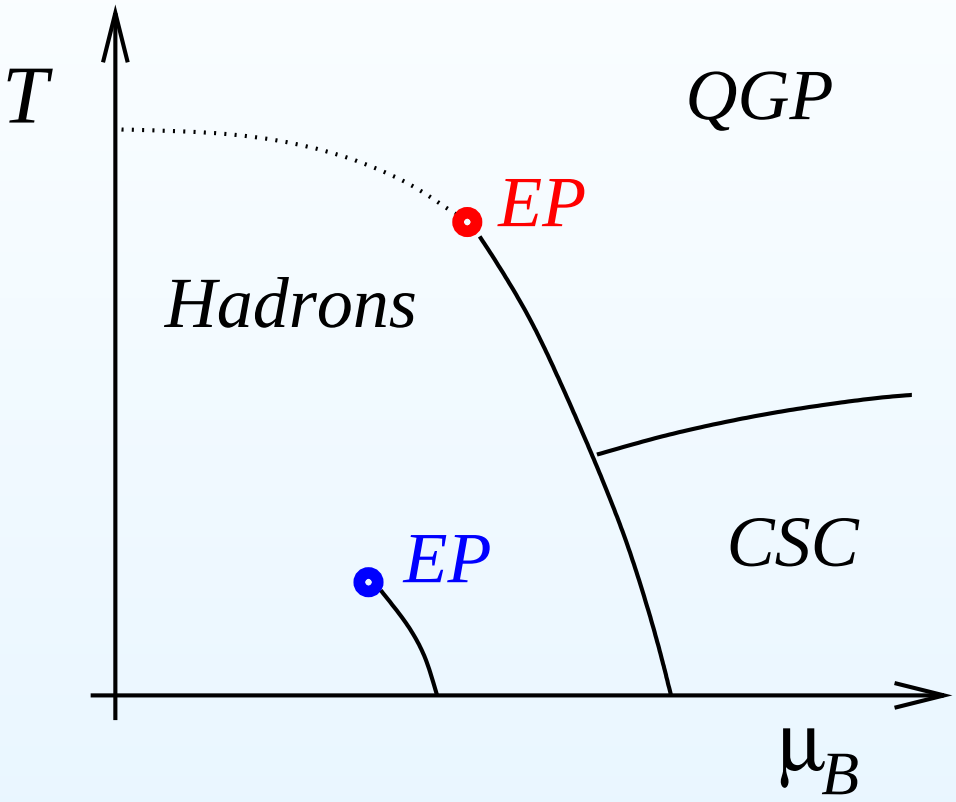
  ▷ Ansatz $\longrightarrow$ order parameters

$$\begin{cases} \sigma_1 &= \langle \bar{u} u \rangle \\ \sigma_2 &= \langle \bar{d} d \rangle \\ \rho &= \frac{1}{2}(\langle \bar{u} \gamma_5 d \rangle - \langle \bar{d} \gamma_5 u \rangle) \end{cases}$$

  ▷ Study like a Landau-Ginzburg model

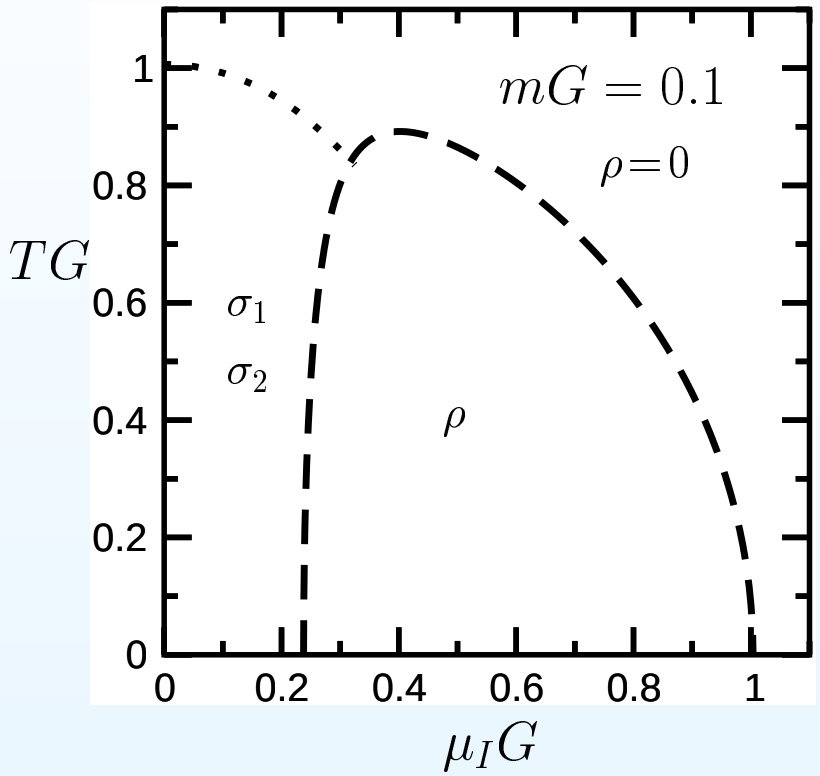
Random Matrix: $\mu_B \neq 0,\ \mu_I = 0$

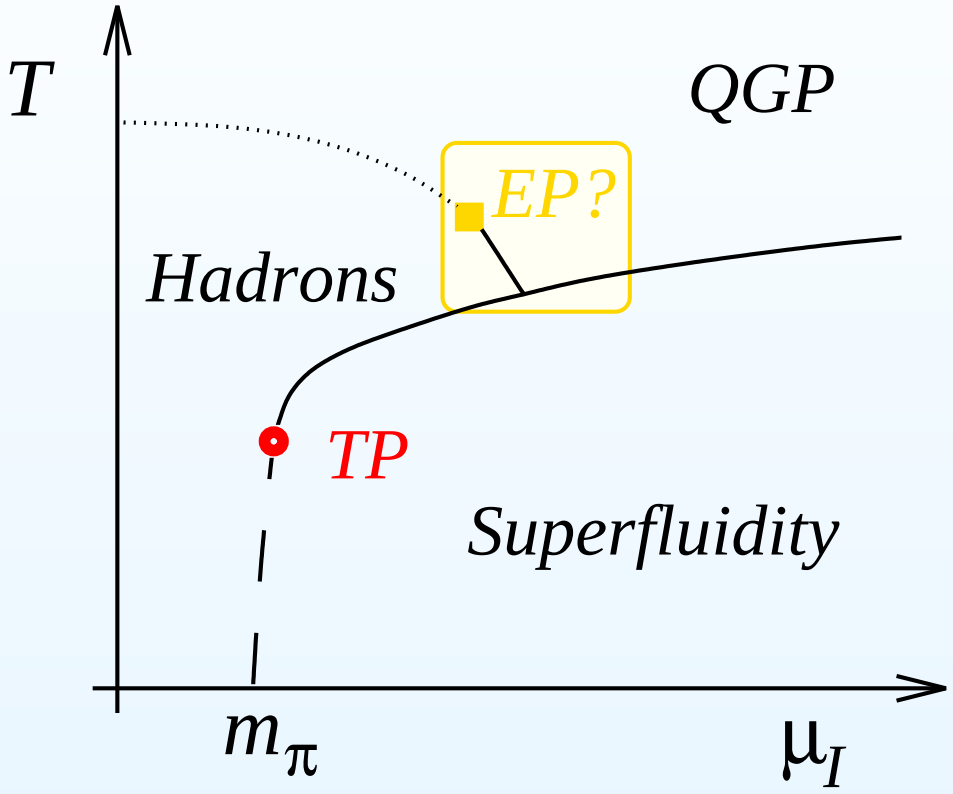




- NO CSC, but Critical endpoint
- Lattice, Nambu–Jona-Lasinio, Ladder QCD

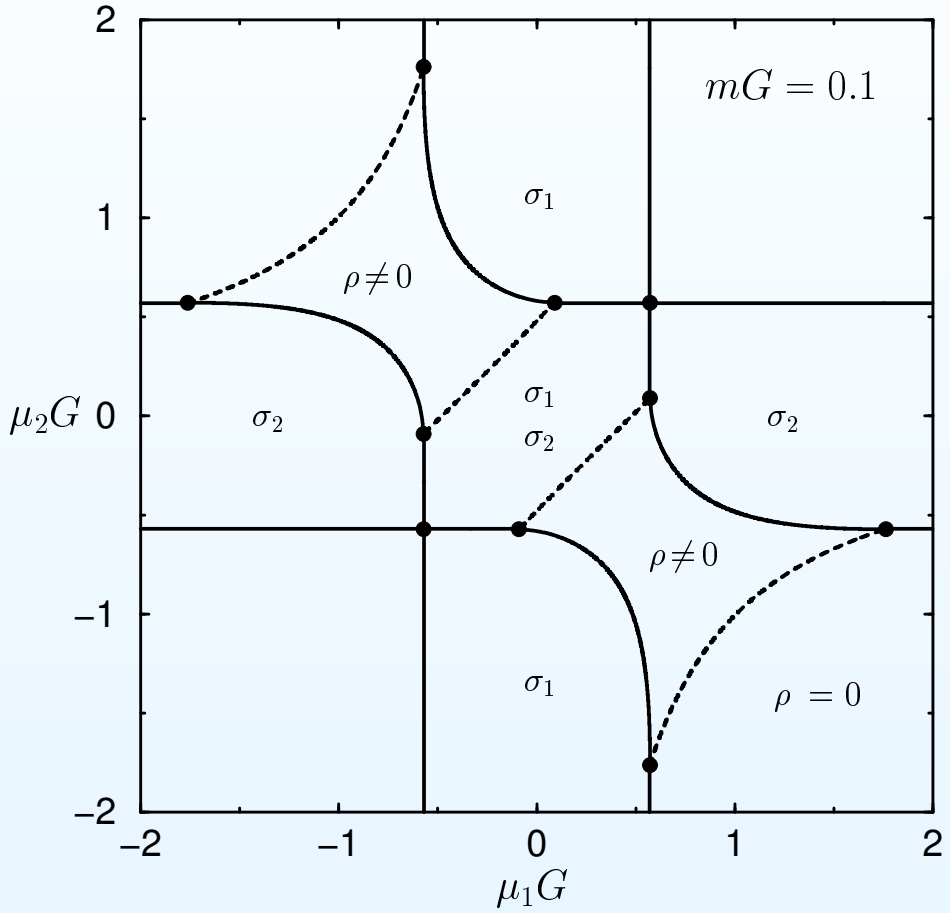
Random Matrix: $\mu_B = 0,\ \mu_I \neq 0$





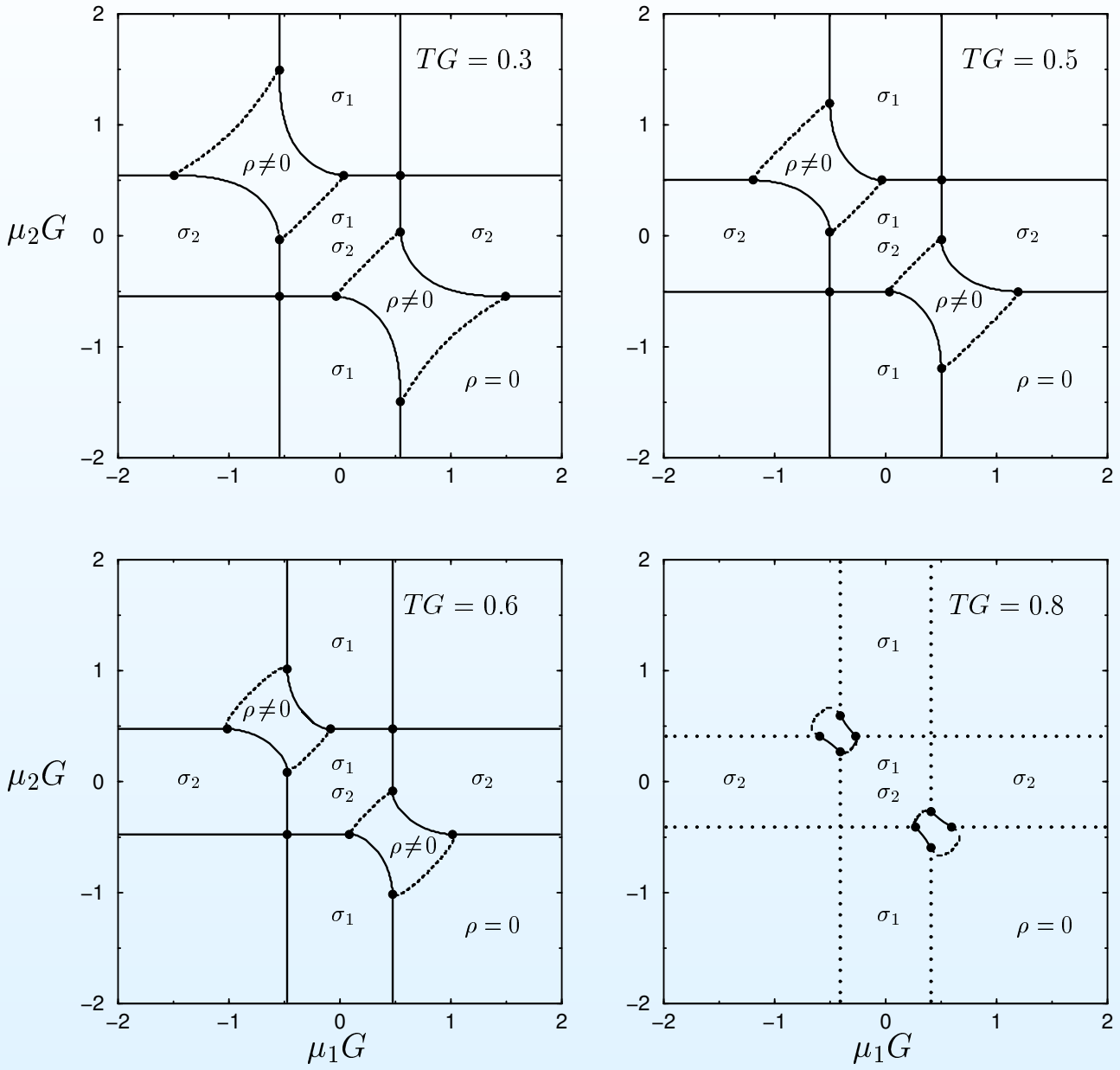
- Superfluidity, but NO Tricritical Point
- Lattice, Chiral Perturbation Theory

Random Matrix: $\mu_I, \mu_B \neq 0, T = 0$

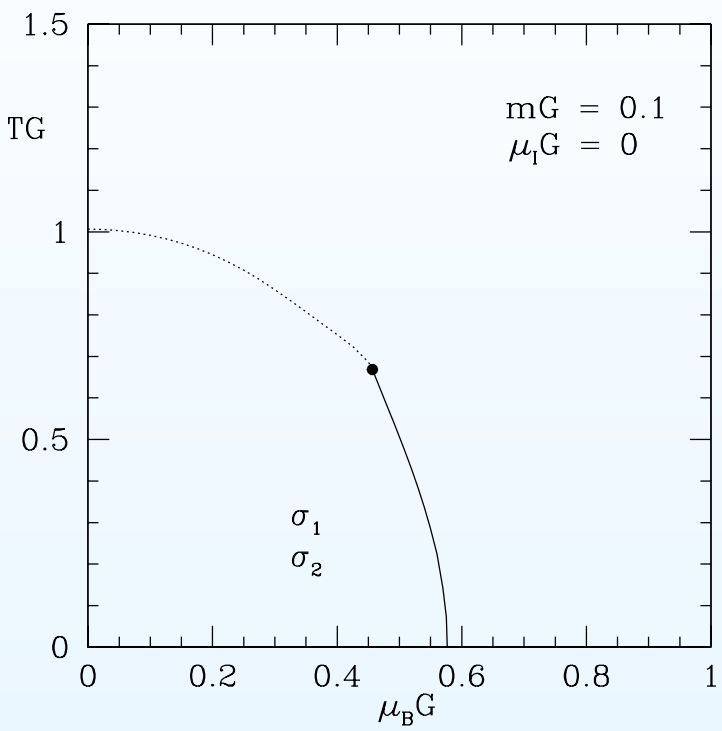


- Hadronic phase, Superfluid phase
- High $\mu_f \Rightarrow \langle \bar{q}_f q_f \rangle \ll 1$

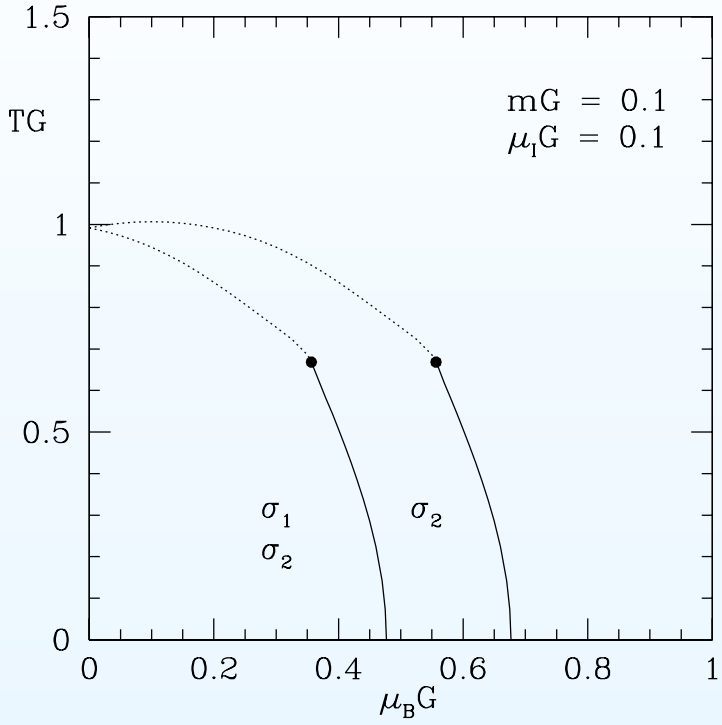
Random Matrix: $\mu_I, \mu_B, T \neq 0$



Random Matrix: $\mu_I, \mu_B, T \neq 0$



$\mu_I \Longrightarrow$



- ▷ Doubling: phase transition lines, critical endpts
- ▷ Critical endpoint at lower μ_B for fixed $\mu_I < m_\pi$

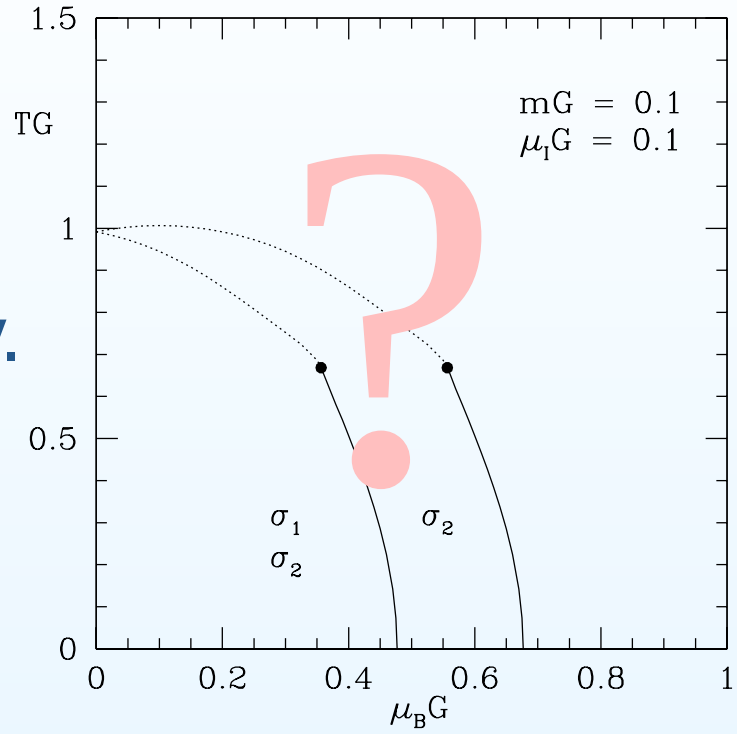
$\Rightarrow$ RHIC: 2 crossovers or first order phase trans.

$\Rightarrow$ Study of critical endpoint easier

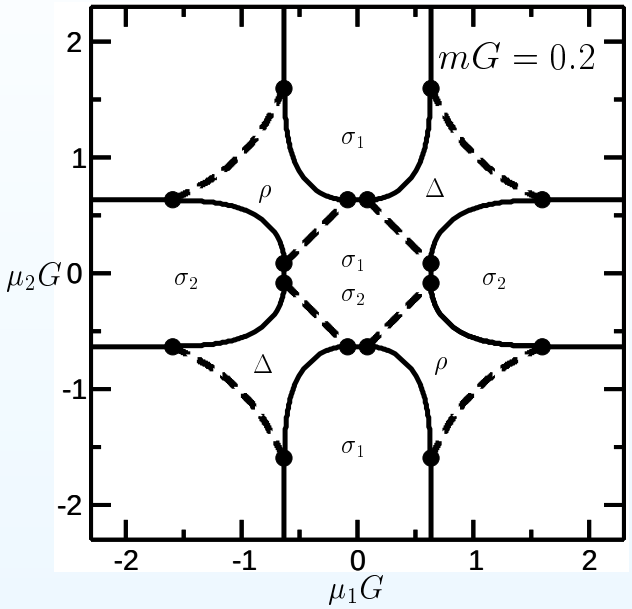
How to test these results?

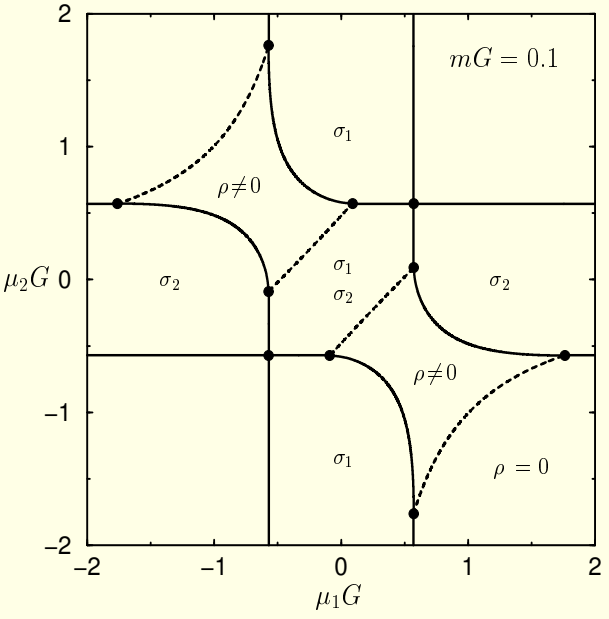
- NJL model if flavor-mix. four-fermion interaction $\lesssim 10-15\%$ of non-flav. mixing term
- Ladder QCD
- Lattice
  - ▷ 3-color QCD: methods used for low μ_B
  - ▷ 2-color QCD: large m_π, fixed $\mu_I < m_\pi$

  → Random Matrix model for 2-color QCD

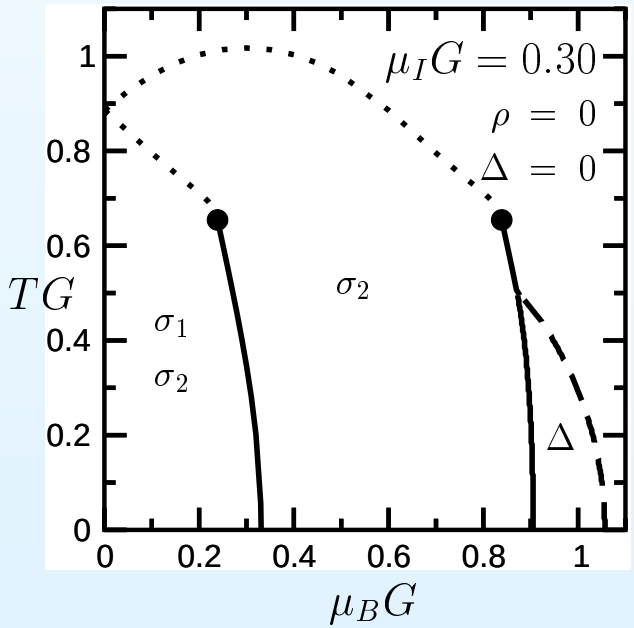


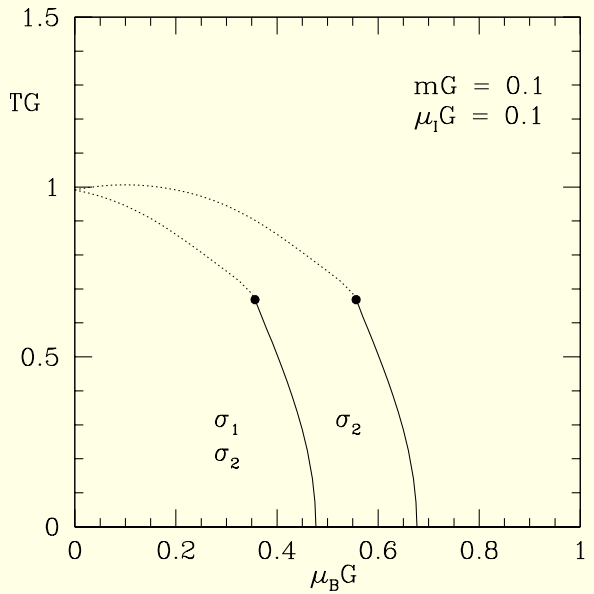
Random Matrix model for 2-color QCD





$N_C = 3$





$N_C = 3$

Conclusions and Outlook

- Influence of small μ_I on QCD phase diagram
  - ▷ Doubling: phase trans., critical endpoints
  - ▷ Critical endpoint at lower μ_B, same T
- Predictions can be tested on lattice
  - ▷ 3-color or 2-color QCD
- Consequences for RHIC:
  - ▷ Two crossovers or two first order phase transitions at low μ_B
  - ▷ Critical end point at lower $\mu_B \Rightarrow$ more important