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QCD Thermodynamics from Imaginary μ

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Motivation

- Imaginary chemical potential
- "Analyticity" of the pseudo-critical line
- $T_c(\mu)$: $N_f = 2, 3, 4$ results and comparison with other methods
- Critical endpoint and its quark mass dependence for $N_f = 3$
- Outlook on $N_f = 2 + 1$
- Conclusions

Motivation

Fermion determinant complex for SU(3), $\mu \neq 0$

$\Rightarrow$ no standard Monte Carlo importance sampling

Recent numerical methods:

Fodor, Katz
Allton et al.
de Forcrand, O.P.

No solution to sign problem

but tricks to ease or side-step it $\Rightarrow$approximations, problems?

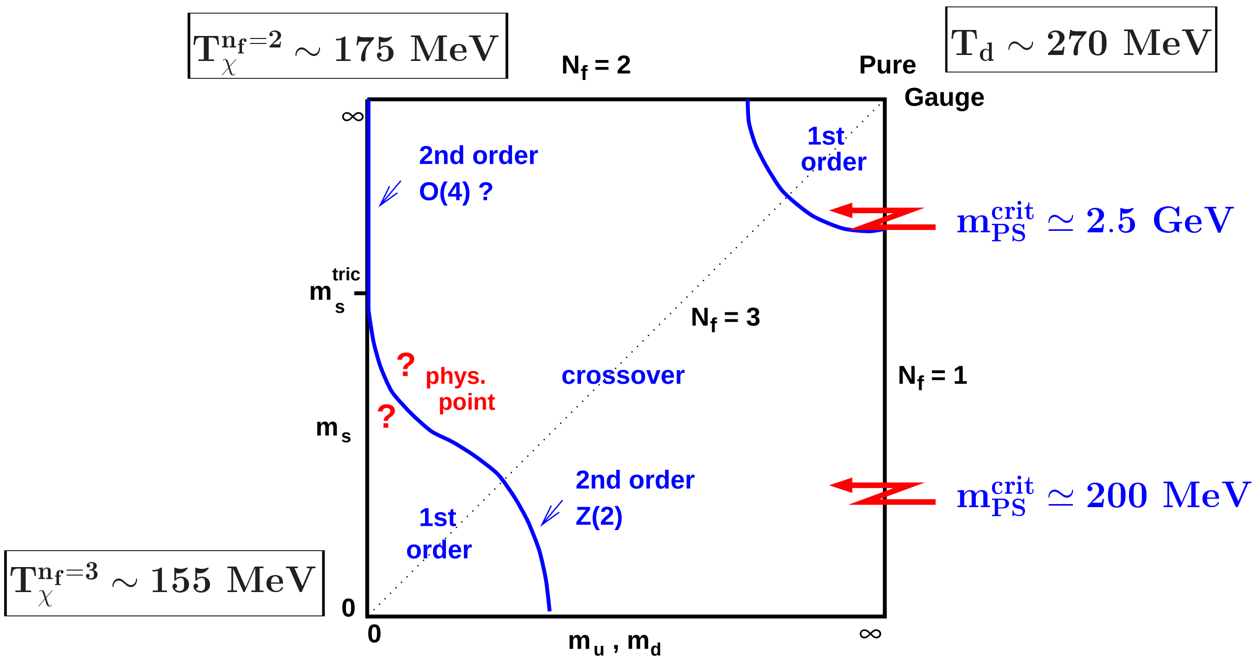
- reweighting: potential overlap problem, error analysis
- reweighting and/or Taylor expansion: dto., convergence
- imaginary μ: limited range of applicability (convergence)

$\Rightarrow$**Pursue all, cross check to control errors**

Lattice QCD at finite temperature

Karsch hep-lat/0109017

3-flavour phase diagram



The critical temperature

$$\text{pure gauge}: \quad T_c = (271 \pm 2)\ \text{MeV}$$

chiral limit of

$$N_f = 2: \quad T_c = \begin{cases} (171 \pm 4)\ \text{MeV}, & \text{clover-impr. Wilson} \\ (173 \pm 8)\ \text{MeV}, & \text{improved KS} \end{cases}$$

$$N_f = 3: \quad T_c = (154 \pm 8)\ \text{MeV}, \quad \text{improved KS}$$

N.B: pure gauge: cont. limit reached,

fermions: $a \approx 0.3$ fm, still coarse

Imaginary μ / fixed baryon density

Weiss;Miller,Redlich;
Dagotto et al;
Hasenfratz,Toussaint

$$Z_B(T,V) = \frac{1}{2\pi}\int_0^{2\pi} d\nu\, \mathrm{e}^{-\mathrm{i}\nu B/T} Z(\mu = \mathrm{i}\nu, T, V)$$

measure of Z positive, standard MC possible

I. compute Z_B by numerical Fourier trafo

Alford,Kapustin,Wilczek

Hubbard model at large T and small B

does not work in thermodynamic limit, QCD?

II. analytic continuation of observables

Lombardo

$\langle\bar{\psi}\psi\rangle$ at real and imag. μ in strong coupling

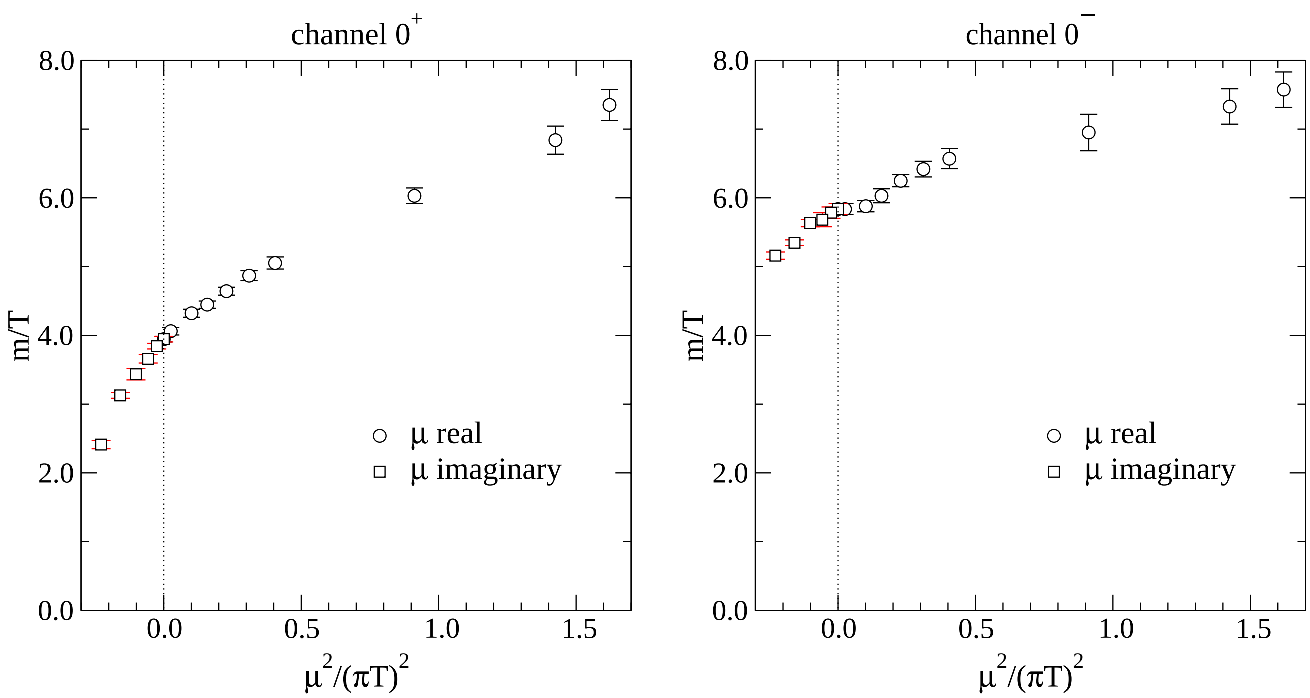
Test: screening masses from dim. red.

Hart,Laine,O.P.

3d eff. action for Matsubara zero-modes ($T \gtrsim 2T_c$):

$$S \to S + \mathrm{i}z \int d^3x \, \mathrm{Tr}\, A_0^3; \quad z = \frac{\mu}{T}\frac{N_f}{3\pi^2} \quad \left(\ll \frac{\mu}{T}\right)$$

complex, but sign prob. mild, physical volumes possible



$M(\mu)$ analytic in μ (no massless modes, no transition)

$$\frac{M}{T} = c_0 + c_1 \left(\frac{\mu}{\pi T}\right)^2 + c_2 \left(\frac{\mu}{\pi T}\right)^4 + \ldots$$

fit c_0, c_1, c_2 to data from real and imag. μ

works for $\mu \lesssim 1.5T \Rightarrow$ Critical line in 4d?

QCD at complex μ: general properties

$$Z(V,\mu,T) = \mathrm{Tr}\left(\mathrm{e}^{-(\hat{H}-\mu\hat{Q})/T}\right); \quad \mu = \mu_R + \mathrm{i}\mu_I$$

$\Rightarrow$Taylor expansion is in $\bar{\mu} = \mu/T$

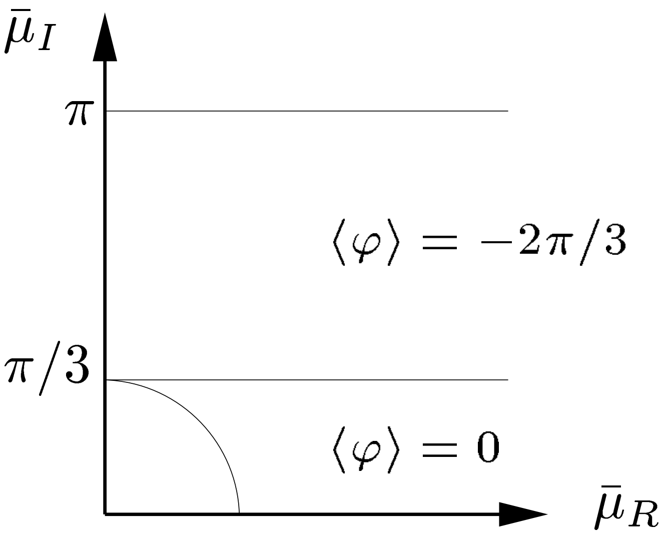
μ term breaks $\mathcal{T},\mathcal{C}$; $\quad \mathcal{T}$ compensated by $\mu \to -\mu$

$$\Rightarrow \boxed{Z(\bar{\mu}) = Z(-\bar{\mu})}$$

periodicity: $\quad Z(3)$ transf equivalent to shift in μ_I Roberge, Weiss

$$\Rightarrow \boxed{Z(\bar{\mu}_R,\bar{\mu}_I) = Z(\bar{\mu}_R,\bar{\mu}_I + 2\pi/N)}$$

$Z(3)$ sectors identified by Polyakov loop $\langle P(x)\rangle = |\langle P(x)\rangle|\mathrm{e}^{\mathrm{i}\langle\varphi\rangle}$



Z(3)-transitions:
$\bar{\mu}_I^c = \frac{2\pi}{3}\left(n+\frac{1}{2}\right)$

pert./strong coupling:
1rst order for deconf. phase
crossover for conf. phase

within arc:

$$\langle O\rangle = \sum_n^N c_n \bar{\mu}_I^{2n} \Rightarrow \mu_I \longrightarrow i\mu_I$$

Analyticity of the (pseudo-) critical line   with P. de Forcrand

phase transition from maximum of susceptibilites:

$$\chi(\beta,\bar\mu,V) = VN_t\left\langle(\mathcal{O}-\langle\mathcal{O}\rangle)^2\right\rangle, \quad \mathcal{O}\in\{\text{plaq},\bar\psi\psi,|P(x)|\}$$

finite volume: suscept. finite and analytic in all cases

- Location of transition:

  Critical line $\beta_c(\bar\mu)$ defined by peak $\chi_{max}\equiv\chi(\bar\mu_c,\beta_c)$

$$\left.\frac{\partial\chi}{\partial\beta}\right|_{\bar\mu_c,\beta_c}=0, \qquad \left.\frac{\partial^2\chi}{\partial\beta^2}\right|_{\bar\mu_c,\beta_c}<0.$$

  Implicit function theorem:

  $\chi(\beta,\bar\mu)$ analytic $\Rightarrow\beta_c(\bar\mu)$ analytic!

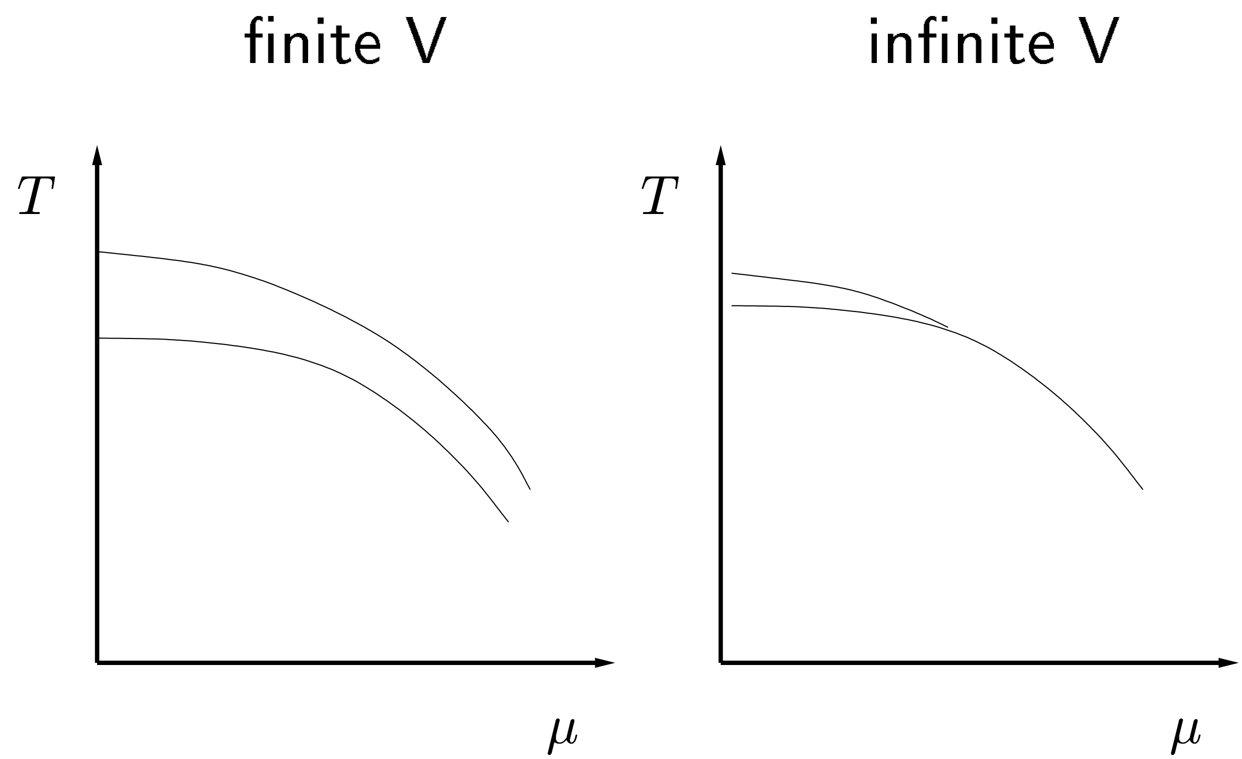
  symmetries: $\chi(\bar\mu)=\chi(-\bar\mu)$ $\Rightarrow$ $\beta_c(\bar\mu)=\beta_c(-\bar\mu)$

$$\boxed{\beta_c(\bar\mu)=\sum_n c_n(a\bar\mu)^{2n}}$$

Approaching the thermodynamic limit

Def. of $\beta_c(\bar{\mu})$ not unique on finite V (e.g. $\partial_{\bar{\mu}}\chi = 0$)

different definitions:



Crit. line unique in thermodynamic limit! (not for crossover)

For large V it is approached arbitrarily well by $\partial_\beta \chi = 0$

- Order of transition:

  finite volume scaling: $(\beta_c(V) - \beta_c(\infty)) \sim V^{-\sigma}$

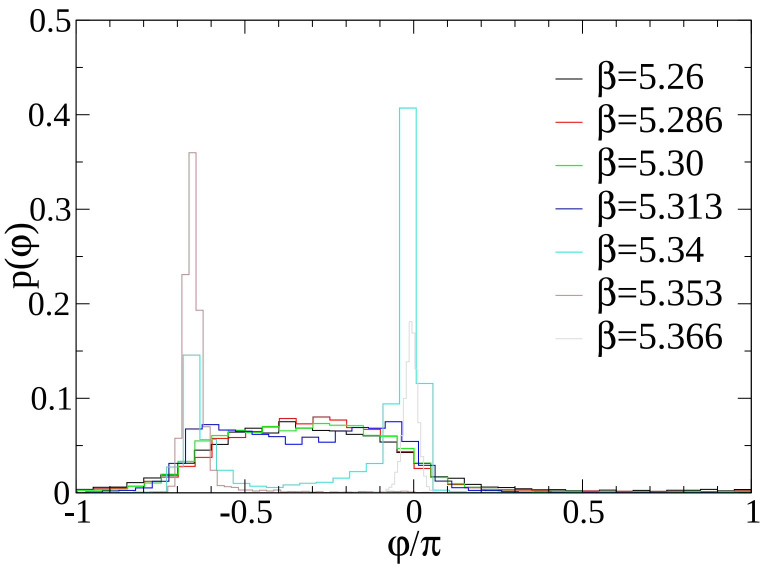
$\sigma = 1$	1st order
$\sigma < 1$	2nd order
$\sigma = 0$	crossover

$N_f = 2$ results

$8^3 \times 4$, KS fermions, $m_\pi \approx 300$ MeV

($T_c(\mu = 0) \sim 170$ MeV, $a \sim 0.3$ fm)

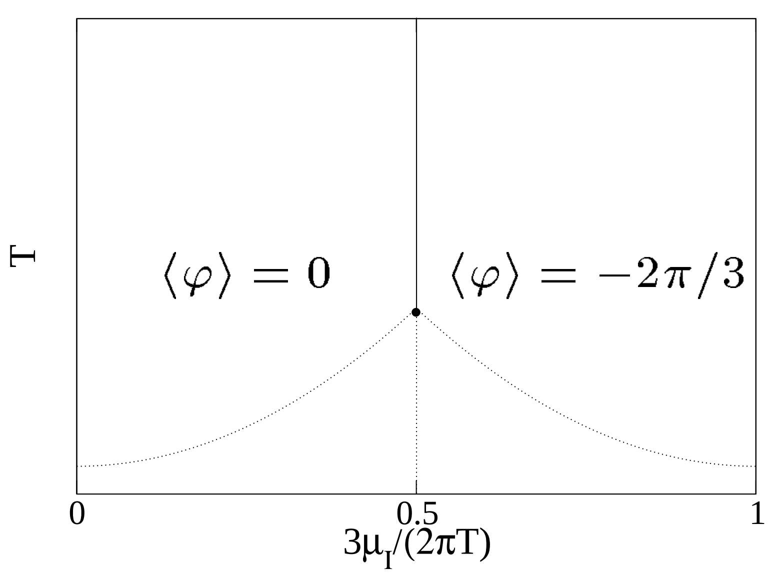
Z(3) transition: phase of $P(x)$ for $(a\mu_I)^c = \pi/12$



1rst order for deconf. phase

continuous for conf. phase

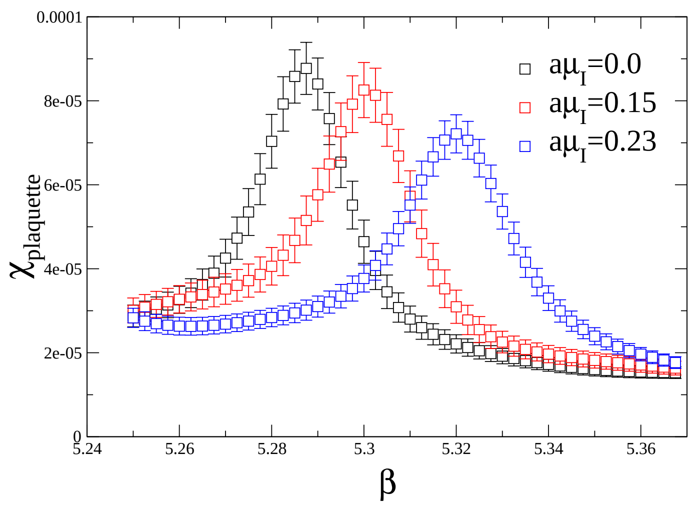
⇒schematic phase diagram: (cf. $N_f = 4$, D'Elia, Lombardo)

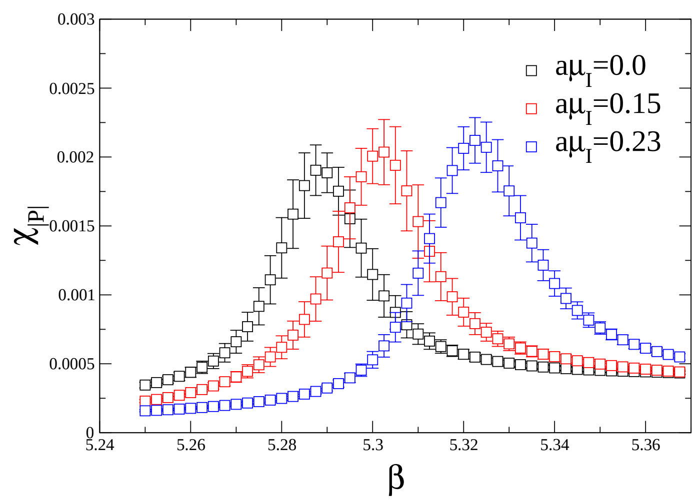


The deconfinement line $T_c(\mu)$: susceptibilities

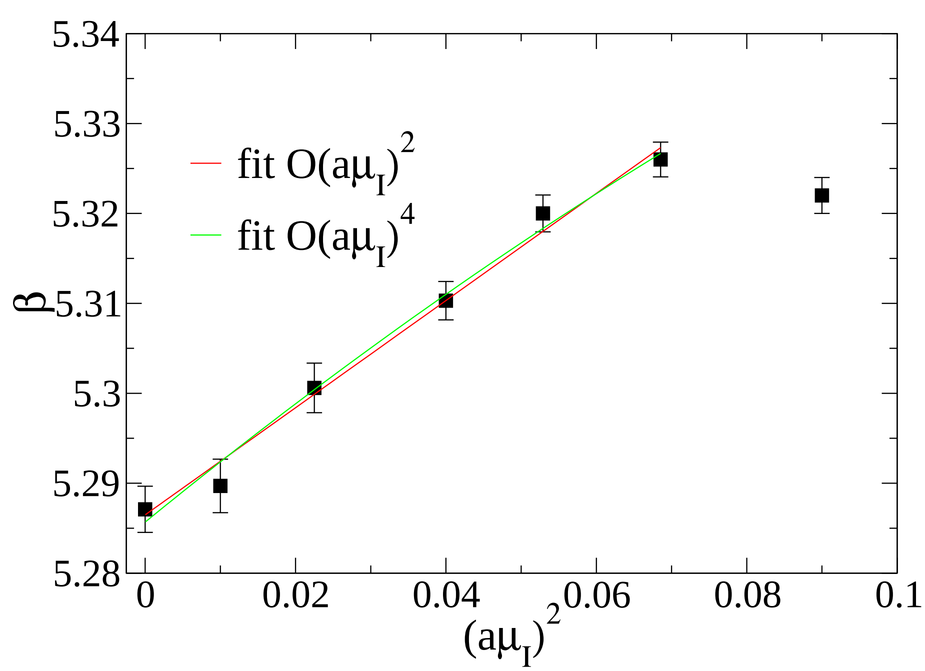
plaquette

Polyakov loop



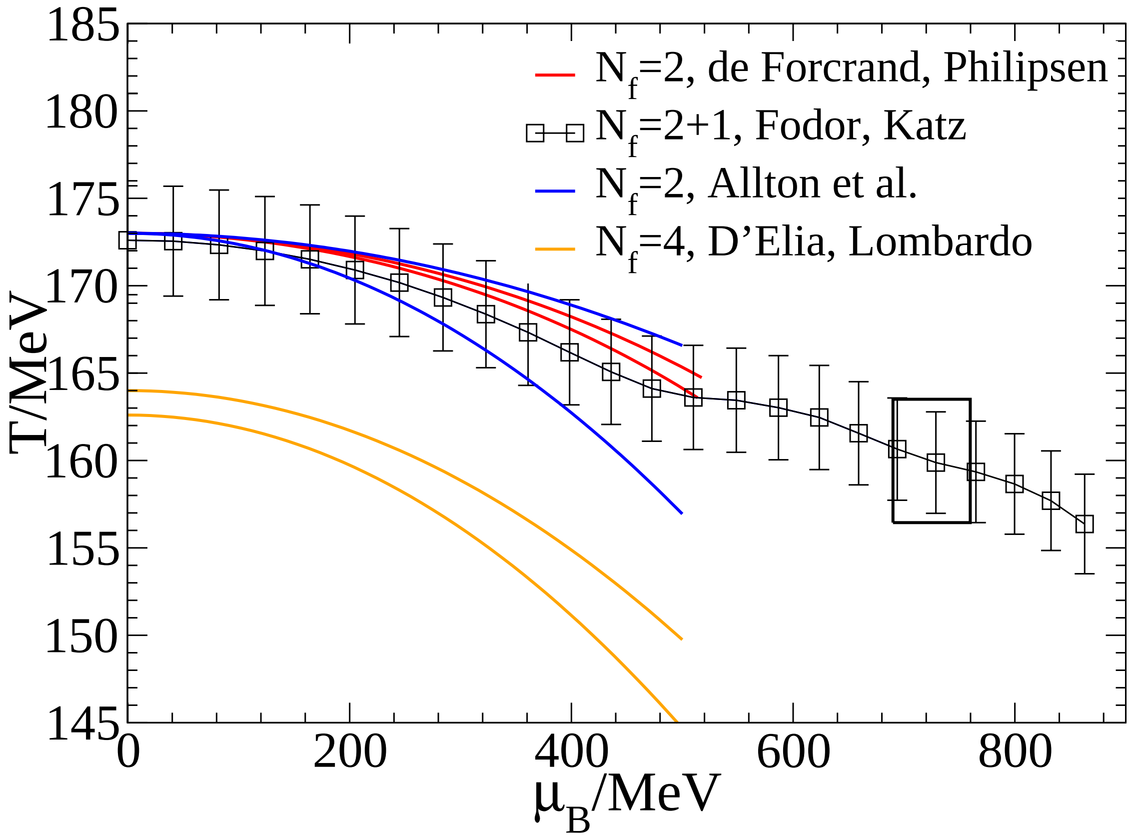


Lattice phase diagram for $\mu = i\mu_I$



control over Taylor expansion, no volume restrictions!

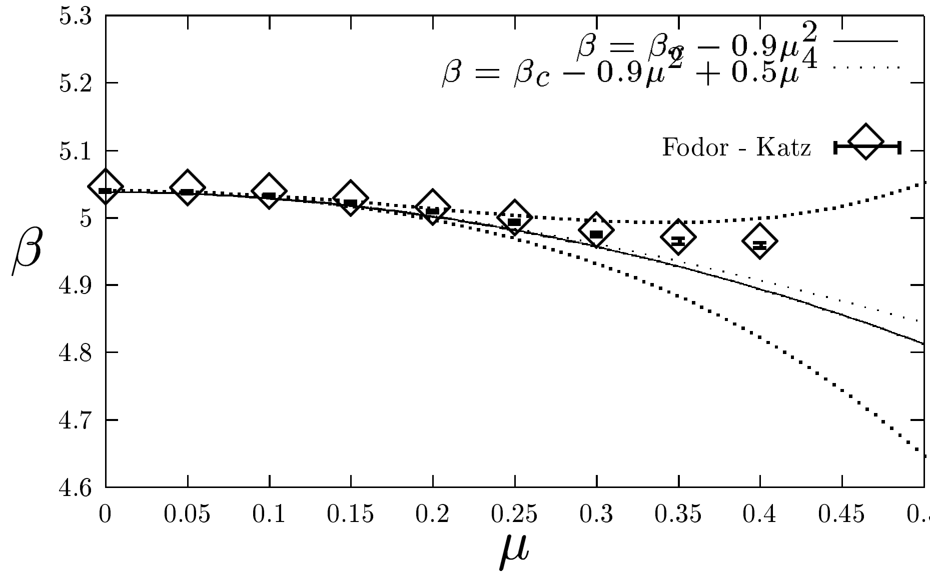
Comparison:



$a \sim 0.3$ fm in all calculations

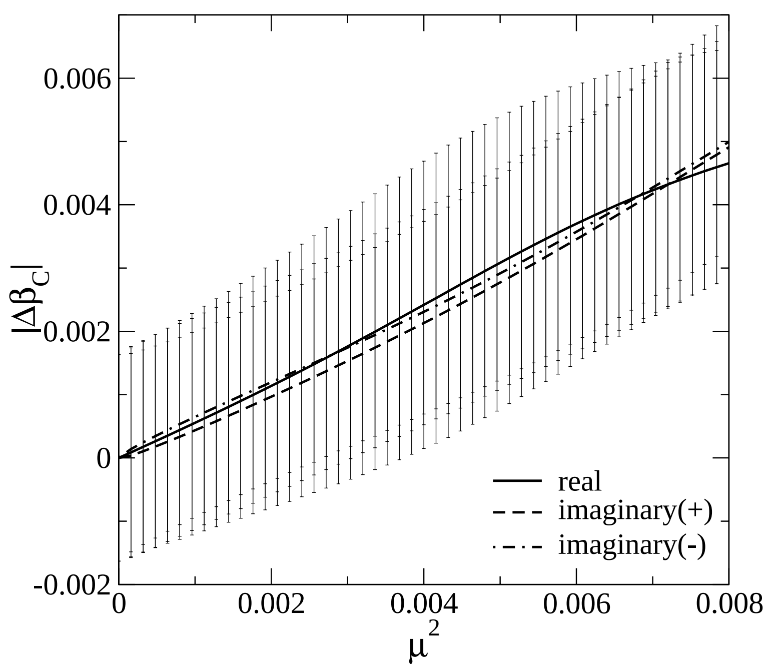
Method	N_f, m_q	largest lattice
reweighting (FK)	2+1, $am_u = 0.025$, $am_s = 8am_u$	$8^3 \times 4$
rew. +Taylor (A et al.)	2 , $am = 0.1$	$16^3 \times 4$
imag. μ (FP)	2, $am = 0.025$	$8^3 \times 4$
imag. μ (EL)	4, $am = 0.05$	$16^3 \times 4$

Comparison: $N_f = 4$, imag. μ vs. reweighting D'Elia, Lombardo



Comparison: analyt. continuation ok in SU(2) Giudice Papa

Comparison: real μ vs. imag. μ to leading order Allton et al.



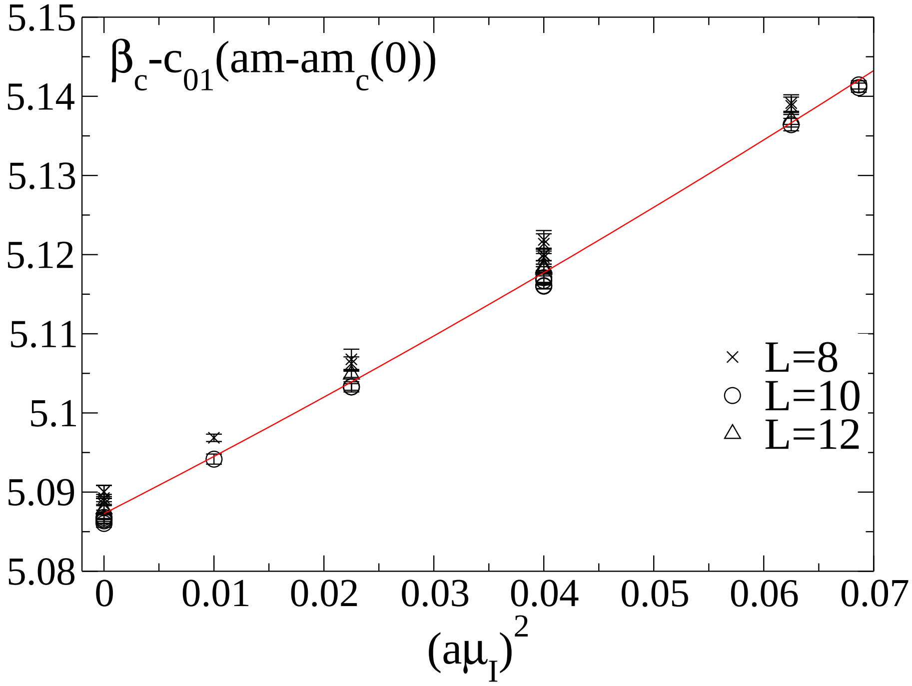
$N_f = 3$ **results,** $T_c(\mu, m)$**:**

de Forcrand, O.P.

vary quark masses m, much more stats.

check for NLO terms in Taylor series:

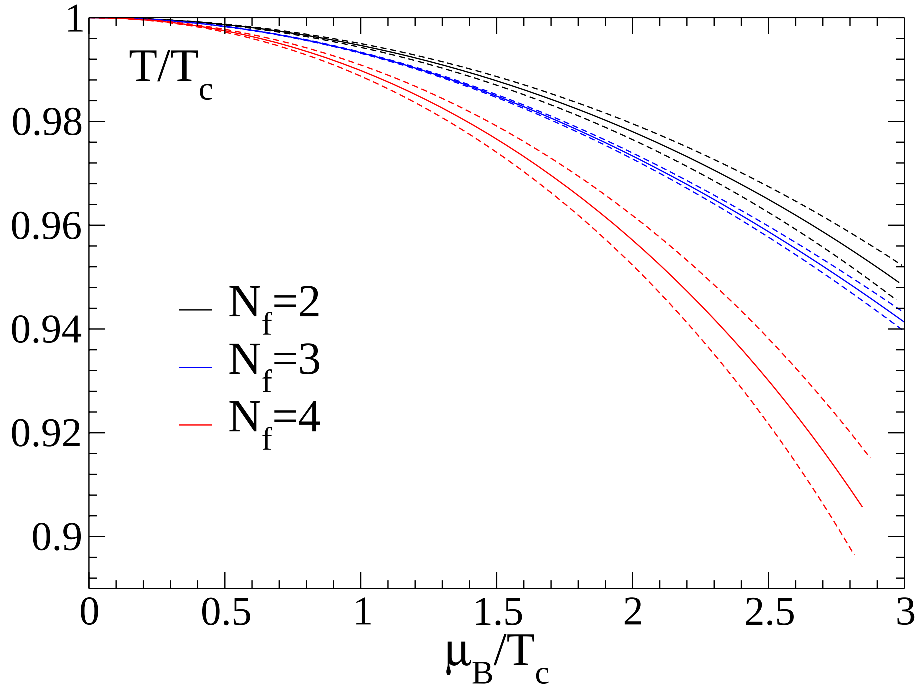
$$\beta_c(a\mu, am) = \sum_{k,l=0} c_{kl}\,(a\mu)^{2k}\,(am)^l$$



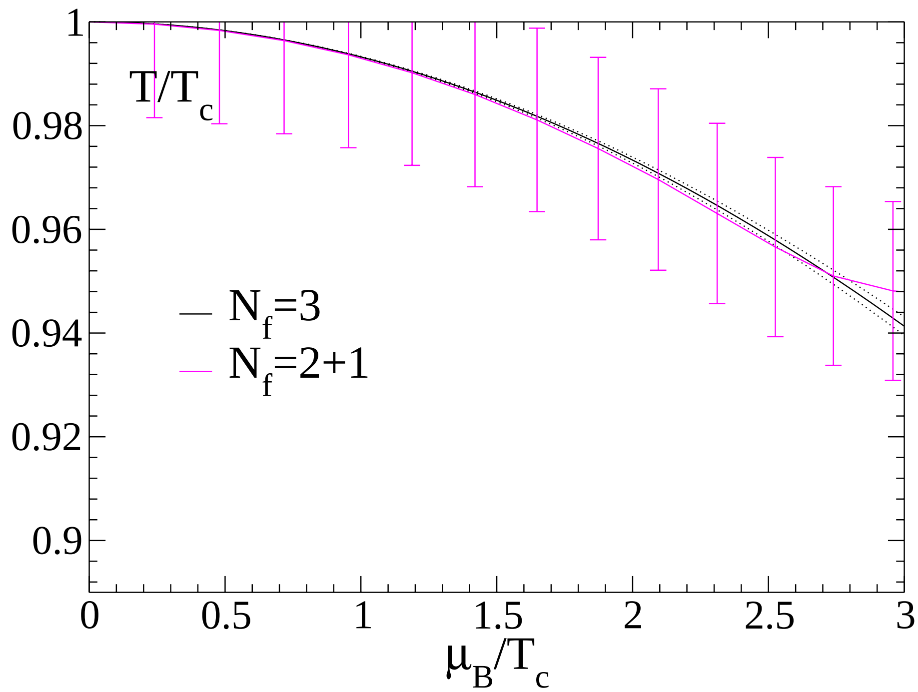
⇒evidence for μ^4! no evidence for $\mathcal{O}(m^2, \mu^2 m)$

$$\frac{T_c(\mu, m)}{T_c(\mu = 0, m_c(0))} = 1 + 1.94(2)\left(\frac{m - m_c(0)}{\pi T_c}\right) + 0.602(9)\left(\frac{\mu}{\pi T_c(0, m)}\right)^2 + 0.23(9)\left(\frac{\mu}{\pi T_c(0, m)}\right)^4$$

N_f dependence



N.B.: only $N_f = 3$ has μ^4 correction

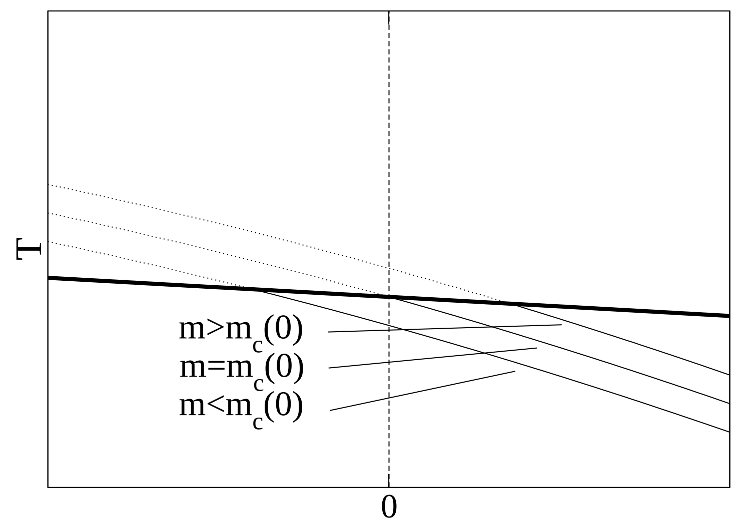


The critical endpoint Phase diag. 3d: (T, μ, m)

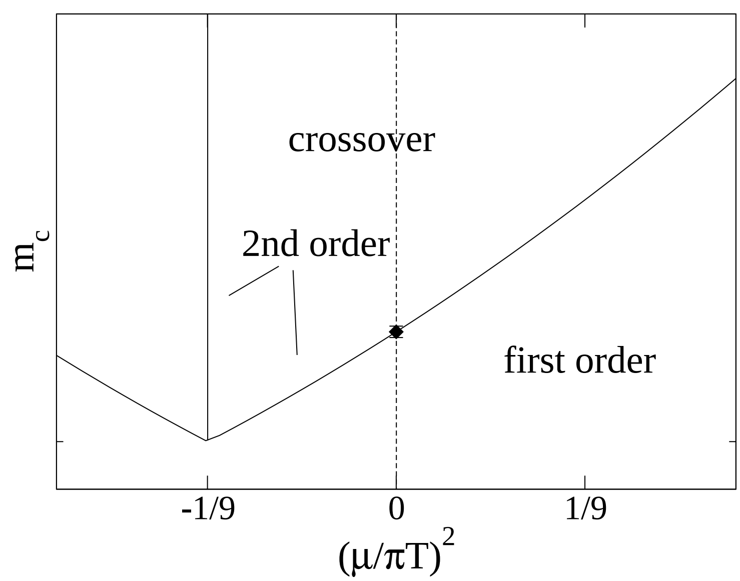
•confined/deconfined ⇒pseudo-crit. surface $T_c(\mu, m)$

On this surface,
•1.O./crossover ⇒line of crit.points $T^*(\mu) = T_c(\mu, m_c(\mu))$

Project on (T, μ) :



Project on (m, μ) :



$m = 0$ ⇒true chiral phase transition

Expect: $\frac{m_c(\mu)}{m_c(\mu=0)} = 1 + c_1 \left(\frac{\mu}{\pi T}\right)^2$ ⇒ $\boxed{c_1 \leq 9}$

Criticality: cumulant ratios, **3d Ising universality**:

$$B_4(m_c,\mu_c) = \frac{\langle(\delta\bar{\psi}\psi)^4\rangle}{\langle(\delta\bar{\psi}\psi)^2\rangle^2} \to 1.604, \quad V \to \infty$$

($<,>\Rightarrow$ first-order, crossover)

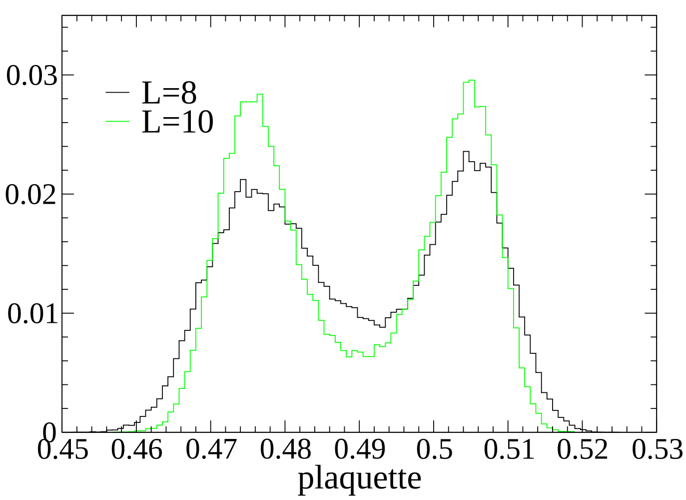
fit data to Taylor series about crit. point

$$B_4(am,a\mu) = 1.604 + B\left(am - am_c(0) + A(a\mu)^2\right) + \ldots$$

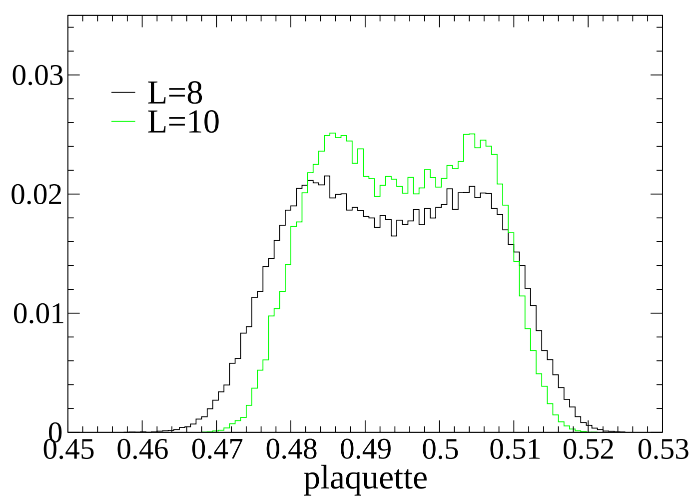
Chain rule: $\frac{dam_c}{d(a\mu)^2} = -\frac{\partial B_4}{\partial(a\mu)^2}\left(\frac{\partial B_4}{\partial am}\right)^{-1} = -A$

need VERY LONG MC runs for sufficient tunneling statistics

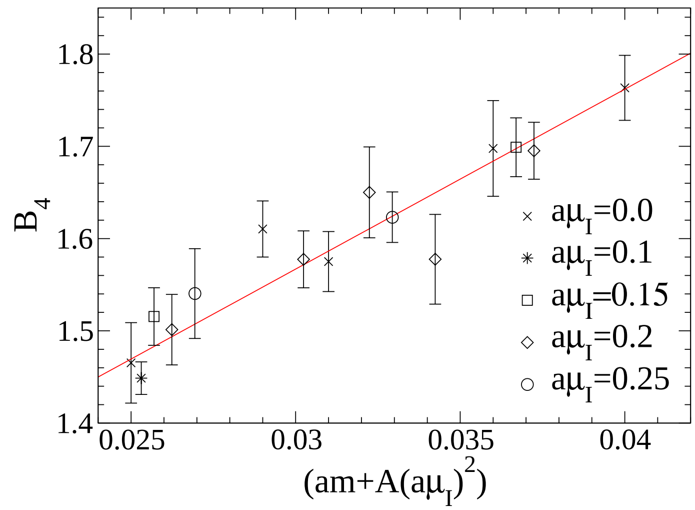
first-order



crossover



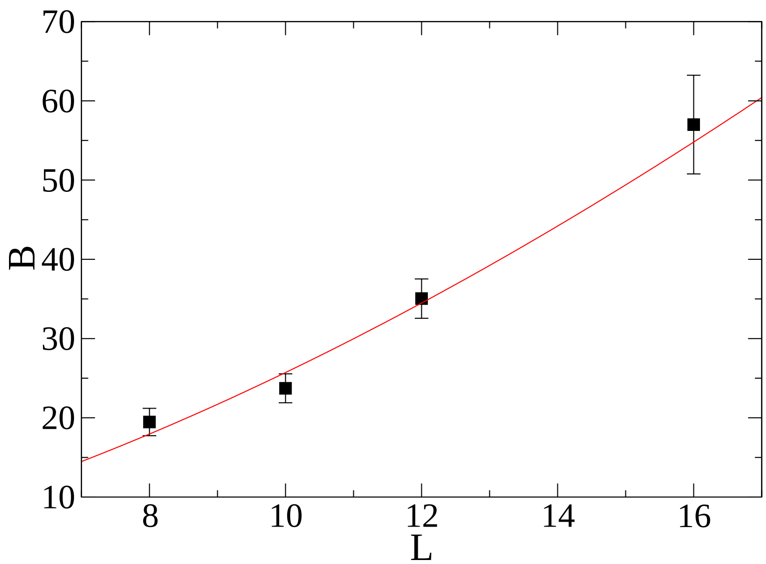
$$B_4(am, a\mu) = 1.604 + B\left(am - am_c(0) + A(a\mu)^2\right)$$



$m_c(0)$ in agreement with previous result for $\mu = 0$

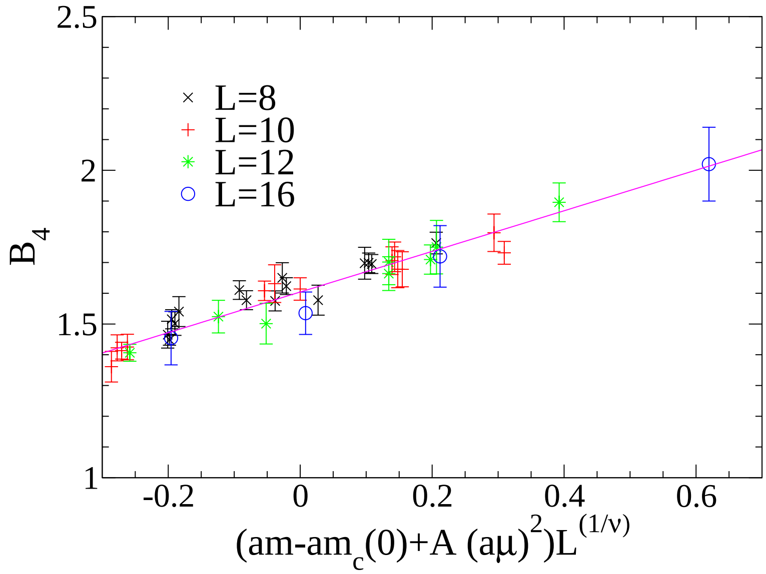
Ising FSS $(\beta = \beta_c(m, \mu))$: $\xi \propto |m - m_c(\mu)|^{-\nu}$

$\Rightarrow B_4(L/\xi) = B_4\left((L/\xi)^{1/\nu}\right)$



$\nu = 0.62(3)$

$\nu(Ising) = 0.63$



Critical quark mass, μ_c:

$$\Rightarrow \frac{m_c(\mu)}{m_c(\mu=0)} = 1 + 0.84(36)\left(\frac{\mu}{\pi T}\right)^2 + \ldots$$

tentative comparison

Taylor exp. rew., improved action: $c_1 \sim 290....2500$ Allton et al.

⇒our critical point at larger μ_c

tiny change in m ⇒huge change in μ_c

Outlook for $N_f = 2+1$

Phase diag. 4d: $(T, \mu, m_{u,d}, m_s)$

Two observables: $B_4^{u,d}(m_{u,d}, m_s, \mu), B_4^s(m_{u,d}, m_s, \mu)$

expand about $N_f = 3, \mu = 0$ critical quark mass m_c

$$B_4^i(m_{u,d}, m_s, \mu) = \sum_{n,l,k} b_{nlk}^i (m_{u,d} - m_c)^n (m_s - m_c)^l (\mu)^{2k}$$

$$B_4^{u,d}(m, m, \mu) = B_4^s(m, m, \mu) = B_4^{N_f=3}(m, \mu)$$

⇒constraints: μ^{2k}-coeffs. equal in all three!

⇒leading order reduces to $\mu = 0$ investigation

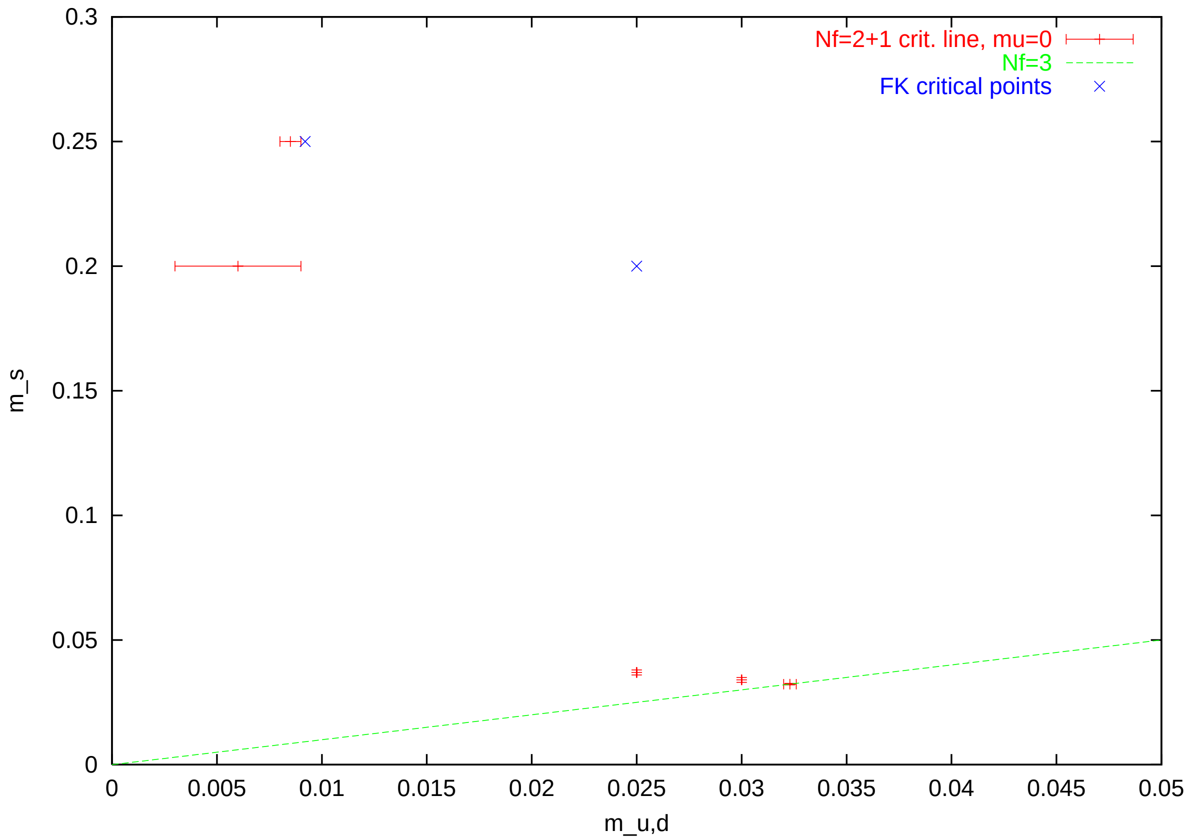
⇒**strategy:**

I. find $(m_{u,d}, m_s)_c$ for $\mu = 0$

II. repeat analysis for (T, μ, m_s) or $(T, \mu, m_{u,d})$

Preliminary results:

$(m_s, m_{u,d})$ phase-diagram, $\mu = 0$:



$\Rightarrow$strong non-linearities in $m_{u,d}$

$\Rightarrow$qualitatively consistent with our $N_f = 3$ results

Conclusions

Simulations of small baryon densities possible at finite T

⇒analytic continuation of non-singular observables,
control over systematics

- location of transition line consistent with other approaches
- (pseudo-) critical line $T_c(m, \mu)$ extremely flat, small quark mass dependence
- critical point for physical QCD not yet under control
  ⇒strong quark mass dependence

Hard work (continuum limit!), but high T phase diagram in reach!