Ginzburg-Landau approach to color superconductivity

Kei Iida (RIKEN BNL Research Center)
Gordon Baym (Illinois), Motoi Tachibana (RIKEN)
Taeko Matsuura, Tetsuo Hatsuda (Tokyo)

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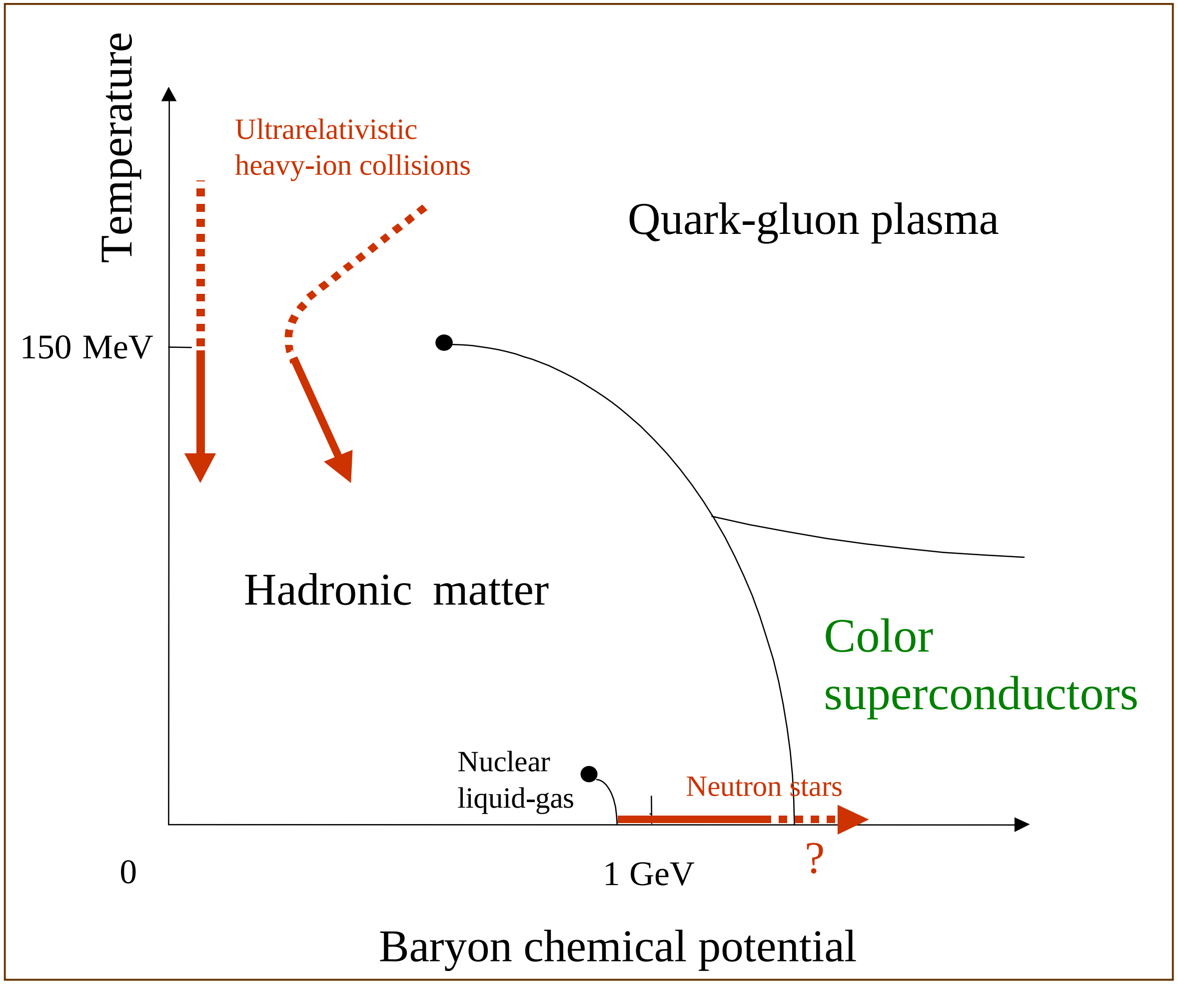
References

Iida & Baym, PRD **63** (2001) 074018; **66** (2002) 059903(E).
Iida & Baym, PRD **66** (2002) 014015.
Matsuura, Iida, Hatsuda, & Baym, PRD **69** (2004) 074012.
Iida, Matsuura, Tachibana, & Hatsuda, hep-ph/0312363.

Introduction

QCD phase diagram



Crucial features of color superconductors

Superfluidity

— presence of a condensate of quark Cooper pairs and superfluid baryon density (n_s)

Color Meissner effects

— transverse color fields screened in a spatial scale of order the London penetration depth ~ $(\mu / g^2 n_s)^{1/2}$

Introduction (contd.)

What happens if quark matter occurs in nature, e.g., in stars

Possible presence of a diquark condensate

— *at lower densities* (μ~1-2 GeV)

Strong coupling regime vs. weak coupling regime where one-gluon exchange induces Cooper pairing in color antitriplet channel

— *color neutral*

Differences in the chemical potential between colors

— *subject to stellar magnetic fields and rotation*

Supercurrents and vortices

— *at finite temperatures* (T <~ 100 MeV)

Thermal fluctuations in gauge and diquark fields

— *affected by finite strange quark mass and electric neutrality*

Splitting of the critical temperature

Focus of this work

General Ginzburg-Landau analysis near T_c

— allowing us to systematically examine the above features

Homogeneous superfluid phases

Ref. Iida & Baym, PRD **63** (2001) 074018; **66** (2002) 059903(E).

System

massless quarks of *uds* flavors and *RGB* colors
temperature T, baryon chemical potential μ
restored chiral symmetry
Fermi momenta common to all colors and flavors

Cooper pairing

Even parity, same chirality, J=0, quark-quark pairing
antisymmetric in color and flavor space

Corresponding on-shell gap at relative pair momentum $|\mathbf{k}|=k_F$:

$$(\phi_+)_{abij} = \varepsilon_{abc}\varepsilon_{ijl}(\mathbf{d}_c)_l$$

a,b: colors of paired quarks
i,j: flavors of paired quarks
$\mathbf{d}_c$: complex vector in flavor space

Homogeneous Ginzburg-Landau free energy near T_c

$$\Delta\Omega \equiv \Omega_s - \Omega_n = \alpha^+ \,\mathrm{Tr}\left(\phi_+^\dagger\phi_+\right) + \beta_1^+\left[\mathrm{Tr}\left(\phi_+^\dagger\phi_+\right)\right]^2 + \beta_2^+\,\mathrm{Tr}\left[\left(\phi_+^\dagger\phi_+\right)^2\right]$$

$$= \bar{\alpha}\sum_a |\mathbf{d}_a|^2 + \beta_1\left(\sum_a |\mathbf{d}_a|^2\right)^2 + \beta_2 \sum_{ab}\left|\mathbf{d}_a^*\cdot\mathbf{d}_b\right|^2 \qquad \text{with} \quad \bar{\alpha} = 4\alpha^+,\ \beta_1 = 16\beta_1^+ + 2\beta_2^+,\ \beta_2 = 2\beta_2^+$$

Homogeneous superfluid phases (contd.)

Candidates of energetically favorable pairing states

Ref. Pisarski & Rischke, PRL **83** (1999) 37; Schäfer, NPB **575** (2000) 269.

Two optimal states determined from energy minimization

1. 2-flavor color superconducting (2SC) state

Ref. Bailin & Love, Phys. Rep. **107** (1984) 325.

$$\mathbf{d}_R \parallel \mathbf{d}_G \parallel \mathbf{d}_B$$

- Gapped quarks: two colors and two flavors (“anisotropic”)
- Gluo-electromagnetic properties:

Long wavelength gluons → 2SC condensate at T=0 → 3 types: propagating; 5 types: screened

2. Color-flavor locked (CFL) state

Ref. Alford, Rajagopal, & Wilczek, NPB **537** (1999) 443.

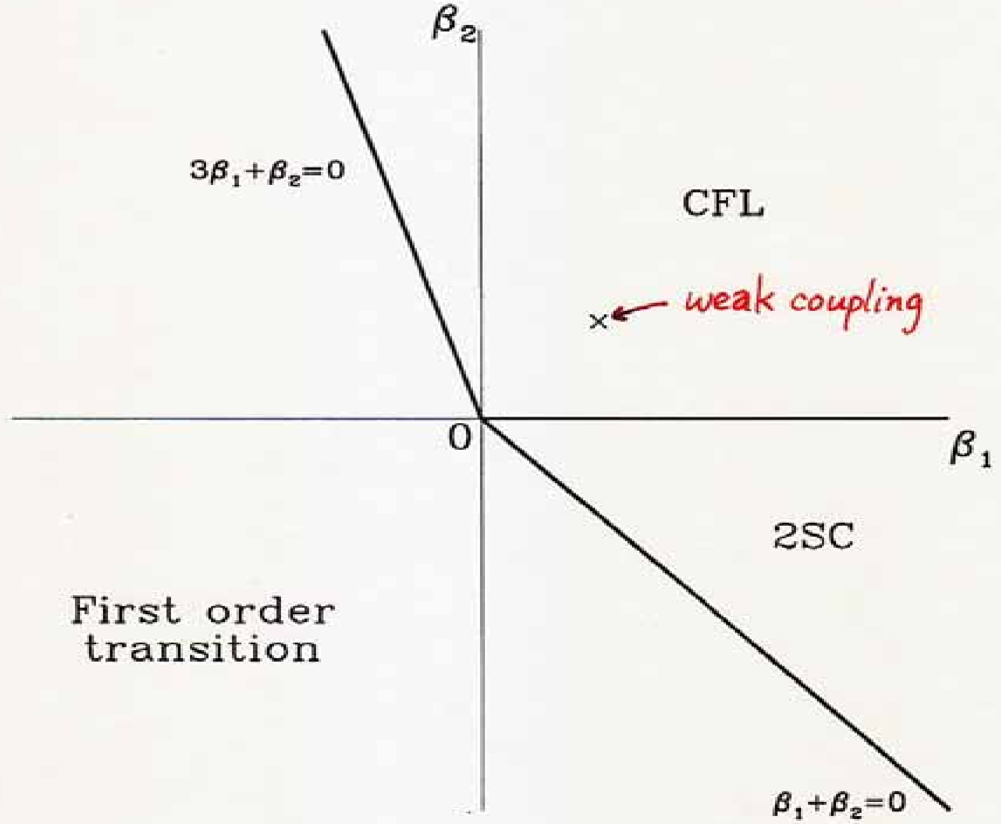
$$\mathbf{d}_R^* \cdot \mathbf{d}_G = \mathbf{d}_G^* \cdot \mathbf{d}_B = \mathbf{d}_B^* \cdot \mathbf{d}_R = 0\,,\ \left|\mathbf{d}_R\right| = \left|\mathbf{d}_G\right| = \left|\mathbf{d}_B\right|$$

- Gapped quarks: three colors and three flavors (“isotropic”)
- Gluo-electromagnetic properties:

Long wavelength gluons → CFL condensate at T=0 → 8 types: screened

Cf. The CFL state is more favorable in the weak coupling limit.

Homogeneous superfluid phases (contd.)



In weak coupling

$$\beta_1 = \beta_2 = \frac{7\zeta(3)}{8(\pi T_c)^2} N\left(\frac{\mu}{3}\right)$$

$$\bar{\alpha} = 4N\left(\frac{\mu}{3}\right)\frac{T-T_c}{T_c}$$

with $N\left(\frac{\mu}{3}\right) = \frac{1}{2\pi^2}\left(\frac{\mu}{3}\right)^2$

Homogeneous superfluid phases (contd.)

In an overall color singlet state

Ref. Iwasaki & Iwado, PLB **350** (1995) 163.

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Generally, chemical potential differences, $\tilde{\mu}_{ab}$, from $\delta_{ab}\,\mu/3$ arise

such that $\tilde{\mu}_{aa} = \mu_a - \dfrac{\mu}{3}, \quad \sum_a \tilde{\mu}_{aa} = 0$.

Then, the Ginzburg-Landau free energy reads

$$\Delta\Omega = \Omega_0 + \Omega_{\mathrm{CN}}$$

$$\Omega_0 = \bar{\alpha}\sum_a |\mathbf{d}_a|^2 + \beta_1\left(\sum_a |\mathbf{d}_a|^2\right)^2 + \beta_2\sum_{ab}|\mathbf{d}_a^* \cdot \mathbf{d}_b|^2$$

$$\Omega_{\mathrm{CN}} = 3\sigma\sum_{ab}|\tilde{\mu}_{ab}|^2 - 2\chi\sum_{ab}\left(\mathbf{d}_a^* \cdot \mathbf{d}_b\right)\tilde{\mu}_{ab} \quad .$$

Color neutrality condition

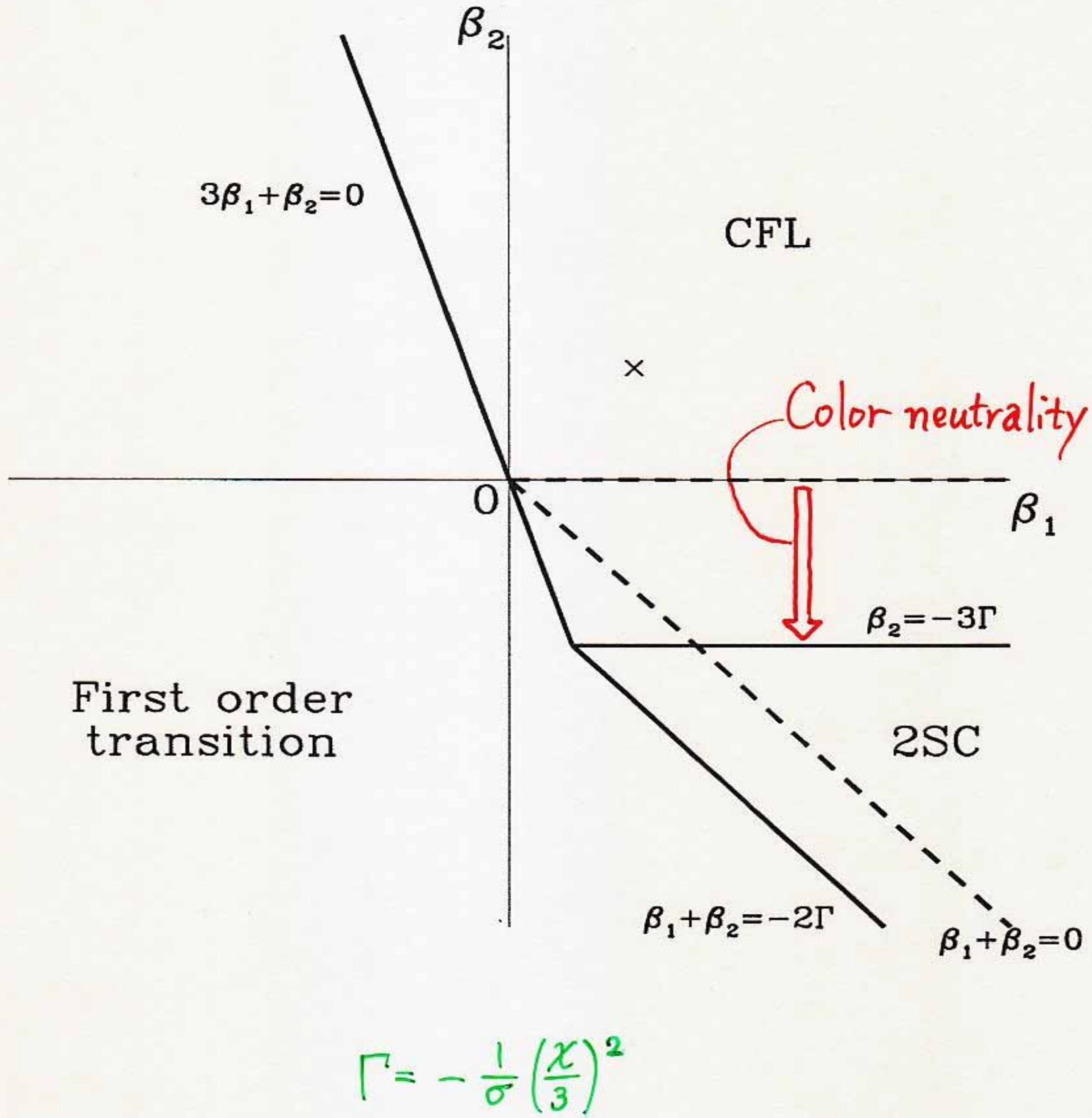
$$\frac{\partial\Delta\Omega}{\partial\tilde{\mu}_{RR}} = \frac{\partial\Delta\Omega}{\partial\tilde{\mu}_{GG}} = \frac{\partial\Delta\Omega}{\partial\tilde{\mu}_{BB}},$$

$$\frac{\partial\Delta\Omega}{\partial\tilde{\mu}_{ab}} = 0 \ \text{ for } a \neq b\,.$$

$$\tilde{\mu}_{ab} = \frac{\chi}{9\sigma}\left[3\left(\mathbf{d}_a \cdot \mathbf{d}_b^*\right) - \delta_{ab}\sum_c |\mathbf{d}_c|^2\right]$$

When the order parameter is anisotropic in color space as in 2SC, the color having a larger gap has a smaller chemical potential, leading to smaller condensation energy.

Homogeneous superfluid phases (contd.)



Responses to magnetic fields and rotation

Ref. Iida & Baym, PRD **66** (2002) 014015.

Gradient energy in the absence of real photon and gluon fields

In inhomogeneous states of wavelengths larger than T_c^{-1}

— ϕ_+ depends on the center-of-mass coordinate **r** of the pair.

— To second order in $\nabla\phi_+$, the gradient energy reads $\Omega_\mathrm{g} = \frac{1}{2} K_\mathrm{T} \mathrm{Tr}\left(\partial_l \phi_+ \partial_l \phi_+^\dagger\right).$

Superfluid density and superfluid momentum density

Ref. Josephson, Phys. Lett. **21** (1966) 608.

Superfluid mass (baryon) density ρ_s (n_s): $\rho_\mathrm{s} = \mu\, n_\mathrm{s} = \frac{4}{9}\mu^2 K_\mathrm{T} \mathrm{Tr}\left(\phi_+ \phi_+^\dagger\right)$

Superfluid momentum (baryon current) density $\mathbf{g}_s$ ($\mathbf{j}_s$): $\mathbf{g}_\mathrm{s} = \mu\, \mathbf{j}_\mathrm{s} = -\frac{i}{3} K_\mathrm{T} \mu\, \mathrm{Tr}\left(\phi_+ \nabla \phi_+^\dagger - \phi_+^\dagger \nabla \phi_+\right)$

Gradient energy in the presence of real photon and gluon fields

Gradient energy invariant under color $SU(3)$ and electromagnetic $U(1)$ local gauge transformations of the quark spinors ψ_{ai} : $\Omega_\mathrm{g} = \frac{1}{2} K_\mathrm{T} \mathrm{Tr}\left[D_l \phi_+ (D_l \phi)^\dagger\right].$

with covariant derivative $D_l \phi_+ \equiv \partial_l \phi_+ + \frac{ig}{2}\left[\left(\lambda^\alpha\right)^* \phi_+ + \phi_+ \lambda^\alpha\right] A_l^\alpha + ieQ\phi_+ A_l$, $Q_{abij} = \delta_{ab}\left(q_i + q_j\right)$

Responses to magnetic fields and rotation (contd.)

Field equations and charged supercurrents

From extremization of $\int d^3r\left(\Omega_0+\Omega_{\rm g}+\frac{1}{4}G^\alpha_{lm}G^\alpha_{lm}+\frac{1}{4}F_{lm}F_{lm}\right)$,

- **Gap equation**

$$-\frac{1}{2}K_{\rm T}D_l\left(D_l\phi_+\right)+\alpha^+\phi_++2\beta_1^+\left[{\rm Tr}\left(\phi_+^\dagger\phi_+\right)\right]\phi_++2\beta_2^+\phi_+\phi_+^\dagger\phi_+=0$$

- **The color Maxwell equation**

$$\begin{aligned}\partial_m G^\alpha_{ml}+gf_{\alpha\beta\gamma}A^\beta_m G^\gamma_{ml}=&-\frac{1}{2}K_{\rm T}g\,{\rm Im}\left\{{\rm Tr}\left[\left(\left(\lambda^\alpha\right)^*\phi_++\phi_+\lambda^\alpha\right)^\dagger\partial_l\phi_+\right]\right\}\\&-\frac{1}{4}K_{\rm T}g^2A_l^\beta\,{\rm Re}\left\{{\rm Tr}\left[\left(\left(\lambda^\alpha\right)^*\phi_++\phi_+\lambda^\alpha\right)\left(\left(\lambda^\beta\right)^*\phi_++\phi_+\lambda^\beta\right)^\dagger\right]\right\}\\&-\frac{1}{2}K_{\rm T}geA_l\,{\rm Re}\left\{{\rm Tr}\left[\left(\left(\lambda^\alpha\right)^*\phi_++\phi_+\lambda^\alpha\right)Q\phi_+^\dagger\right]\right\}\equiv J_l^\alpha\end{aligned}$$

- **The Maxwell equation**

$$\begin{aligned}\partial_m F_{ml}=&-K_{\rm T}e\,{\rm Im}\left[{\rm Tr}\left(Q\phi_+^\dagger\partial_l\phi_+\right)\right]-K_{\rm T}e^2A_l{\rm Tr}\left(Q\phi_+Q\phi_+^\dagger\right)\\&-\frac{1}{2}K_{\rm T}geA_l^\alpha\,{\rm Re}\left\{{\rm Tr}\left[Q\phi_+\left(\left(\lambda^\alpha\right)^*\phi_++\phi_+\lambda^\alpha\right)^\dagger\right]\right\}\equiv J_l\end{aligned}$$

Responses of the color-flavor locked (CFL) condensate

Pairing gap

$$(\mathbf{d}_a)_i = U_{ai}\kappa_{\mathrm{A}},\ \kappa_{\mathrm{A}} = e^{i\varphi}|\kappa_{\mathrm{A}}|$$

Supercurrents associated with U(1)$_B$ phase and photon fields

In the London limit (spatial variation larger than $\xi = \sqrt{K_{\mathrm{T}}/2|\alpha^{+}|}$)

Baryonic: $\mathbf{j}_{\mathrm{s}} = -8K_{\mathrm{T}}|\kappa_{\mathrm{A}}|^2\nabla\varphi$ — $U(1)_{\mathrm{B}}$ phase induced

Color: $\mathbf{J}^8 = -K_{\mathrm{T}}g|\kappa_{\mathrm{A}}|^2\left(\nabla\varphi_8 + 2g\mathbf{A}^8 + \frac{4e}{\sqrt{3}}\mathbf{A}\right)$

Electric: $\mathbf{J} = \frac{2e}{\sqrt{3}g}\mathbf{J}^8$

Photon-gluon mixed fields Ref. Gorbar, PRD **62** (2000) 014007.

$$\mathcal{A} \equiv \frac{\sqrt{3}g\mathbf{A} - 2e\mathbf{A}^8}{3g_8},\ \mathcal{A}^8 \equiv \frac{\sqrt{3}g\mathbf{A}^8 + 2e\mathbf{A}}{3g_8},\ g_8 = \frac{1}{3}\sqrt{3g^2 + 4e^2}$$

Corresponding supercurrents:

$\mathcal{J} = 0$ — free propagation

$\mathcal{J}^8 = -K_{\mathrm{T}}g_8|\kappa_{\mathrm{A}}|^2\left(\sqrt{3}\nabla\varphi_8 + 6g_8\mathcal{A}^8\right)$ — mixed Meissner

$SU(3)_{\mathrm{c+f}}$ phase

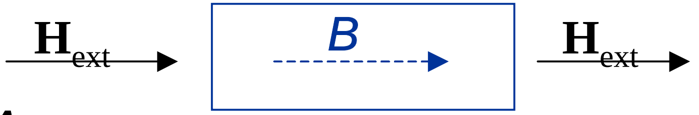
Responses of the CFL condensate (contd.)

Response to magnetic fields

Low fields: imperfect diamagnetism

Ref. Alford, Berges, & Rajagopal, NPB **571** (2000) 269.

Uniformly applied weak $\mathbf{H}_{\text{ext}}$



- The most part is propagating in the form of $B = \nabla \times A$
- A fraction of $\mathbf{H}_{\text{ext}}$ included in $B^8 = \nabla \times A^8$ is screened in a length scale $\lambda_{\text{CFL}} = \left(\sqrt{6K_{\text{T}}}\, g_8 |\kappa_{\text{A}}|\right)^{-1}$

High fields: possible $SU(3)_{c+f}$ vortices

Ref. Giannakis & Ren, NPB **669** (2003) 462.

- Vortices can appear such that $\oint dl \cdot \left(A^8 + \lambda_{\text{CFL}}^2 J^8\right) = 2\pi / g_8$

  when the system is Type II ($\kappa_{\text{CFL}} \equiv \lambda_{\text{CFL}}/\xi > 1/\sqrt{2} \times O(1)$).

- Complicated magnetic and supercurrent structure of a vortex

- Critical field $H_{\text{c}} = \dfrac{3}{2e\xi\lambda_{\text{CFL}}}$

  Weak coupling expressions:

$$\xi \approx 0.26 \left(\frac{100\ \text{MeV}}{T_{\text{c}}}\right)\left(1 - \frac{T}{T_{\text{c}}}\right)^{-1/2} \text{fm}$$

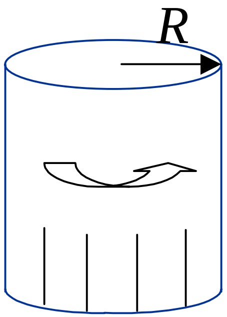
$$\lambda_{\text{CFL}} \approx 1.7 \left(\frac{\sqrt{3}}{g_8}\right)\left(\frac{300\ \text{MeV}}{\mu/3}\right)\left(1 - \frac{T}{T_{\text{c}}}\right)^{-1/2} \text{fm}$$

} Type II in stars ?

$$H_{\text{c}} \approx 2.2 \times 10^{19} \left(\frac{g_8}{\sqrt{3}}\right)\left(\frac{T_{\text{c}}}{100\ \text{MeV}}\right)\left(\frac{\mu/3}{300\ \text{MeV}}\right)\left(1 - \frac{T}{T_{\text{c}}}\right) \text{G} \gg 10^{12}\ \text{G}$$

Responses of the CFL condensate (contd.)

Response to rotation



Transformation to the rotating frame

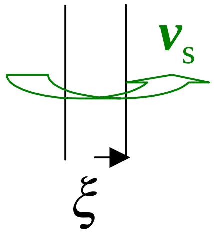
— under constant angular velocity **ω**,

$$\Omega_s \to \Omega_s - \mathbf{L}_s \cdot \boldsymbol{\omega}, \quad \mathbf{L}_s = \mathbf{r} \times \mathbf{g}_s$$

$$\Omega_g = 6K_T\left[\left|\left(\nabla + \frac{2i\mu}{3}\boldsymbol{\omega}\times\mathbf{r}\right)\kappa_A\right|^2 + \frac{1}{2}g_8^2|\kappa_A|^2|A^8|^2\right] - \frac{1}{2}\rho_s|\boldsymbol{\omega}\times\mathbf{r}|^2$$

A triangular lattice of singly quantized vortices

— superflow pattern simulating corotation of the condensate



Quantization condition on vorticity: $\oint dl \cdot \mathbf{v}_s = 2\pi \dfrac{3}{2\mu}$

Number of vortices (calculated from net circulation)

$$N_v = \frac{2}{3}\mu R^2 \omega$$

$$\approx 6.4\times10^{18}\left(\frac{1\,\text{ms}}{P_{\text{rot}}}\right)\left(\frac{\mu/3}{300\,\text{MeV}}\right)\left(\frac{R}{10\,\text{km}}\right)^2, \quad P_{\text{rot}} = \frac{2\pi}{\omega}$$

— just like rotating superfluid helium and Bose-Einstein condensates of alkali atoms

Responses of the isoscalar (2SC) condensate

Pairing gap

$$(\mathbf{d}_a)_i = e^{i\varphi}|\mathbf{d}|\delta_{aB}\delta_{is}$$

Supercurrents associated with $U(1)_B$ phase and photon fields

In the London limit (spatial variation larger than $\xi = \sqrt{K_T/2|\alpha^+|}$)

Baryonic: $\mathbf{j}_s = -\frac{8}{3}K_T|\mathbf{d}|^2\left(\nabla\varphi + \frac{g}{\sqrt{3}}\mathbf{A}^8 + \frac{e}{3}\mathbf{A}\right)$ — $U(1)_B$ phase induced

Color: $\mathbf{J}^8 = -\frac{4}{\sqrt{3}}K_T g|\mathbf{d}|^2\left(\nabla\varphi + \frac{g}{\sqrt{3}}\mathbf{A}^8 + \frac{e}{3}\mathbf{A}\right)$

Electric: $\mathbf{J} = \frac{e}{\sqrt{3}g}\mathbf{J}^8$

Photon-gluon mixed fields Ref. Gorbar, PRD **62** (2000) 014007.

$$\mathcal{A} \equiv \frac{\sqrt{3}g\mathbf{A} - e\mathbf{A}^8}{3g_8},\quad \mathcal{A}^8 \equiv \frac{\sqrt{3}g\mathbf{A}^8 + e\mathbf{A}}{3g_8},\quad g_8 = \frac{1}{3}\sqrt{3g^2 + e^2}$$

Corresponding supercurrents:

$\mathcal{J} = 0$ — free propagation

$\mathcal{J}^8 = -4K_T g_8|\mathbf{d}|^2\left(\nabla\varphi + g_8\mathcal{A}^8\right)$ — mixed Meissner

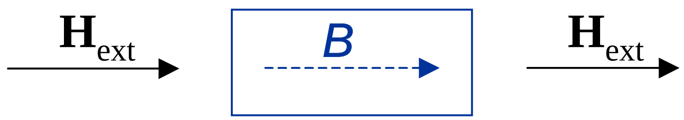
$U(1)_{em}$ phase

Responses of the 2SC condensate (contd.)

Response to magnetic fields

Low fields: imperfect diamagnetism Ref. Alford, Berges, & Rajagopal, NPB **571** (2000) 269.

Uniformly applied weak $\mathbf{H}_{\rm ext}$



- The most part is propagating in the form of $B = \nabla \times A$
- A fraction of $\mathbf{H}_{\rm ext}$ included in $B^8 = \nabla \times A^8$ is screened in a length scale $\lambda_{\rm 2SC} = \left(2\sqrt{K_{\rm T}}\, g_8 |\mathbf{d}|\right)^{-1}$

High fields: possible $U(1)_{\rm em}$ vortices —just like ordinary SC Ref. Blaschke & Sedrakian, nucl-th/0006038.

- Vortices can appear such that $\oint dl \cdot \left(A^8 + \lambda_{\rm 2SC}^2 J^8\right) = 2\pi / g_8$

  when the system is Type II ($\kappa_{\rm 2SC} \equiv \lambda_{\rm 2SC}/\xi > 1/\sqrt{2}$).

- Critical fields: $H_{\rm c} = \dfrac{3}{\sqrt{2} e \xi \lambda_{\rm 2SC}}, H_{\rm c1} = \dfrac{1}{\sqrt{2}\kappa_{\rm 2SC}} H_{\rm c} \ln \kappa_{\rm 2SC}, H_{\rm c2} = \sqrt{2}\kappa_{\rm 2SC} H_{\rm c}$

  Weak coupling expressions:

$$\xi \approx 0.26 \left(\frac{100\ \text{MeV}}{T_{\rm c}}\right)\left(1 - \frac{T}{T_{\rm c}}\right)^{-1/2} \text{fm}$$

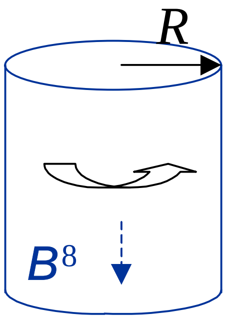
$$\lambda_{\rm 2SC} \approx 1.5 \left(\frac{\sqrt{3}}{g_8}\right)\left(\frac{300\ \text{MeV}}{\mu/3}\right)\left(1 - \frac{T}{T_{\rm c}}\right)^{-1/2} \text{fm}$$

} Type II in stars ?

$$H_{\rm c} \approx 3.6 \times 10^{19} \left(\frac{g_8}{\sqrt{3}}\right)\left(\frac{T_{\rm c}}{100\ \text{MeV}}\right)\left(\frac{\mu/3}{300\ \text{MeV}}\right)\left(1 - \frac{T}{T_{\rm c}}\right) \text{G} \gg 10^{12}\ \text{G}$$

Responses of the 2SC condensate (contd.)

Response to rotation



Transformation to the rotating frame

— under constant angular velocity **ω**,

$$\Omega_{\rm s} \to \Omega_{\rm s} - \mathbf{L}_{\rm s}\cdot\boldsymbol{\omega}\,, \quad \mathbf{L}_{\rm s} = \mathbf{r}\times\mathbf{g}_{\rm s}$$

$$\Omega_{\rm g} = -\frac{1}{2}\rho_{\rm s}\left|\boldsymbol{\omega}\times\mathbf{r}\right|^2 + 2K_{\rm T}\left\{\left[\partial_l - ig_8A_l^8 - \frac{2i\mu}{3}(\boldsymbol{\omega}\times\mathbf{r})_l\right]\mathbf{d}^*\right\}\cdot\left\{\left[\partial_l + ig_8A_l^8 + \frac{2i\mu}{3}(\boldsymbol{\omega}\times\mathbf{r})_l\right]\mathbf{d}\right\}$$

Absent from CFL

London magnetic field generation —just like ordinary SC

Superflow pattern corotating with the vessel:

$$J^8 = \frac{3}{2}g_8 n_{\rm s}\boldsymbol{\omega}\times\mathbf{r}$$

Quantization condition on the vorticity: $\oint dl\cdot\left(A^8 + \lambda_{\rm 2SC}^2 J^8\right) = 0$

London field

$$B^8 = -\frac{4\mu}{3g_8}\boldsymbol{\omega}, \quad A^8 = -\frac{2\mu}{3g_8}\boldsymbol{\omega}\times\mathbf{r}$$

— small: $\left|B^8\right| \approx 0.15\left(\frac{\sqrt{3}}{g_8}\right)\left(\frac{1\,\text{ms}}{P_{\rm rot}}\right)\left(\frac{\mu/3}{300\,\text{MeV}}\right)\,\text{G} \ll 10^{12}\,\text{G}$

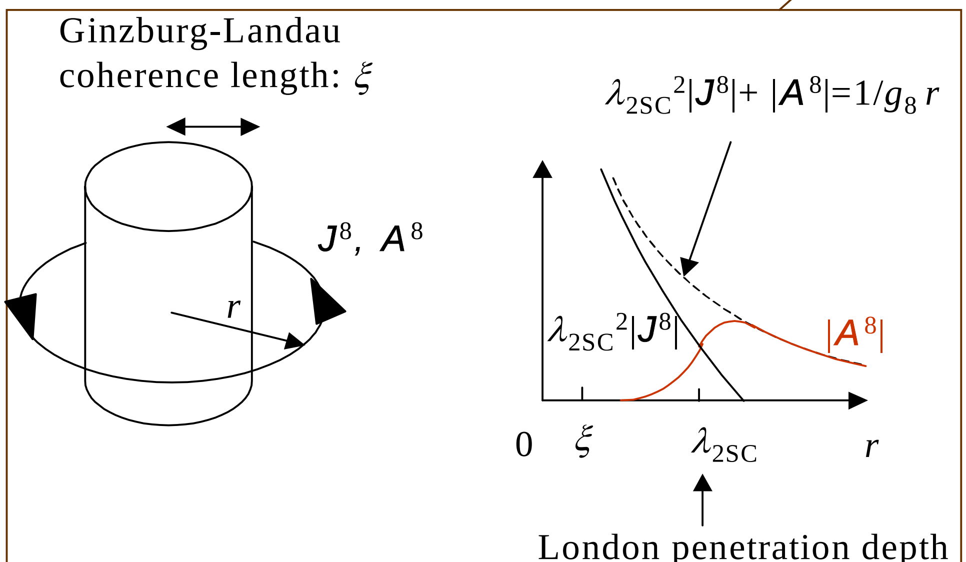
— color dominant: $\left|\mathbf{B}^8\right| = \frac{\sqrt{3}g}{e}\left|\mathbf{B}\right| \gg \left|\mathbf{B}\right| \qquad (\leftarrow B = 0)$

Responses to magnetic fields and rotation (contd.)

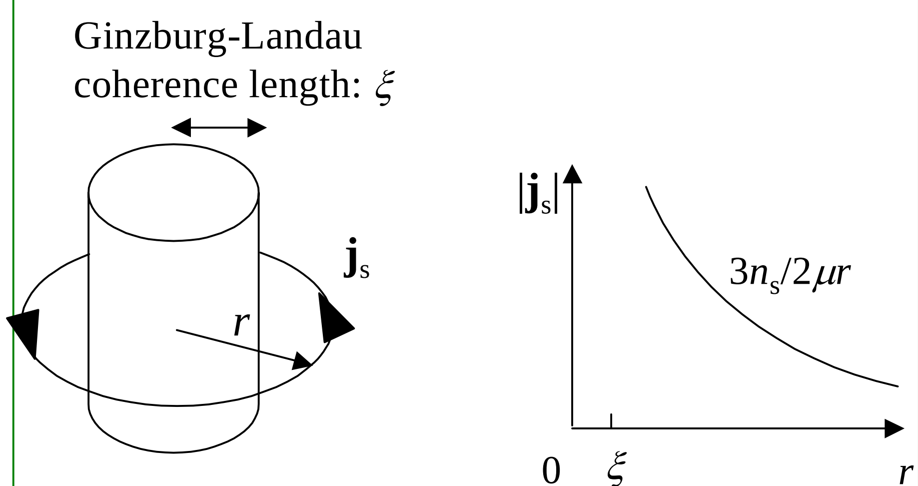
Summary

Responses of homogeneous superfluid quark matter near T_c to magnetic fields and rotation

Phase	Magnetic fields	Rotation
CFL	Partial screening Possible $SU(3)_{c+f}$ vortices	$U(1)_B$ vortices
2SC	Partial screening Possible $U(1)_{em}$ vortices	London field



Magnetic vortex in the 2SC phase



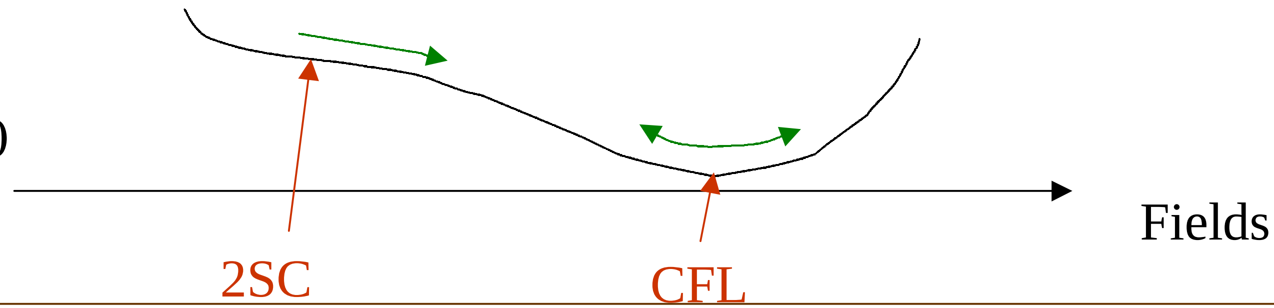
Rotational vortex in the CFL phase

Fluctuation-induced first order transition

Ref. Matsuura et al., PRD **69** (2004) 074012.

Fluctuations in gluon and diquark fields

Energy landscape for $\beta_1=\beta_2>0$



Mean-square thermal fluctuations in the Gaussian approximation where interactions between fluctuations can be ignored:

$$\left\langle \mathbf{A}^{\alpha}\mathbf{A}^{\beta}\right\rangle = \delta_{\alpha\beta}\,2T\int_{|\mathbf{k}|<\Lambda\sim T_c}\frac{d^3k}{(2\pi)^3}\frac{1}{\mathbf{k}^2+(m_{\mathrm{A}})^2_{\alpha\alpha}} = \delta_{\alpha\beta}T\left[\frac{T_{\mathrm{c}}}{\pi^2}-\frac{(m_{\mathrm{A}})_{\alpha\alpha}}{2\pi}+\frac{(m_{\mathrm{A}})^2_{\alpha\alpha}}{\pi^2 T_{\mathrm{c}}}+\cdots\right]$$

$$\left\langle (\delta d)_{\rho}(\delta d)_{\sigma}\right\rangle = \delta_{\rho\sigma}\frac{T}{2K_{\mathrm{T}}}\int_{|\mathbf{k}|<\Lambda\sim T_c}\frac{d^3k}{(2\pi)^3}\frac{1}{\mathbf{k}^2+(m_{\mathrm{d}})^2_{\rho\rho}} = \delta_{\rho\sigma}\frac{T}{4K_{\mathrm{T}}}\left[\frac{T_{\mathrm{c}}}{\pi^2}-\frac{(m_{\mathrm{d}})_{\rho\rho}}{2\pi}+\frac{(m_{\mathrm{d}})^2_{\rho\rho}}{\pi^2 T_{\mathrm{c}}}+\cdots\right]$$

	2SC	CFL	$T>T_c$
m_d^2	$2(\beta_1+\beta_2)\lvert\mathbf{d}\rvert^2/K_T$ (1) $-\beta_2\,\lvert\mathbf{d}\rvert^2/K_T$ (8), 0 (9)	$2(3\beta_1+\beta_2)\lvert\kappa_A\rvert^2/K_T$ (1) $2\beta_2\,\lvert\kappa_A\rvert^2/K_T$ (8), 0 (9)	$2\alpha'/K_T$ (18)
m_A^2	$4K_T g^2\lvert\mathbf{d}\rvert^2/3$ (1) $K_T g^2\lvert\mathbf{d}\rvert^2$ (4), 0 (3)	$2K_T g^2\lvert\kappa_A\rvert^2$ (8)	0 (8)

Fluctuation-induced first order transition (contd.)

The region in which the Gaussian approximation is valid

1. Weak coupling regime

$$\frac{|\beta_i|}{32\pi^2 K_{\mathrm{T}}^2} \ll 1 \quad \text{and} \quad \frac{2\alpha_s}{\pi} \ll 1, \quad \text{with} \quad \alpha_s = \frac{g^2}{4\pi}$$

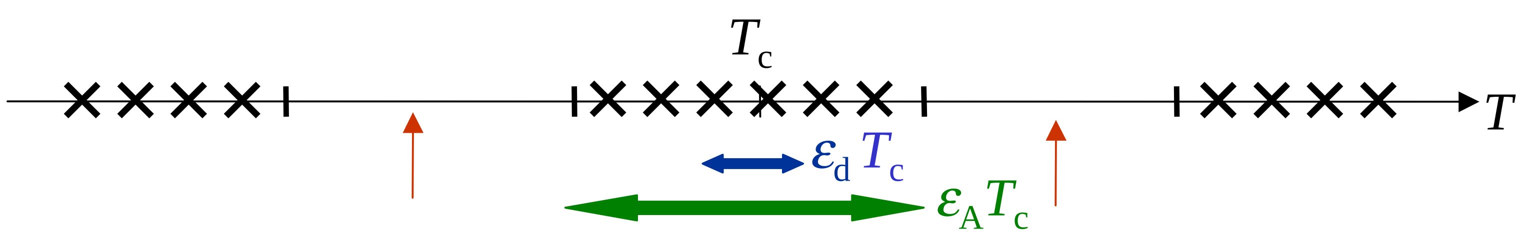
Then, $m_{\mathrm{A}} \gg m_{\mathrm{d}}$. **"Type I"**

2. Outside critical regions Ref. Ginzburg, Fiz. Tverd. Tela **2** (1960) 2031.

$$\left|\frac{T - T_{\mathrm{c}}}{T_{\mathrm{c}}}\right| \gg \varepsilon_{\mathrm{d}} \approx 10^{-2}\left(\frac{T_{\mathrm{c}}}{100\ \mathrm{MeV}}\right)^4\left(\frac{1\,\mathrm{GeV}}{\mu}\right)^4 \quad \text{for diquark fields}$$

$$\left|\frac{T - T_{\mathrm{c}}}{T_{\mathrm{c}}}\right| \gg \varepsilon_{\mathrm{A}} \approx \frac{\alpha_s}{10}\left(\frac{T_{\mathrm{c}}}{100\ \mathrm{MeV}}\right)^2\left(\frac{1\,\mathrm{GeV}}{\mu}\right)^2 \quad \text{for gluon fields}$$

The valid temperature region in weak coupling:



Fluctuations in gluon fields are more important.

Cf. Fluctuation effects on the phase transition in ordinary superconductors are hard to probe experimentally since the critical region for the order parameter is very small.

Fluctuation-induced first order transition (contd.)

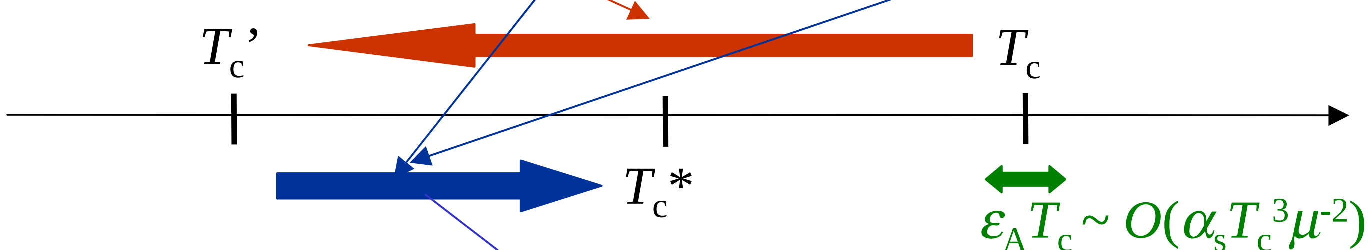
Effects of fluctuations in gluon fields —— the weak coupling CFL condensate ——

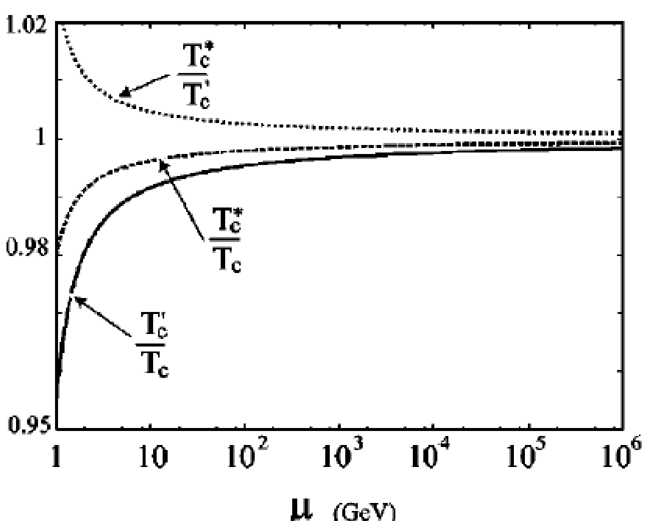
Free energy

$$\Delta\Omega = 3\bar{\alpha}|\kappa_A|^2 + 3(3\beta_1+\beta_2)|\kappa_A|^4 + \frac{1}{2}\sum_{\alpha} \int_0^{(m_A^2)_{\alpha\alpha}} d(m_A^2)_{\alpha\alpha} \langle \mathbf{A}^\alpha \mathbf{A}^\alpha \rangle$$

$$= \left(3\bar{\alpha} + \frac{32}{\pi}TT_c K_T \alpha_s\right)|\kappa_A|^2 - \frac{8\sqrt{2}}{3\pi}T(4\pi K_T \alpha_s)^{3/2}|\kappa_A|^3 + \left[3(3\beta_1+\beta_2) + 128\left(\frac{T}{T_c}\right)K_T^2\alpha_s^2\right]|\kappa_A|^4$$

Transition temperature





Weak first order transition

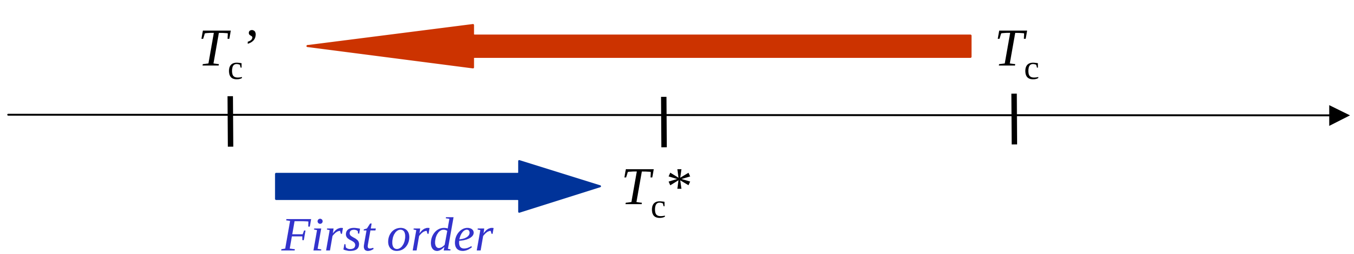
$$T_c' < T_c^* < T_c\,, \quad \frac{T_c^* - T_c'}{T_c'} \sim O(\alpha_s)$$

Fluctuation-induced first order transition (contd.)

Comparison with other works—— the weak coupling regime——

Short-wavelength fluctuations beyond the Ginzburg-Landau framework can be important in strong Type-I superconductors (Ren's talk).

	Approach	$m_A(k=0)$ at $T= T_c^*$	$(T_c^* - T_c')/T_c'$
Bailin & Love	GL ($k< T_c$)	$m_A(k=0)$ » T_c	$O\left(\alpha_s^3 \mu^2 T_c^{-2}\right)$ » 1
Present	GL ($k< T_c$)	$m_A(k=0) \sim T_c$	$O\left(\alpha_s\right)$
Giannakis et al. (Ren's talk)	CJT (incl. Pippard regime)	$m_A(k=0)$ » T_c	$O\left(\alpha_s^{1/2}\right)$



Melting pattern of diquark condensates in neutral quark matter

What are the color superconducting phases like in a more realistic situation?

Color and electric neutrality

Weak equilibrium with an electron gas

Nonzero quark masses (especially strange quark mass)

Instantons ...

T=0

· in weak coupling: CFL with possible "η" condensation   Ref. Kryjevski et al., hep-ph/0312363

· in strong coupling: many other possibilities

2SC, gapless 2SC, gapless CFL, CFL with "K" condensation, Sarma, deformed Fermi surface, phase separation/mixed phase, LOFF, 1-color pairing, uSC, ...

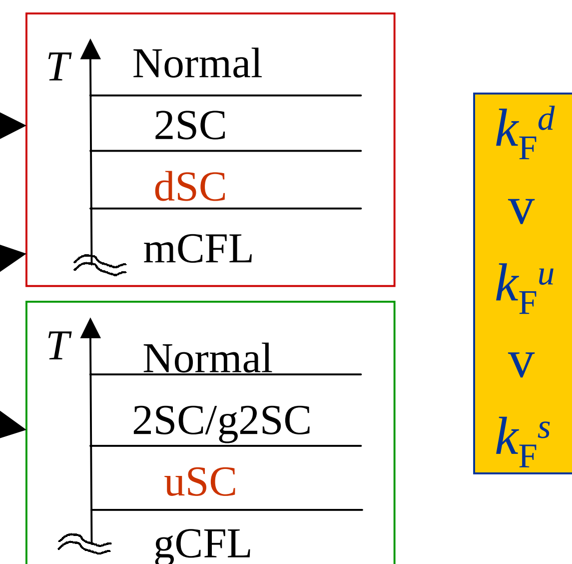
Near T_c

· in weak coupling (Tachibana's talk)

Ref. Iida et al., hep-ph/0312363

· in strong coupling (Kouvaris' talk)   ?

Ref. Rüster et al., hep-ph/0405170



Cf. The physics looks simpler near T_c than at T=0.

Melting pattern of diquark condensates in neutral quark matter (contd.)

Corrections to the Ginzburg-Landau free energy in the weak coupling and massless limit

Strange quark mass

$$\delta k_F^s = -\frac{3m_s^2}{2\mu}$$

$$\delta\mu_i = -q_i\mu_e, \quad \mu_e = \frac{3m_s^2}{4\mu}$$

Electric neutrality and weak equilibrium

Corrections up to $O(m_s^2)$

$$8N(\mu/3)\sigma\sum_a\left[|\mathbf{d}_a|^2 - |(\mathbf{d}_a)_s|^2\right]$$

$$4N(\mu/3)\sigma\sum_a\left[\frac{1}{3}|\mathbf{d}_a|^2 - |(\mathbf{d}_a)_u|^2\right]$$

with

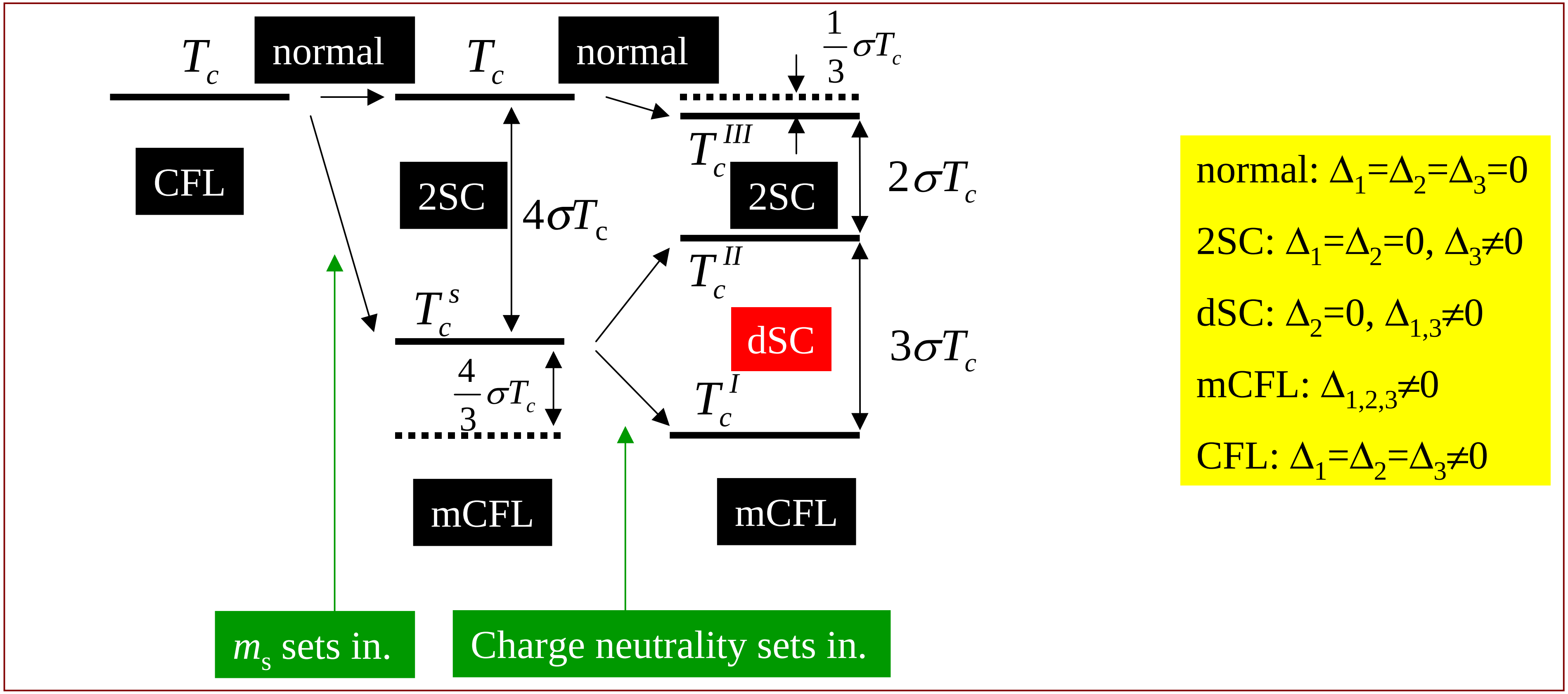
$$\sigma \equiv \frac{3\pi^2}{8\sqrt{2}}\frac{m_s^2}{g}\left(\frac{3}{\mu}\right)^2$$

Magnetic interactions

Melting pattern of diquark condensates in neutral quark matter (contd.)

Splitting of the critical temperature in weak coupling

Pairing ansatz $$(\mathbf{d}_a)_i = \begin{pmatrix} \Delta_1 & 0 & 0 \\ 0 & \Delta_2 & 0 \\ 0 & 0 & \Delta_3 \end{pmatrix}$$



Cf. Similar splitting of the critical temperature is also seen in superfluid ^{3}He under magnetic fields:

A phase $\rightarrow$ normal at H=0 vs. A_2 phase $\rightarrow$ A_1 phase $\rightarrow$ normal at $H\neq0$.

Conclusion

Ginzburg-Landau approach to color superconductivity helps us examine

- Phase diagrams near T_c
- Responses to external magnetic fields and rotation
- Effects of the thermally fluctuating color magnetic fields on the phase transition of weak-coupling Type I color superconductors
- Effects of non-zero strange quark mass and electric charge neutrality on melting pattern of diquark condensates

Many questions remain

- μ dependence of the parameters (α^+, β_1, β_2, K_T, g) in the free energy at lower densities?
- Effects of the thermally fluctuating diquark and color magnetic fields on the phase transition of strong-coupling color superconductors ?

  Ref. Pisarski, PRC **62** (2000) 035202.

  —— crossover, second order, or first order ?

- Competition with other possible order parameters ?