Deconfinement phase transition with various gauge groups

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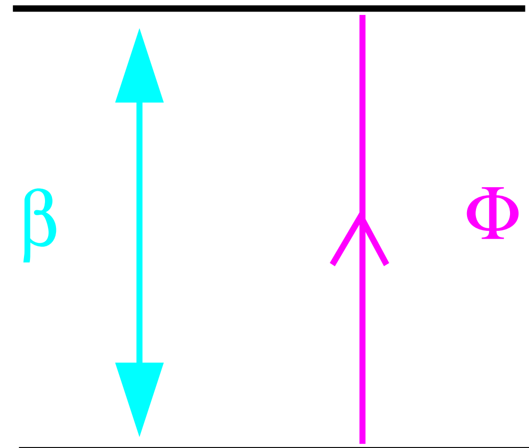
- deconfinement in Yang-Mills theory
- phase transition & center symmetry
- universal behavior & Svetitsky-Yaffe conjecture
- $SU(N)$ gauge theory
- $Sp(N)$ gauge theory
- conjecture when transition universal

Michele Pepe (Bern) and Uwe-Jens Wiese (Bern & MIT)

Deconfinement & center symmetry



Finite temperature $\beta = 1/T$

Physical quantities periodic in time

$A_\mu(x)$ periodic up to gauge trans. $\Omega(x)$

Gauge group G twist $\Omega(x) \to z\,\Omega(x)$

$z \in H$ (center of G) Global symmetry

Polyakov loop $\Phi = \mathrm{Tr}\mathcal{P}\exp(i \int dx_4 A_4(x))$ $\Phi \to z\,\Phi$

$\langle\Phi\rangle = \exp(-\beta F)$ free energy F of static quark

$\langle\Phi\rangle = 0 \Rightarrow F \to \infty$ confined center symmetry intact

$\langle\Phi\rangle \neq 0 \Rightarrow F$ finite deconfined center symmetry broken

Svetitsky-Yaffe conjecture: if $(d+1)$-dim. gauge theory has 2nd order deconfinement phase transition $\Rightarrow$ belongs to universality class of d-dim. scalar field theory with symmetry H

2nd order $\xi \propto (T - T_{\mathrm{crit}})^{-\nu} \to \infty$ ν critical exponent

$\xi \gg \beta$ dimensional reduction

conjecture does NOT say phase transition MUST be 2nd order

$SU(N)$ gauge theory

$SU(N)$ Yang-Mills in 4-dim. center of $SU(N)$ is $Z(N)$

$N = 2$ 2nd order phase transition

critical exponents same as 3-dim. $Z(2)$ symmetric scalar field theory i.e. Ising universality class

$N = 3$ weak 1st order phase transition not universal

$N = 4, 6, 8$ 1st order phase transition stronger as N increases

For $N \geq 5$ $\exists$ 3-dim. $Z(N)$ universality class $U(1)$ XY model

Pisarski-Tytgat conjecture: $SU(N = \infty)$ has 2nd order deconfinement phase transition, $SU(3)$ with 1st order transition is a small deviation from $N = \infty$ — seems not to be true

$SU(N)$ Yang-Mills in 3-dim.

$N = 2, 3, 4(?)$ 2nd order phase transition

belong to universality classes of 2-dim. $Z(2), Z(3), Z(4)$ symmetric scalar field theory

All cases support Svetitsky-Yaffe conjecture

4-dim. 2nd order phase transitions rare

Other possibilities?

$Sp(N)$ gauge theory

Symplectic groups $\quad Sp(N) \subset SU(2N) \quad U \in Sp(N)$

$$U^* = JUJ^\dagger \quad J = \begin{pmatrix} 0 & 1\!\!1 \\ -1\!\!1 & 0 \end{pmatrix} \quad \Rightarrow \quad U = \begin{pmatrix} W & X \\ -X^* & W^* \end{pmatrix}$$

$Sp(N)$ has $(2N+1)N$ generators and rank N

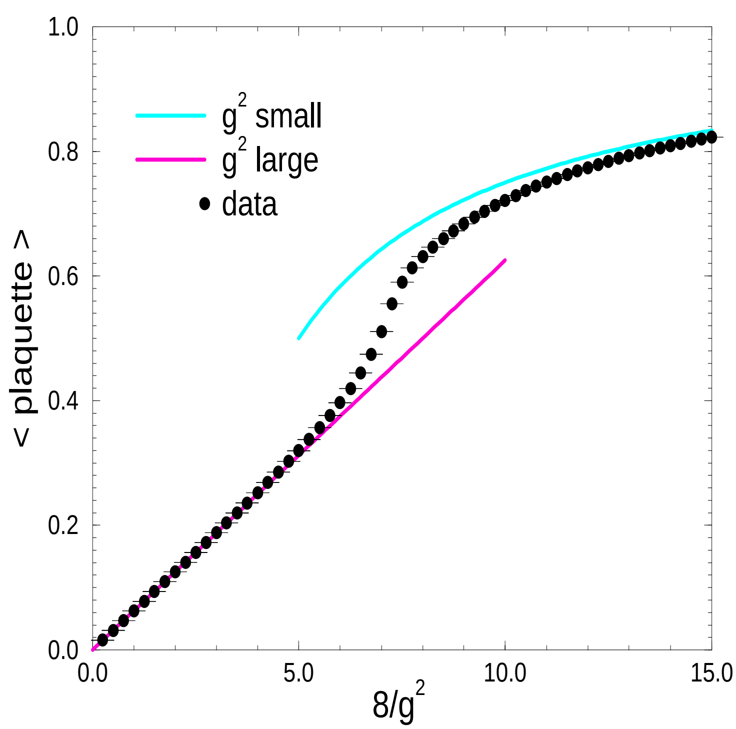
Special cases $\quad Sp(1) = SU(2) \quad Sp(2) \simeq SO(5)$

center of $Sp(N)$ is $Z(2)$ possibly 2nd order phase transition?

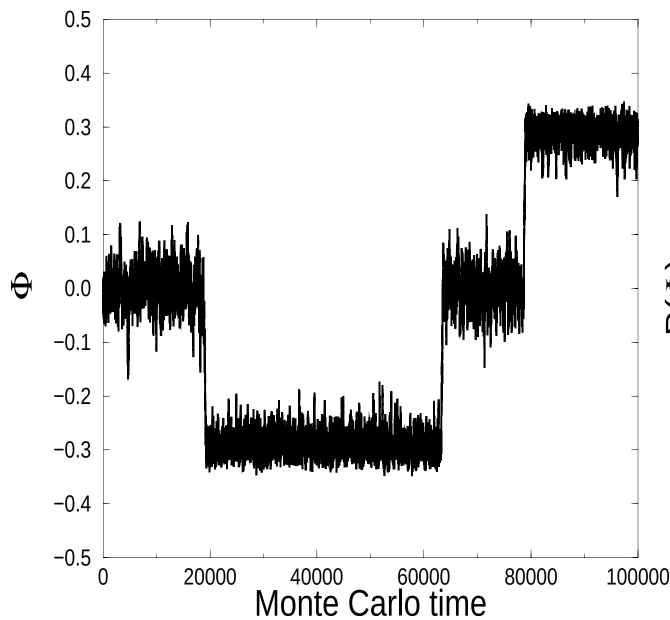
lattice $Sp(N)$ Yang-Mills

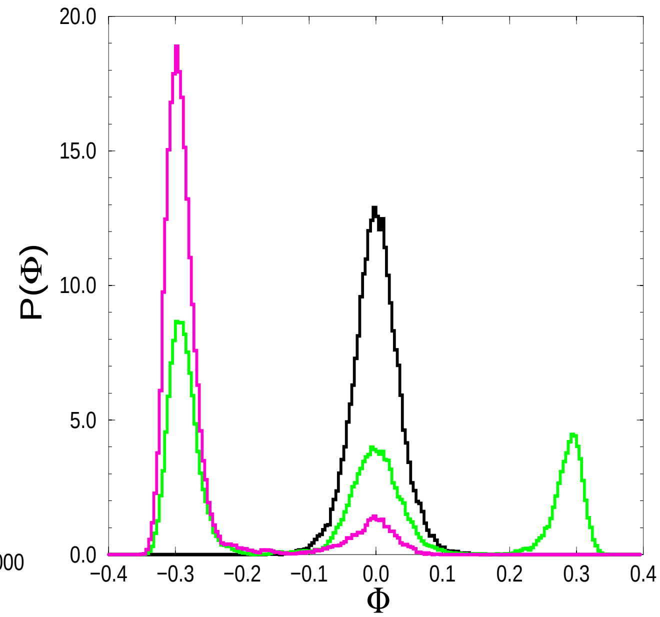
$Sp(2)$ in 4-dim.

no sign of any bulk transition separating strong and weak coupling

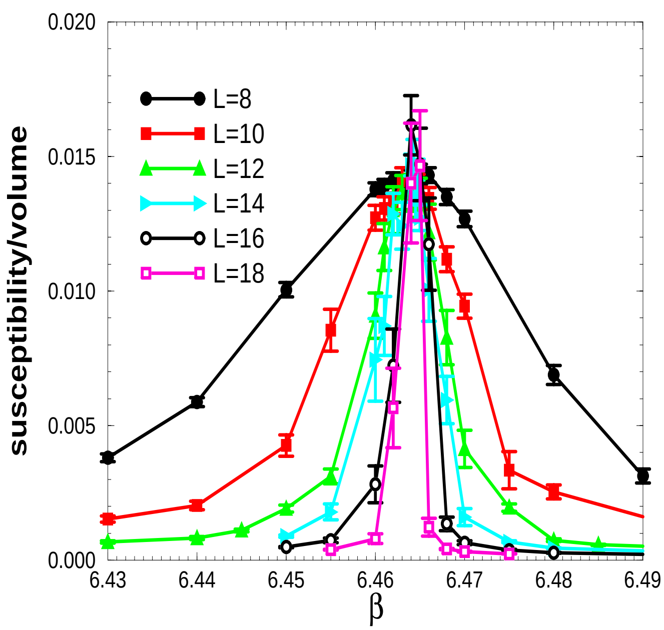


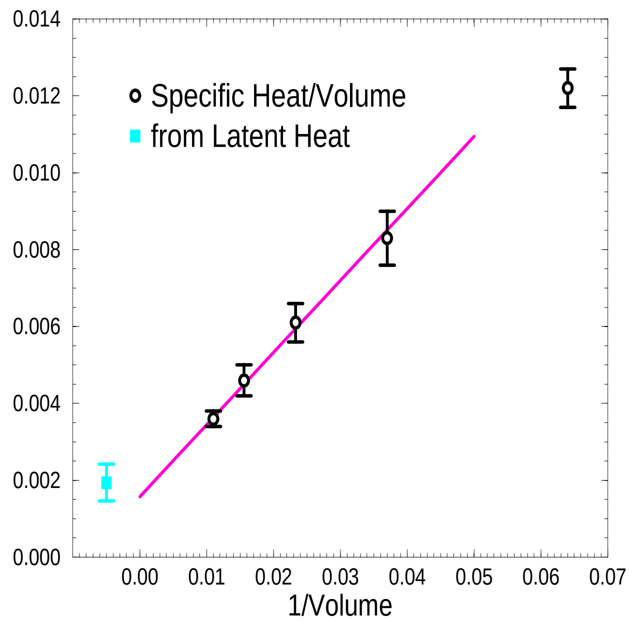
$Sp(2)$ in 4-dim.





Looks like 1st order deconfinement phase transition





susceptibility $\chi = V(\langle\Phi^2\rangle - \langle|\Phi|\rangle^2) \propto V$ at $T_{\rm crit}$

specific heat/volume extrapolation agrees with non-zero latent heat

phase transition is 1st order

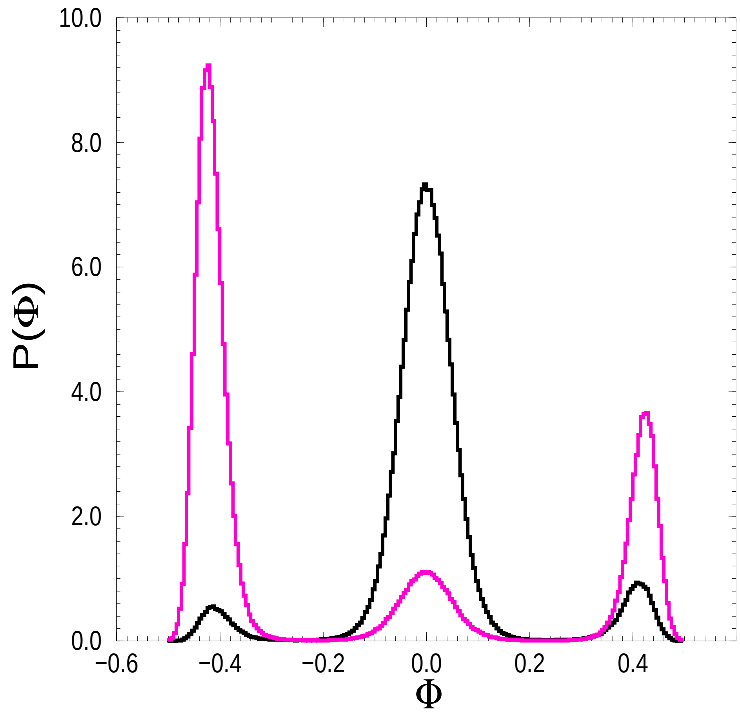
Conjecture for 4-dim.

$Sp(3)$ in 4-dim. also has 1st order deconfinement phase transition

What other possibilities?

$SO(N)$ groups

Exceptional groups



$SO(3) \simeq SU(2) \qquad SO(4) \simeq SU(2) \times SU(2)$

$SO(5) \simeq Sp(2) \qquad SO(6) \simeq SU(4)$

$G(2), E(8), F(4)$ have trivial center — expect no phase transition

$E(6), E(7)$ have center $Z(3), Z(2)$ respectively

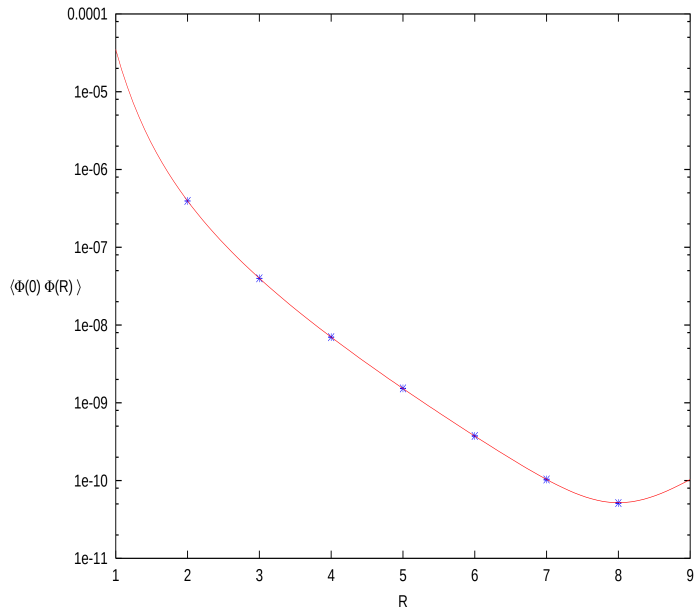
Conjecture: only $Sp(1) = SU(2) \simeq SO(3)$ has universal 2nd order deconfinement phase transition in 4-dim.

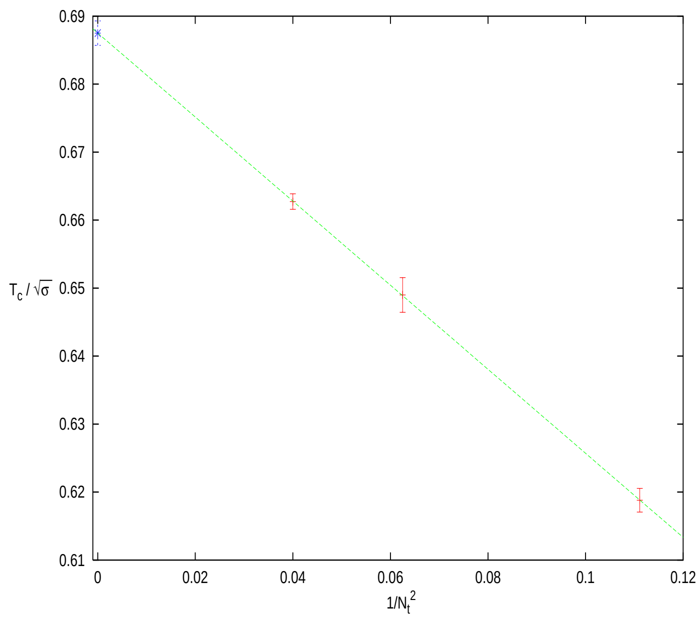
size of group (number of "gluons") makes transition 1st order

$Sp(2)$: 10 generators $\qquad Sp(3)$: 21 generators

center of group does not predict order of phase transition

1st order transition is physical





$$
\begin{aligned}
T_C/\sqrt{\sigma} &= 0.688(2) \text{ for 4-dim. } Sp(2) \\
&= 0.709(3) \text{ for 4-dim. } SU(2) = Sp(1) \\
&= 0.646(3) \text{ for 4-dim. } SU(3)
\end{aligned}
$$

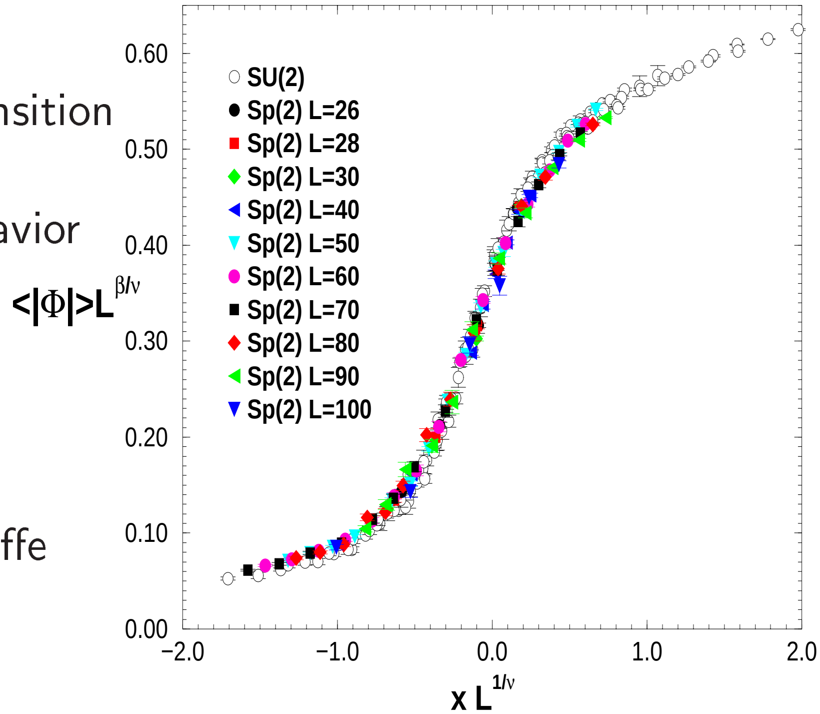
$Sp(2)$ seems to be closer to $SU(2) = Sp(1)$ than $SU(3)$ is

$Sp(N)$ in 3-dim.

$Sp(2)$ has 2nd order transition

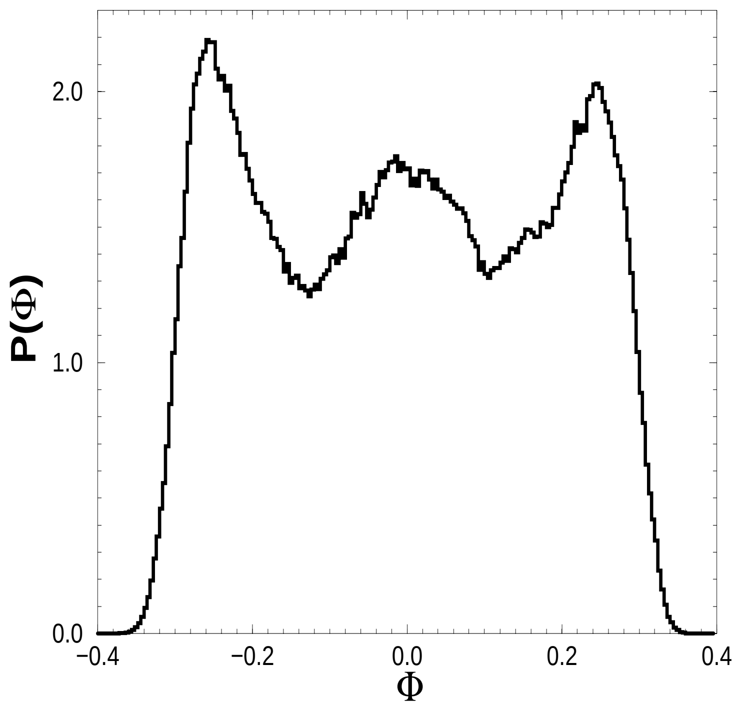
$\langle|\Phi|\rangle L^{\beta/\nu}$ universal behavior with 2-dim. Ising critical exponents β, ν

SUPPORTS Svetitsky-Yaffe conjecture

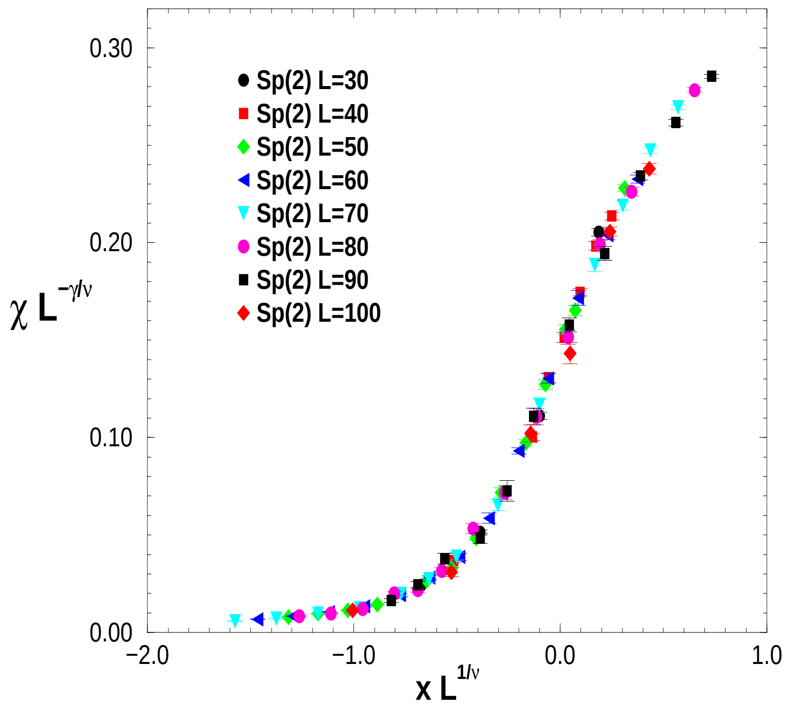


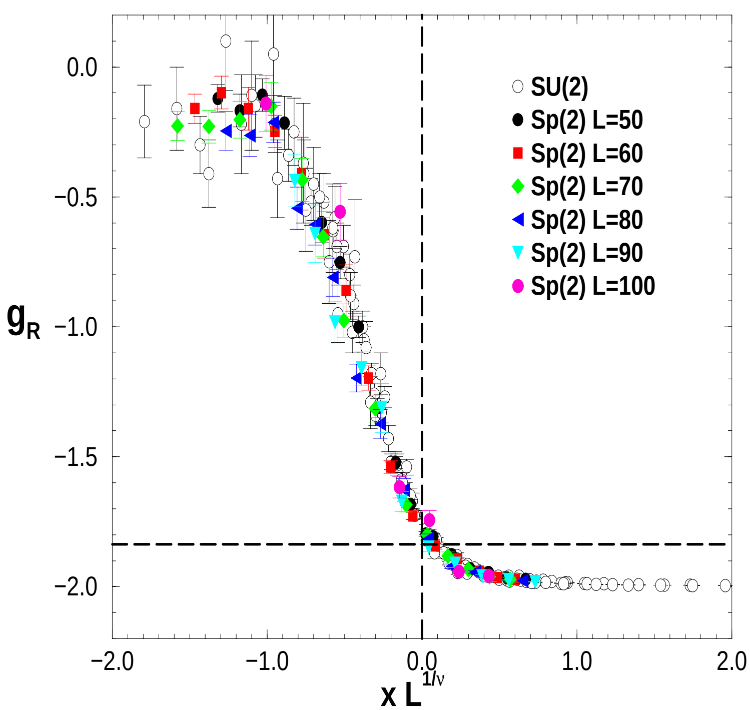
$Sp(3)$ has weak 1st order transition

Similar to 4-dim.
"large" groups have 1st order transition



$Sp(2)$ in 3-dim.





Critical exponents of 2-d Ising universality class

$\nu = 1 \qquad \beta = 1/8 \qquad \gamma = 7/4$

Summary

2nd order deconfinement phase transitions with universal behavior rare

conjecture that only $Sp(1) = SU(2) \simeq SO(3)$ have universal deconfinement transitions in 4-dim.

$Sp(1) = SU(2), Sp(2), SU(3)$ and maybe $SU(4)$ universal deconfinement in 3-dim.

phase transition driven by size of group, not by center

interesting to study more gauge groups systematically