

Infrared QCD and the renormalisation group

Daniel F. Litim

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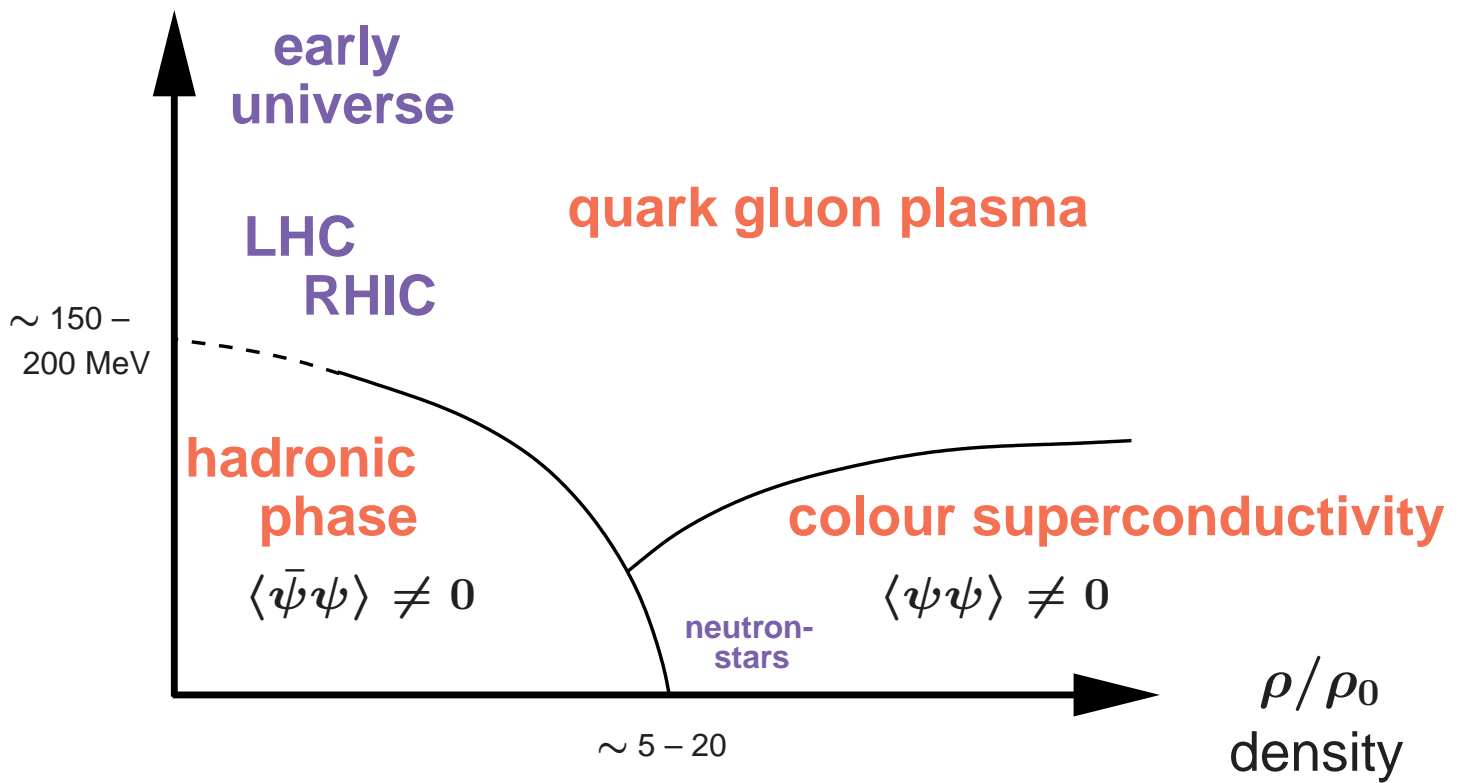
- **motivation**
 - QCD phase diagram
- **renormalisation group**
 - momentum cutoff
 - flow equation
- **applications to QCD**
 - infrared analysis in Landau gauge
 - signatures of confinement

in collaboration with

S. Nedelko (Dubna), J.M.Pawlowski (Erlangen), L. von Smekal (Erlangen)

Phase diagram of QCD

T temperature



- high temperature QCD
- high density QCD
- QCD at strong coupling

QCD at strong coupling

- relevant for:

confinement

chiral symmetry breaking

confinement-deconfinement phase transition

matter at realistic densities

hot QCD dynamics of soft excitations

- lattice simulations

ideal for

- time independent observables
- finite temperature

problematic for

- dynamics, non-equilibrium
- high baryonic density

- renormalisation group methods

analytic approach

complementary to lattice

Exact Renormalisation Group: very flexible

Exact Renormalisation Group

- **goal:** calculation of quantum effective action $\Gamma[\phi]$
- successive integrating-out of momentum modes via $S[\phi] \rightarrow S[\phi] + \Delta S_k[\phi]$, with

$$\Delta S_k[\phi] = \int_q \phi(q) \mathbf{R}(q) \phi(-q)$$

leads to scale-dependent effective action Γ_k with momentum modes $q^2 > k^2$ integrated out.

- infinitesimal integration \Rightarrow flow equation:

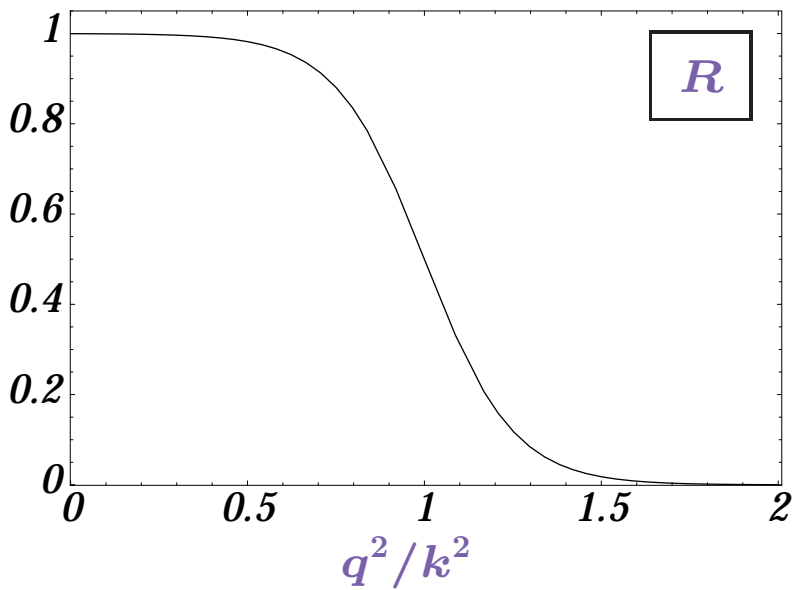
$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left(\frac{\delta^2 \Gamma_k[\phi]}{\delta \phi \delta \phi} + \mathbf{R} \right)^{-1} \partial_t \mathbf{R}$$

initial condition: classical action $\Gamma_{k \rightarrow \Lambda} = S$

endpoint: quantum effective action $\Gamma_{k \rightarrow 0} \equiv \Gamma$

flow is infrared finite and ultraviolet finite

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left(\frac{\delta^2 \Gamma_k[\phi]}{\delta \phi \delta \phi} + R \right)^{-1} \partial_t R = \frac{1}{2} \text{circle with a cross}$$

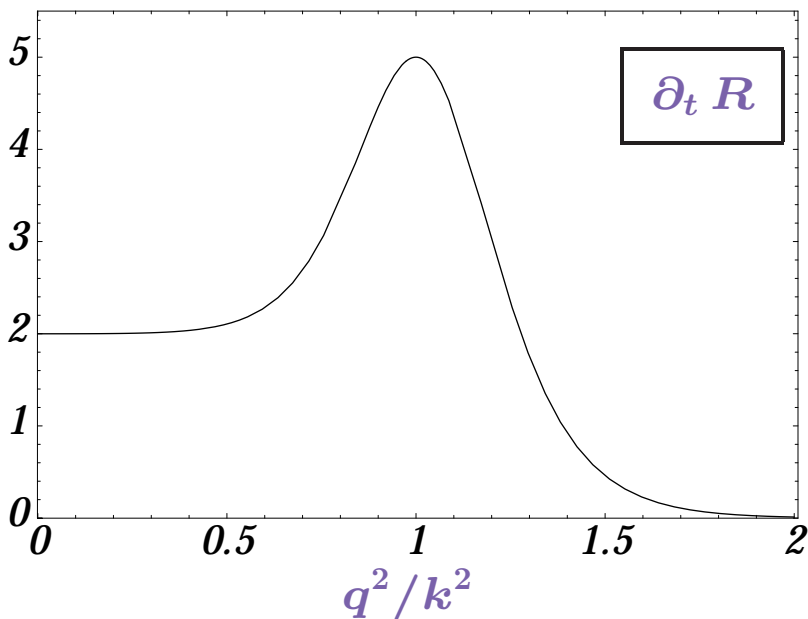


- infrared cutoff R

$$R(q^2 \rightarrow 0) \approx k^2$$

$$R(k \rightarrow 0) \rightarrow 0$$

$$R(q^2 \rightarrow \infty) \rightarrow 0$$



- locality
momentum integration
peaked about

$$q^2 \approx k^2$$

Example: scalar fields at criticality

- real scalar field theory Ising universality class
- scaling behaviour

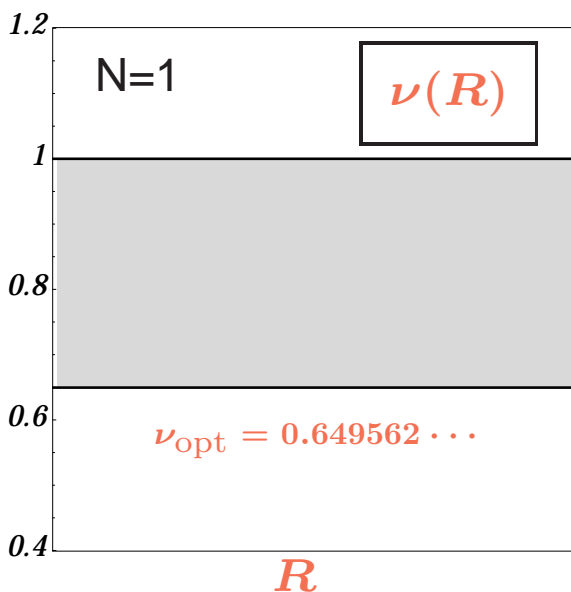
flow equation for effective potential u in $3d$:

$$\partial_t u + 3u - \rho u' = \int_0^\infty dy \frac{-y^{5/2} r'(y)}{y(1+r) + u' + 2\rho u''}$$

scaling variables $y = \frac{q^2}{k^2}$, $\rho = \frac{\phi^2}{2k}$, $u(\rho) = \frac{U(\phi)}{k^3}$, $r = \frac{R}{q^2}$

- critical exponent ν

deduced from Wilson-Fisher fixed point $\partial_t u' = 0$



- optimisation for ν
- global extrema exist

$$\nu_{\text{opt}} = \min_R \nu(R) .$$

- optimisation entails PMS
- physical value ($N=1$)

$$\nu_{\text{phys}} \approx 0.63 .$$

Gauge symmetry

- gauge symmetry \Rightarrow link between Greens functions

Ward identity:

$$D_\mu \frac{\delta \Gamma}{\delta A_\mu} = \text{quantum corrections}$$

- flow equation:

momentum cutoff quadratic in the fields $\sim \int A R_k A$ is, a priori, incompatible with non-linear gauge symmetry.

modified Ward identity:

$$D_\mu \frac{\delta \Gamma_k}{\delta A_\mu} = \text{quantum corrections} + \text{cutoff terms}$$

$$\text{cutoff terms} \sim \langle D_\mu \frac{\delta}{\delta A_\mu} \Delta S_k \rangle \neq 0$$

- gauge invariance of physical Greens functions:

required in the infrared limit at $k = 0$.

not mandatory at $k \neq 0$.

Confinement in Landau gauge QCD

- Kugo-Ojima confinement criterion

gluonic mass gap and absence of Higgs mechanism
 \Leftrightarrow momentum behaviour of two-point functions:

$$\Gamma_A^{(2)}(p) = p^{2(1-2\kappa_C)}, \quad \Gamma_C^{(2)}(p) = p^{2(1+\kappa_C)}$$

confinement: $\kappa_C \geq 0$

- Schwinger-Dyson equation

infrared coefficients $\kappa_C \geq 0, \alpha_s$

inclusion of quarks

problems: RG scaling and renormalisation

- stochastic quantisation

infrared coefficients $\kappa_C \geq 0, \alpha_s$

resolution of Gribov problem

problems: RG scaling and renormalisation

- lattice

infrared behaviour of gluon propagator

problem: finite size scaling

- flow equations

heavy quark potential

effective quark interactions

problems: access to infrared regime

Flows in Landau gauge QCD

- infrared analysis

consider Greens functions for the regime

$$k^2 \ll p^2 \ll \Lambda_{\text{QCD}}^2$$

- deep infrared region
- physics already integrated-in
- trivial cutoff dependence

- fixed point behaviour

Greens functions in the deep infrared regime

$$\Gamma_k^{(n)}(p_i \ll \Lambda_{\text{QCD}}) = z_n \cdot \hat{\Gamma}^{(n)}(p_i, p_i^2/k^2)$$

- z_n is k -independent up to RG scalings
- k -dependence of $\hat{\Gamma}^{(n)}$ only via the ratio p^2/k^2

Technical details

- truncation

general ghost and gluon two-point functions

vertices $\Gamma^{(3)}$, $\Gamma^{(4)}$ from Slavnov-Taylor identities

parametrisation ($x = p^2/k^2$)

$$\Gamma_{k,A}^{(2)}(p) = z_A \cdot x^{\kappa_A} (1 + \delta Z_A(x)) \cdot p^2 \cdot \Pi(p)$$

$$\Gamma_{k,C}^{(2)}(p) = z_C \cdot x^{\kappa_C} (1 + \delta Z_C(x)) \cdot p^2$$

$$\Gamma_k^{(n)}(p) = z_n \cdot S_{cl}^{(n)}$$

- RG scaling implies

$$\kappa_A = -2\kappa_C, \quad \alpha_s = \frac{g^2}{4\pi} \frac{1}{z_A z_C^2}$$

- integrated flow

analyse $\Gamma_k^{(2)}(p) - \Gamma_0^{(2)}(p) = \int_0^k \frac{dk'}{k'} \partial_{t'} \Gamma_{k'}^{(2)}$

trade scale- for momentum integration $\frac{dk'}{k'} = -\frac{dx'}{2x'}$

\Rightarrow deduce infrared parameters

Infrared analysis of QCD

- fixed point equation

evaluate the integrated flow in the deep infrared regime:

$$\delta Z(x) = \frac{\alpha_s}{\pi^2} N \int_x^\infty \frac{dx'}{x'^{2+\kappa}} f(x'; x, \kappa, \delta Z)$$

$\delta Z(x \rightarrow \infty) \rightarrow 0$ physical regime

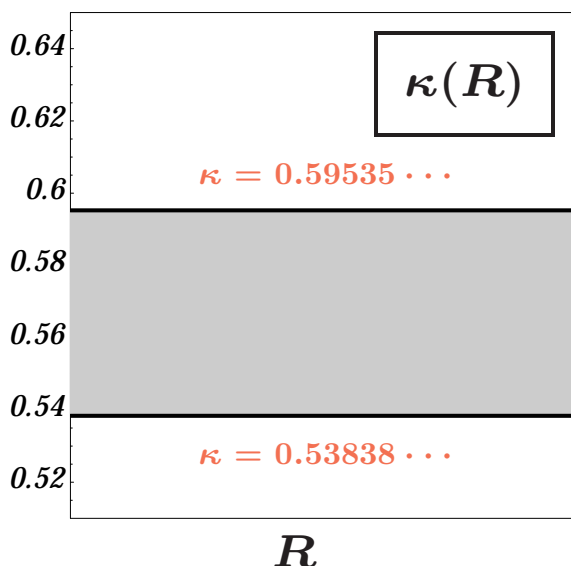
$\delta Z(x \rightarrow 0) \rightarrow -1 + \dots$ cutoff regime

\Rightarrow deduce κ , α_s and δZ from the limit $x \rightarrow 0$.

- infrared coefficients

zeroth order: $\kappa_C = 0.59535$ $\alpha_s = 2.9717$

unique, identical to Dyson-Schwinger result



full iteration:

global extrema exist

$$\kappa_{\text{opt}} = \text{extr}_R \kappa(R)$$

maximum located at the non-iterated result

Infrared analysis (cont'd)

- solution for δZ

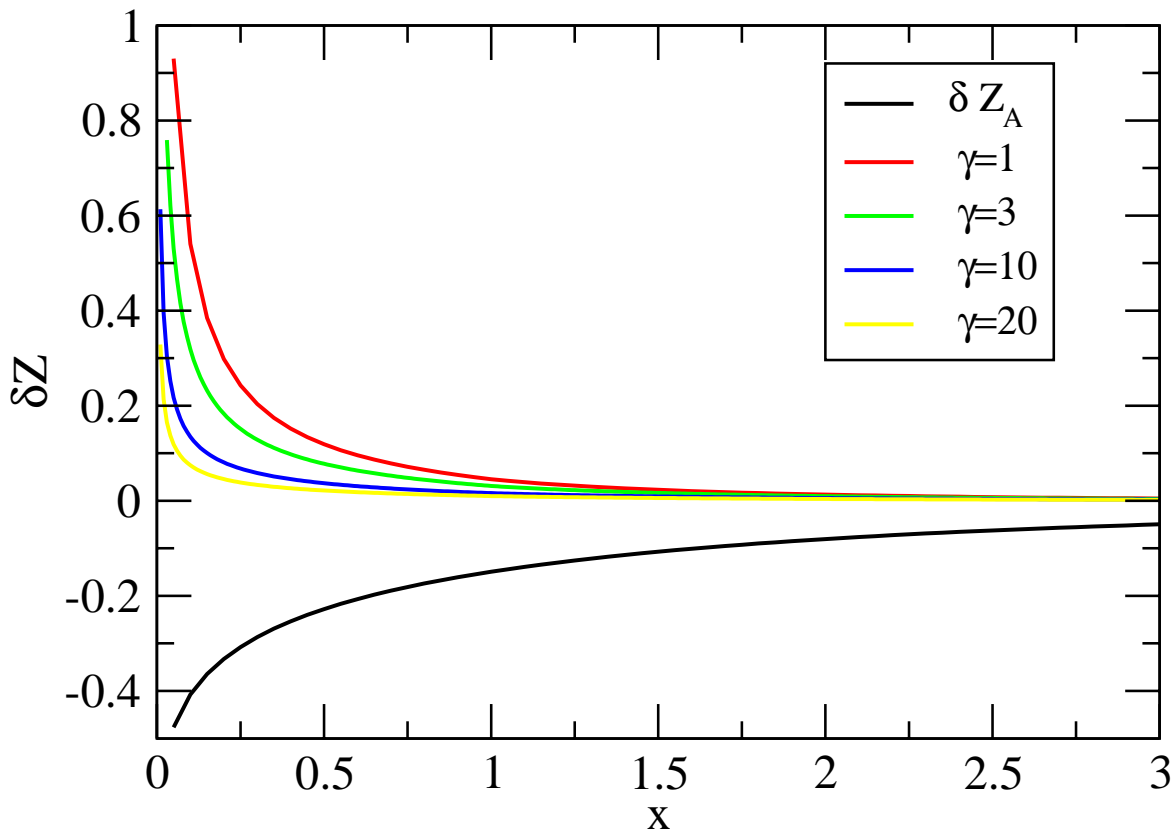
zeroth order:

solution is R independent

full iteration:

optimal propagators near non-iterated solution

$$r(x) = \frac{\gamma}{x(1+x)}$$



Conclusions and outlook

- **infrared regime of QCD**

fixed point behaviour

analytical access

infrared coefficients κ_C and α_s

support for Kugo-Ojima scenario

full vertex functions, fermions

- **finite temperature**

deconfinement phase transition

thermal pressure

- **finite density**

inclusion of quarks, chemical potential