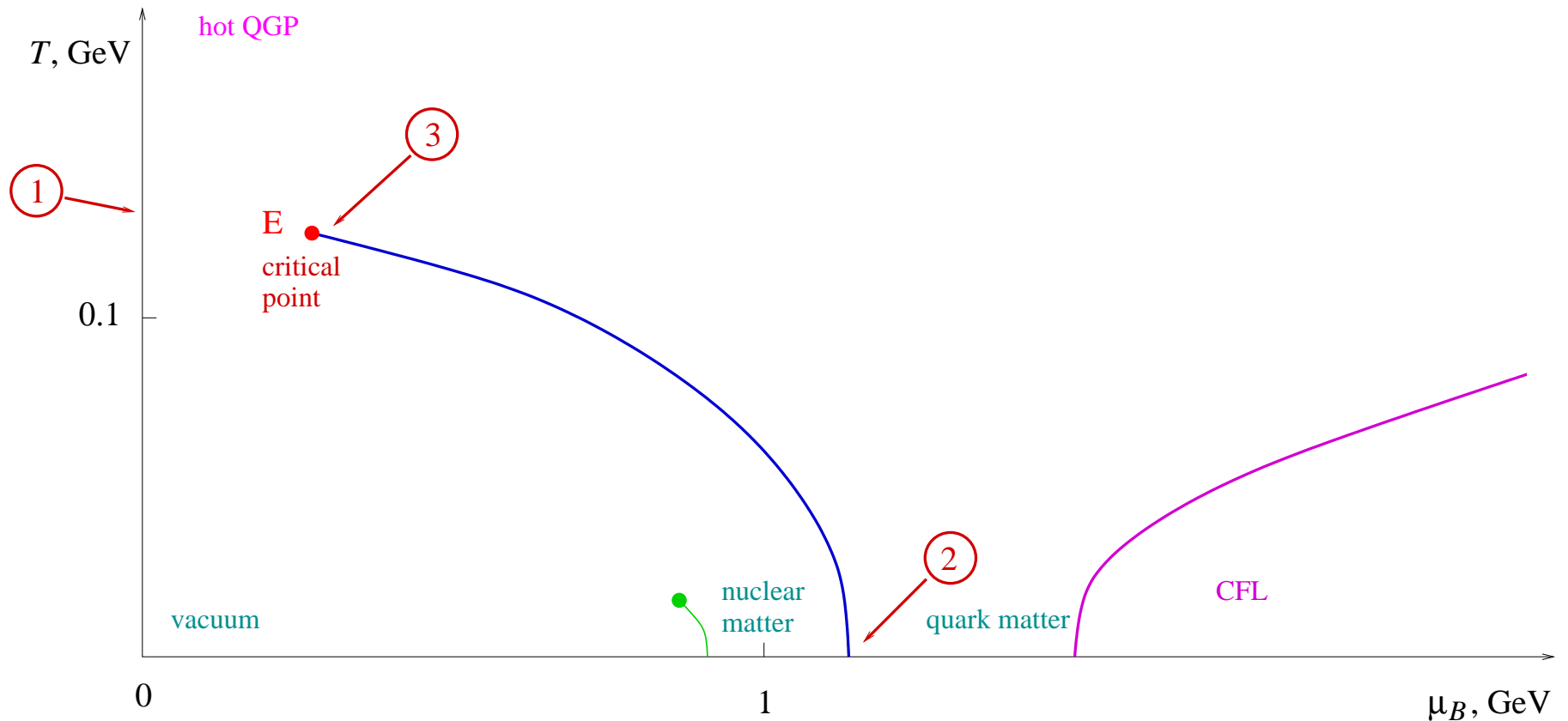


# QCD critical point: fluctuations, hydrodynamics and universality class

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# QCD critical point



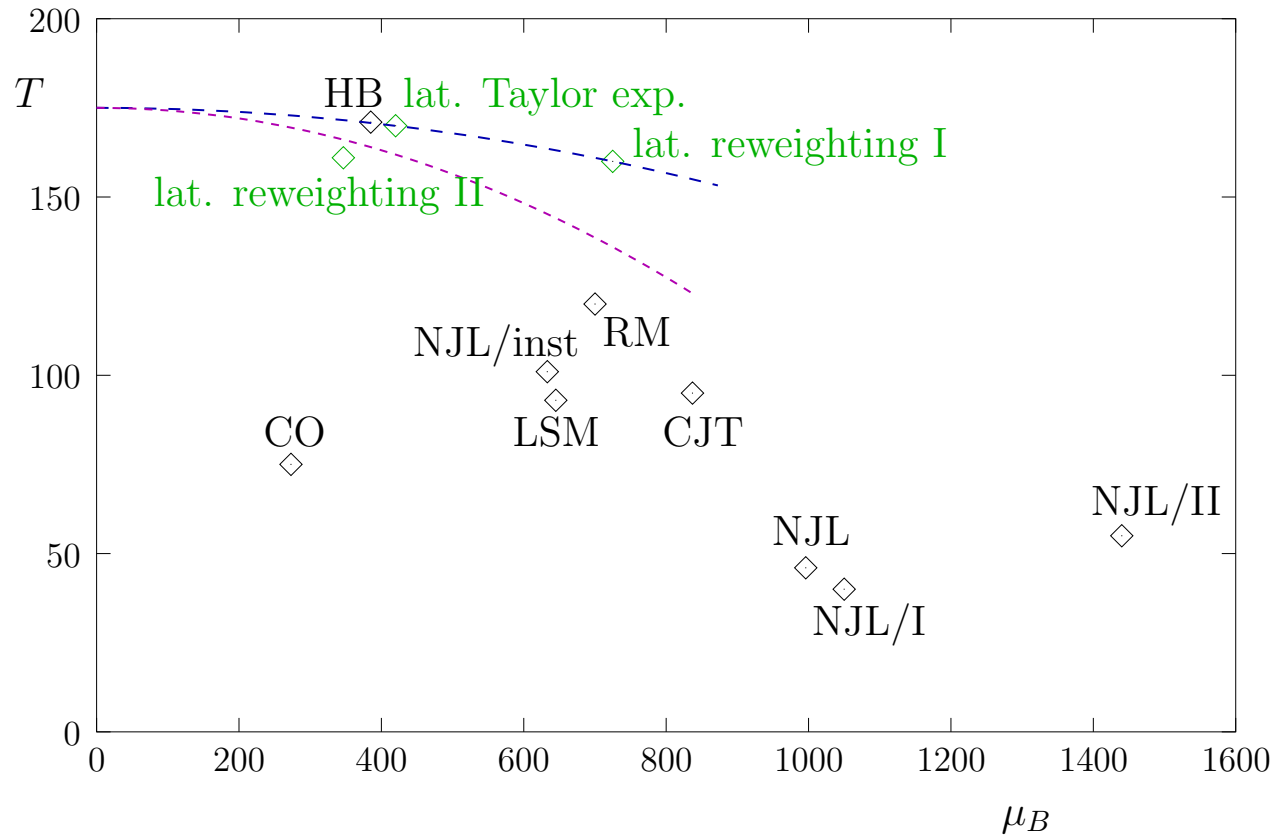
- ① Lattice at  $\mu = 0 \rightarrow$  crossover. ② Sign problem. Models: 1st order transition.  
① + ② = ③: critical point **E**. (As in water at  $p = 221\text{bar}$ ,  $T = 373^\circ\text{C}$  – critical opalescence.)

Where is point **E**? Challenge to theory and experiment.

# Location of the CP (theory)

Source	$(T, \mu_B)$ , MeV	Comments	Label
MIT Bag/QGP	none	<i>only 1st order</i>	—
Asakawa, Yazaki '89	(40, 1050)	NJL, CASE I	NJL/I
“	(55, 1440)	NJL, CASE II	NJL/II
Barducci, <i>et al</i> '89-94	(75, 273) <sub>TCP</sub>	composite operator	CO
Berges, Rajagopal '98	(101, 633) <sub>TCP</sub>	instanton NJL	NJL/inst
Halasz, <i>et al</i> '98	(120, 700) <sub>TCP</sub>	random matrix	RM
Scavenius, <i>et al</i> '01	(93,645)	linear $\sigma$ -model	LSM
“	(46,996)	NJL	NJL
Fodor, Katz '01	(160, 725)	lattice reweighting I	
Hatta, Ikeda, '02	(95, 837)	effective potential (CJT)	CJT
Antoniou, Kapoyannis '02	(171, 385)	hadronic bootstrap	HB
Ejiri, <i>et al</i> '03	(?,420)	lattice Taylor expansion	
Fodor, Katz '04	(162, 360)	lattice reweighting II	

# Location of the CP (theory)

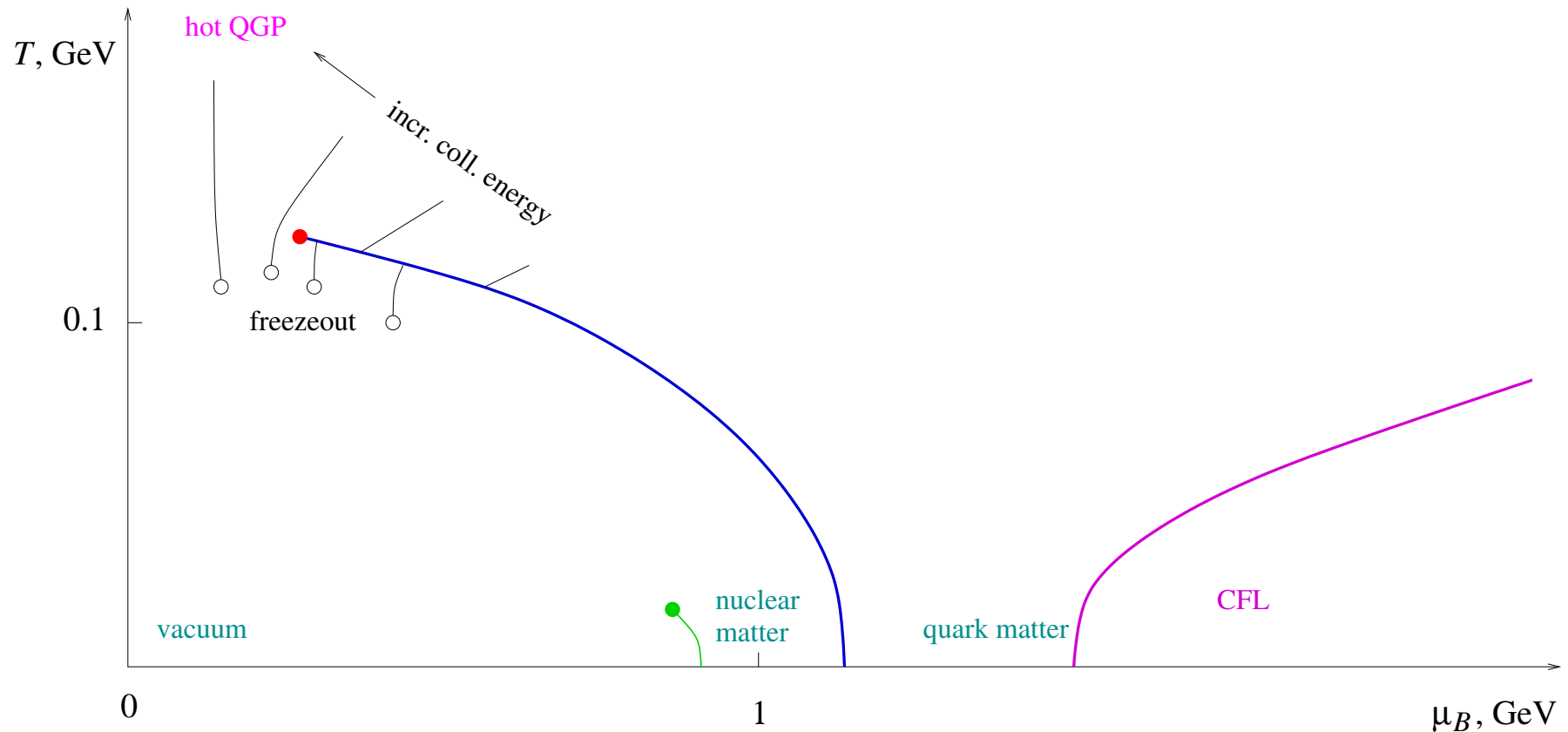


Slope: Allton et al '02 (Taylor exp.), de Forcrand, Philipsen '02 ( $\text{Im}\mu$ ):

$$\frac{T(\mu_B)}{T(0)} \approx 1 - .00(5 - 8) \left( \frac{\mu_B}{T(0)} \right)^2$$

Allton et al '03: **.013**. Quark mass dependence?

# Locating the point experimentally



Strategy: let fireball cool at different  $\mu_B$  (baryon per entropy).

Control parameter: **collision energy**.

(focusing)

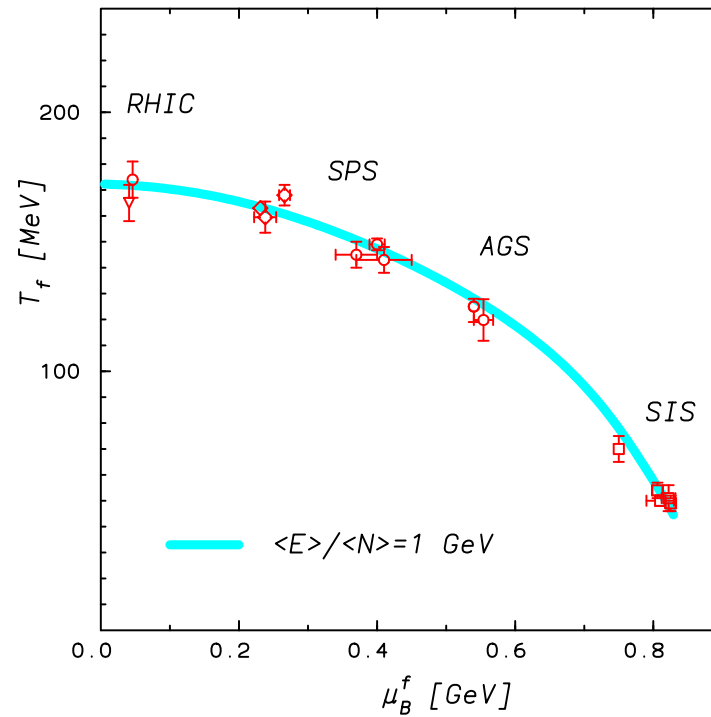
AGS @  $\sqrt{s} = 5A\text{GeV}$  :  $\mu_B \approx 500\text{MeV}$ ;

SPS @  $\sqrt{s} = 17A\text{GeV}$  :  $\mu_B \approx 200\text{MeV}$ ;

RHIC @  $\sqrt{s} \sim 30 - 200A\text{GeV}$ :  $\mu < 200\text{MeV}$ .

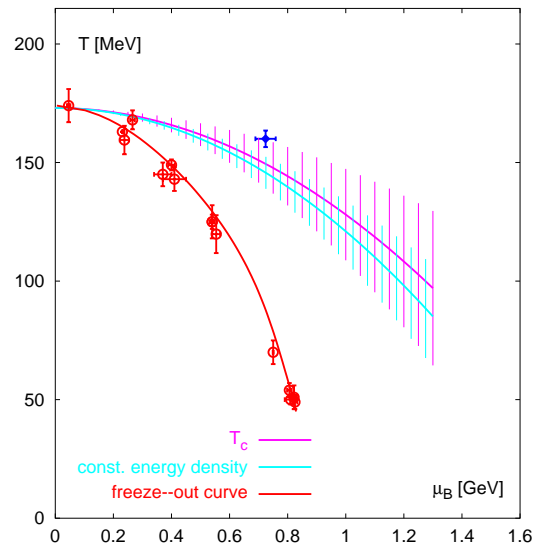
**Non-monotonic** signatures

# Freezeout (experimental domain)

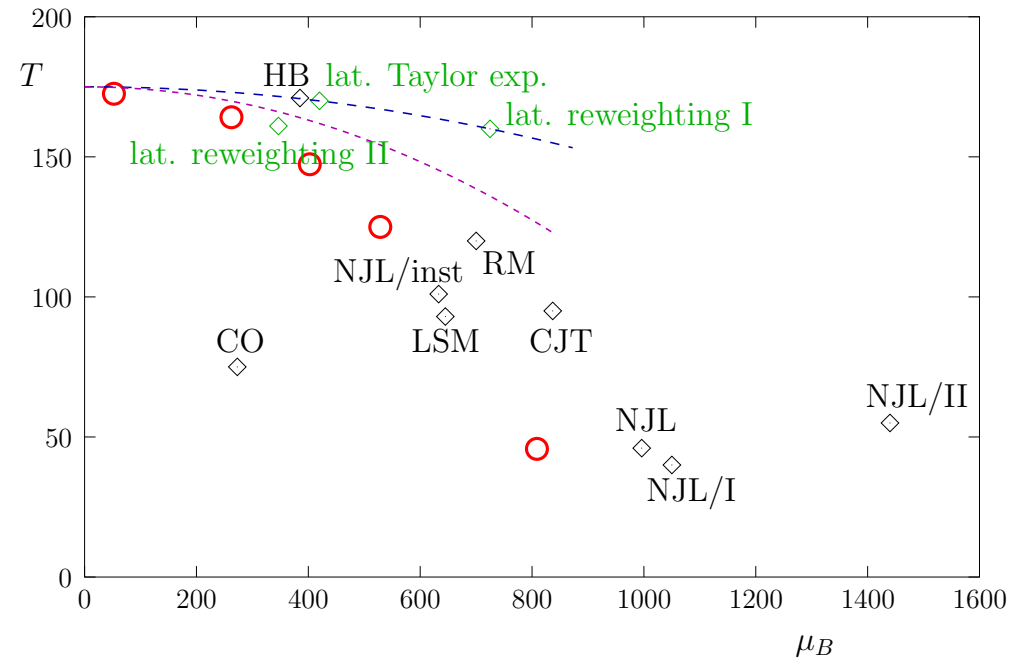


(from Braun-Munzinger, Redlich, Stachel)

# Freezeout vs critical point predictions



(Braun-Munzinger, Redlich, Stachel)



Models predict lower  $T_c(\mu)$ , closer to experimental domain.

So does most recent lattice data.

$T_{\text{freezeout}}$  higher for smaller ions.

# Fluctuations: the idea



$$\frac{\partial E}{\partial T} = -\frac{\partial^2 \mathcal{F}}{\partial T^2} = \frac{1}{T^2} \langle (\delta E)^2 \rangle,$$

$$\frac{\partial Q}{\partial \mu} = -\frac{\partial^2 \mathcal{F}}{\partial \mu^2} = \frac{1}{T} \langle (\delta Q)^2 \rangle$$

$Q$  = baryon charge, electric charge.

- Susceptibilities diverge  $\Rightarrow$  fluctuations grow towards the critical point.
- Fluctuations can be measured in heavy ion collision experiments.

# Magnitude of fluctuations

Scaling and universality of critical phenomena:  $\chi \sim \xi^{\text{power}}$ .

How big can  $\xi$  grow? (MS, Rajagopal, Shuryak; Berdnikov, Rajagopal)

Limits:

- Proximity of the critical point
- Finite size of the system  $\xi < 6$  fm.
- Finite *time*:  $\tau \sim 10$  fm.

$$\xi \sim \tau^{1/z}$$

$z$  – dynamical critical exponent.

# Dynamic scaling and universality

Hohenberg, Halperin Rev. Mod. Phys. 49, 435 (1977)

Near the c.p. Typical relaxation time scale in the system diverge (critical slowing down):

$$\tau \sim \xi^z$$

(dynamic scaling).

The exponent  $z$  is determined by the *dynamic* universality class.

Systems with equivalent *static* critical behavior are not always in the same *dynamic* universality class.

Example: Ising model and liquid-gas phase transition.

Simple reason: the order parameter (density) in the latter is a conserved quantity  $\Rightarrow$  relaxes slower at large distance scales (by diffusion), then in the Ising model (local relaxation).

# Dynamic universality class of QCD critical point

Son, MS, hep-ph/0401052

Statics: symmetry and dimensionality  $\rightarrow$  Ising model (same as liquid-gas).

Dynamic universality class depends on relevant hydrodynamic modes.

Modes which relax arbitrarily slowly: densities of conserved quantities, order parameter.

- The fluctuations of the energy and momentum densities:  $\varepsilon \equiv T^{00} - \langle T^{00} \rangle$ , and  $\pi^i \equiv T^{0i}$ ;
- The fluctuations of the baryon number density,  $n \equiv \bar{q}\gamma^0 q - \langle \bar{q}\gamma^0 q \rangle$ ;
- The chiral condensate  $\sigma \equiv \bar{q}q - \langle \bar{q}q \rangle$ .

Is it liquid gas (H), or is it Ising model (A), or is it another universality class altogether? (Berdnikov, Rajagopal: model C)

# Statics

$\sigma$  and  $n$  mix.

$$F[\sigma, n] = \int d\mathbf{x} \left[ V(\sigma, n) + \frac{a}{2}(\nabla\sigma)^2 + b(\nabla\sigma)(\nabla n) + \frac{c}{2}(\nabla n)^2 \right],$$

$$V(\sigma, n) = \frac{A}{2}\sigma^2 + B\sigma n + \frac{C}{2}n^2 + \text{terms of higher orders.}$$

At the critical point  $\Delta \equiv AC - B^2 \rightarrow 0$ .

One flat direction:  $(\sigma, n) \sim (-B, A)$ . Only one critical mode.

Susceptibilities, e.g.:

$$\langle \sigma_{q \rightarrow 0}^2 \rangle = \frac{TC}{\Delta}, \quad \langle n_{q \rightarrow 0}^2 \rangle = T\chi_B = \frac{TA}{\Delta}.$$

Either  $\sigma$  or  $n$  can be used as an order parameter (both jump across 1st order phase transition).

# Dynamic equations

$$\begin{aligned}\dot{\sigma} &= -\Gamma \frac{\delta F}{\delta \sigma} + \tilde{\lambda} \nabla^2 \frac{\delta F}{\delta n} + \xi_{\sigma}, \\ \dot{n} &= \tilde{\lambda} \nabla^2 \frac{\delta F}{\delta \sigma} + \lambda \nabla^2 \frac{\delta F}{\delta n} + \xi_n.\end{aligned}$$

with

$$\begin{aligned}\langle \xi_{\sigma}(x) \xi_{\sigma}(y) \rangle &= 2T\Gamma \delta^4(x - y), \\ \langle \xi_{\sigma}(x) \xi_n(y) \rangle &= -2T\tilde{\lambda} \delta(t - t') \nabla^2 \delta^3(\mathbf{x} - \mathbf{y}), \\ \langle \xi_n(x) \xi_n(y) \rangle &= -2T\lambda \delta(t - t') \nabla^2 \delta^3(\mathbf{x} - \mathbf{y}).\end{aligned}$$

Using  $F$ :

$$\begin{aligned}\dot{\sigma} &= -\Gamma A \sigma - \Gamma B n, \\ \dot{n} &= (\tilde{\lambda} A + \lambda B) \nabla^2 \sigma + (\tilde{\lambda} B + \lambda C) \nabla^2 n.\end{aligned}$$

(+ noise)

# Modes

$$\det \begin{vmatrix} \Gamma A - i\omega & \Gamma B \\ (\tilde{\lambda}A + \lambda B)\mathbf{q}^2 & (\tilde{\lambda}B + \lambda C)\mathbf{q}^2 - i\omega \end{vmatrix} = 0.$$

Near critical point:

$$\begin{aligned} \omega_1 &= -i\lambda \frac{\Delta}{A} \mathbf{q}^2. \\ \omega_2 &= -i\Gamma A. \end{aligned}$$

Diffusive:  $(\sigma, n) \sim (-B, A)$ , relaxational:  $(\sigma, n) \sim (1, 0)$ .

Critical mode (flat direction) is the diffusive mode,  $\sigma = (-B/A)n$ .

Sigma alone ( $n = 0$ ) relaxes on a finite time scale *even at c.p* ( $\Gamma$  is finite).

Conclusion: only one hydrodynamic mode after  $\sigma$  and  $n$  mixing, and it is diffusive.

Diffusion constant:

$$D = \lambda \frac{\Delta}{A} = \lambda C - \lambda \frac{B^2}{A}.$$

$D \rightarrow 0$  at c.p.

# Diffusion constant and critical indices

$$D = \lambda \chi_B^{-1}$$

$$\mu(\mathbf{x}) = \chi_B^{-1} n(\mathbf{x}) \quad \rightarrow \quad \mathbf{E} = -\nabla \mu \quad \rightarrow \quad \mathbf{j}_n = \lambda \mathbf{E} = -\lambda \chi_B^{-1} \nabla n$$

Typical relaxation time

$$\tau \sim D^{-1} \xi^2,$$

$$\text{but } D^{-1} \sim \chi_B \sim \xi^{2-\eta} \quad \rightarrow \quad \tau \sim \xi^{4-\eta} \quad \rightarrow \quad z = 4 - \eta.$$

This is model B.

# Coupling to energy-momentum

Modes to consider:

●  $\epsilon, \pi$

●  $n$

●  $\sigma$

Only one combination of  $n$  and  $\sigma$  is truly hydrodynamic  $\rightarrow \epsilon, \pi$  and  $n$ .

Same as in the liquid-gas dynamic universality class.

This is model H.

# Model H

Review results (Hohenberg, Halperin):

$$D = \lambda \chi_B^{-1} \sim \xi^{x_\lambda} \chi_B^{-1},$$
$$\bar{\eta} \sim \xi^{x_\eta}.$$

$$x_\lambda + x_\eta = 4 - d - \eta.$$

$$x_\eta = \frac{1}{19} \epsilon (1 + 0.238\epsilon + \dots) \approx 0.065, \quad (1)$$

$$x_\lambda = \frac{18}{19} \epsilon (1 - 0.033\epsilon + \dots) \approx 0.916, \quad (2)$$

$$D \sim \xi^{-2+\eta+x_\lambda},$$

$$z = 4 - \eta - x_\lambda \approx 3.$$

# Conclusion

- Mixing between  $\sigma$  and  $n$  (and  $\varepsilon$ ) leaves only one hydrodynamic mode – diffusive.
- Dynamic universality class of QCD c.p. is that of model H.
- $z \approx 3$  ( $> 2$  of model A, but  $< 4$  of model B).
- Finite time constraint is rather strong:  $\xi_{\max} \sim \tau^{1/z}$  is not too large.

# Appendix

# Divergence of $\lambda\bar{\eta}$

$$\mathbf{E} = -\nabla\mu \quad \rightarrow \quad \mathbf{j}_n = \lambda\mathbf{E} \quad (\text{in model B})$$

$$\mathbf{f}_{\text{appl}} \sim n\mathbf{E}L^d$$

$$\mathbf{f}_{\text{visc}} + \mathbf{f}_{\text{appl}} = 0$$

$$\mathbf{f}_{\text{visc}} \sim -\bar{\eta}\rho\mathbf{v}L^{d-2}$$

$$\mathbf{j}_n = n\mathbf{v} \sim \left( \frac{n^2}{\bar{\eta}\rho} L^2 \right) \mathbf{E}. \quad (\text{in model H})$$

$L \rightarrow \infty$  divergence is cut off at  $L \sim \xi$ , because  $n$  is correlated only up to this scale.

$$\langle n^2 \rangle = T\chi_B/L^d \text{ for } L \gg \xi$$

$$\lambda\bar{\eta} \sim \langle n^2 \rangle \xi^2 \sim \chi_B \xi^{2-d} \sim \xi^{4-d-\eta},$$

# Real time correlation functions

$$\langle \sigma_{\omega \mathbf{q}}^2 \rangle = \frac{2T\Gamma}{\omega^2 + \Gamma^2 A^2} + \frac{2TB^2 \lambda \mathbf{q}^2}{A^2(\omega^2 + D^2 \mathbf{q}^4)},$$

$$\langle n_{\omega \mathbf{q}}^2 \rangle = \frac{2T\lambda \mathbf{q}^2}{\omega^2 + D^2 \mathbf{q}^4}.$$