The *hep* and the *hen* processes in EFT

(hep) \( ^3\text{He} + p \rightarrow ^4\text{He} + e^+ + \nu_e \)

(hen) \( ^3\text{He} + n \rightarrow ^4\text{He} + \gamma \)

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TSP et al., PRC67(’03)055206, nucl-th/0208055  
Y.-H. Song and TSP, nucl-th/0311055  

INT2003 at UW, Seattle: Nov. 26
Among the solar burning processes \((4\text{ }p\rightarrow^4\text{He} + 2\text{ }e^+ + 2\nu_e + \gamma\text{'s})\),

\[
\begin{align*}
(pp) & \quad p + p \rightarrow d + e^+ + \nu_e \quad E_\nu = 0 \sim 0.4 \text{ MeV} \\
(pep) & \quad p + e^- + p \rightarrow d + \nu_e \quad E_\nu = 1.4 \text{ MeV} \\
(^8\text{B}) & \quad ^8\text{B} \rightarrow ^8\text{Be} + e^+ + \nu_e \quad E_\nu < 18 \text{ MeV} \\
(hep) & \quad ^3\text{He} + p \rightarrow ^4\text{He} + e^+ + \nu_e \quad E_\nu < 20 \text{ MeV}
\end{align*}
\]

\[\phi(pp-pep) \gg \phi(^8\text{B}) \gg \phi(hep)\]

\textit{pp} produces the dominant solar neutrinos.

\textit{hep} produces the highest-energy solar neutrinos. There can be a significant distortion of the high-end of the \(^8\text{B}\) neutrino spectrum.
**hep history (S-factor in 10^{-23} MeV-b unit):**

**Schematic wave functions**

- ’52 (Salpeter) 630 Single particle model
- ’67 (Werntz) 3.7 Symmetry group consideration
- ’73 (Werntz) 8.1 Better wave functions (P-wave)
- ’83 (Tegner) 4~25 D-state & MEC
- ’89 (Wolfs) 15.3±4.7 analogy to $^3$He+n
- ’91 (Wervelman) 57 $^3$He+n with shell-model

**Modern wave functions**

- ’91 (Carlson et al.) 1.3 VMC with Av14
- ’92 (Schiavilla et al.) 1.4-3.1 VMC with Av28 (N+Δ)
  → $S_0 = 2.3$ (“standard value”)
- ’01 (Marcucci et al) 9.64 CHH with Av18 (N+Δ) + $p$-wave
  PRL84(’00)5959, PRC63(’00)015801
J. Bahcall’s challenge:

“... do not see any way at present to determine from experiment or first principle theoretical calculations a relevant, robust upper limit to the hep production cross section.”

(hep-ex/0002018)

Q: Can effective field theory (EFT) be a breakthrough?

A: Yes (naive considerations: BE(4He)=28 MeV) ... ...
   No (if you know more about the hep)... ... 
   Yes! (the 1st half of my presentation)
What’s wrong with the *hep*?

- Leading order $\langle 1B \rangle$ is highly suppressed.

\[ \left| ^4\text{He} \right\rangle = \left| S_4 : \text{most symmetric} \right\rangle + \cdots \]
\[ \left| ^3\text{He} + p \right\rangle = \left| S_{31} : \text{next-to-most symmetric} \right\rangle + \cdots \]

\[ \langle S_4 | g_A \Sigma_i \sigma_i \tau_i | S_{31} \rangle = 0. \quad : \quad \text{(Gamow-Teller)} \]

→ 1B-LO is small and difficult to evaluate
→ We need realistic (not schematic) wave functions.
→ Meson-exchange current (MEC) plays an important role.
2. Meson-exchange current (MEC) is *not* dominated by the long-ranged one-pion-exchange: short-ranged operators with unknown coefficients plays an important role.

3. There is a substantial cancellation between 1B and MEC. → Errors are amplified.

4. Getting realistic/reliable 4-body wave functions is quite non-trivial. Furthermore we need w.f.s for both scattering states as well as bound states.
Various possible approaches for the *hep*

- Traditional/conventional, phenomenological or *standard nuclear physics approach* (SNPA):
  - Chemtob-Rho type current operators ($\pi, \rho, \omega, \Delta, \ldots$)
  - Phenomenological but very accurate potentials: $\chi^2 \approx 1$
  - State-of-the-art technique for many-body wave functions
  - Extensively tested for many processes with impressive successes
  - Limitations:
    - Not systematic
    - Uncertainties in the short-range physics
Effective field theory (EFT) a la Weinberg

- Consistent and systematic expansion for the current operators (and the potential)

\[ O = \sum O_v = O_0 + O_1 + O_2 + \cdots \]

- Wave functions need infinite summation for a given \( V \), which can be done by solving Schroedinger equation

\[ |\Psi\rangle = |\phi\rangle + G^0 V |\Psi\rangle = (1+G^0 V + G^0 V G^0 V + ...) |\phi\rangle \]

- Limitation: As of now, we do not have accurate enough wave functions for the \textit{hep} process, though great efforts and progresses are being made recently. \textbf{Q}: How much the w.f.’s should be accurate? (see the Discussion)

- How can we go further?
• Hybrid method (of SNPA & EFT)
  \(|\Psi\rangle : SNPA\)
  \(\therefore : EFT\)
  – We can concentrate only on the current operators
  – Better accuracy (inherited from SNPA) for the 1B and the long-ranged contributions
  – Problems (limitations)
    • Model dependence
    • Mismatch/inconsistency
    • Poor control over the short-range physics
• **More-effective EFT (MEEFT, EFT*)**

= Consistent and systematic EFT with the (phenomenological) S NPA wave functions

= hybrid method + renormalization procedure for the short ranged contributions

– The whole problem (of SNPA and hybrid-method) lies in the short-range (SR) physics.

– In EFT, SR physics is described by the local operators,
  
  \[ O_{\text{short}} = \sum_n c_n \nabla^{2n} \delta (r) = c_0 \delta (r) + \cdots \]

– Up to N^4LO (Q^4 compared to the LO), we have only non-derivative contact term, \( C_0 \), for many cases.

– \( \langle \Psi_f | \delta (r) | \Psi_i \rangle \): model(potential)-dependent
– We can then fix the value of $C_0$ so as to reproduce other known experimental data (in many cases in a system with different $A$).

– The value of $C_0$ is model-dependent, which cancels out the model-dependence of $\langle \Psi_f | \delta(r) | \Psi_i \rangle$, so as to have model-independent $\langle \Psi_f | O_{\text{short}} | \Psi_i \rangle$, which is the renormalization condition.
MEEFT Strategy for $M(hep) = \langle \Psi_f | O | \Psi_i \rangle$

$|\Psi\rangle$ : Correlated-hyperspherical-harmonics (CHH) with Argonne Av18 potential
+ Urbana-IX three-nucleon interactions

$O$ : Up to $N^3\text{LO}$ in heavy-baryon chiral-perturbation theory (H BChPT)

Pertinent degrees of freedom: pions and nucleons.
Expansion parameter $= Q/\Lambda_\chi$

$Q$ : typical momentum scale and/or $m_\pi$,
$\Lambda_\chi$ : $m_N$ and/or $4\pi f_\pi$

Weinberg’s power counting rule for irreducible diagrams.
Gamow-Teller channel (*pp* and *hep*)

\[
\vec{A}_{1B} = g_A \sum_i \tau_i \left[ \vec{\sigma}_i + \frac{p_i \vec{\sigma}_i \cdot p_i - \vec{\sigma}_i p_i^2}{2m_N^2} \right] = \text{LO} + \text{N}^2\text{LO}
\]

\[
\vec{A}_{2B} = \sum_{i<j} \left[ \vec{A}_{ij}^{\text{OPE}} + \vec{A}_{ij}^{4F} \right] = \text{N}^3\text{LO}
\]

There is no soft-OPE (which is N^2LO) contributions
\[ \tilde{A}^{\text{OPE}}_{ij} = - \frac{g_A}{2m_N f_\pi^2} \frac{1}{m_\pi^2 + q^2} \left[ - \frac{i}{2} (\tau_i \times \tau_j) \tilde{p} (\vec{\sigma}_i - \vec{\sigma}_j) \cdot \vec{q} 
+ 4 \hat{c}_3 \vec{q} \cdot (\tau_i \vec{\sigma}_i + \tau_j \vec{\sigma}_j) 
+ \left( \hat{c}_4 + \frac{1}{4} \right) (\tau_i \times \tau_j) \vec{q} \times [(\vec{\sigma}_i \times \vec{\sigma}_j) \times \vec{q}] \right] \]

The values of \( c \)'s are determined from the \( \pi \)-N data
\[ \hat{c}_3 = -3.66 \pm 0.08, \quad \hat{c}_4 = 2.11 \pm 0.08 \]
\[
\bar{A}_{ij}^{AF} = -\frac{g_A}{m_N f_\pi^2} \left[ 2 \hat{d}_1 (\tau_i \vec{\sigma}_i + \tau_j \vec{\sigma}_j) + \hat{d}_2 (\tau_i \times \tau_j)(\vec{\sigma}_i \times \vec{\sigma}_j) \right]
\]

Thanks to Pauli principle and the fact that the contact terms are effective only for L=0 states, only one combination is relevant:

\[
\hat{d}^R \equiv \hat{d}_1 + 2\hat{d}_2 + \frac{1}{3} \hat{c}_3 + \frac{2}{3} \hat{c}_4 + \frac{1}{6}
\]

\(\hat{d}^R\) corresponds to \(L_{1A}\) in PDS scheme (Butler et al, PLB549(’02)26))

The same combination enters into \(pp, \ hep\), tritium-\(\beta\) decay (TBD), \(\mu-d\) capture, \(\nu-d\) scattering, … . We use the experimental value of TBD to fix \(\hat{d}^R\), then all the others can be predicted!
To control the short-range physics consistently, we apply the same (Gaussian) regulator

$$j^\mu_\Lambda(\vec{r}) = \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \exp\left(-\frac{\vec{k}^2}{\Lambda^2}\right) J^\mu(\vec{k})$$

for all the $\Lambda=2,3$ and 4 systems, with

$$\Lambda = [500, 600, 800] \text{ MeV}$$

$\hat{d}^R$ is a function of $\Lambda$, and determined for each value of $\Lambda$ to reproduce experimental value of TBD rate.
(Warming up) Results: $M_\Lambda(pp)$

<table>
<thead>
<tr>
<th>$\Lambda$ (MeV)</th>
<th>$\hat{d}^R$</th>
<th>$\langle 1B \rangle$</th>
<th>$\langle 2B \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>1.00</td>
<td>4.85</td>
<td>$0.076 - 0.035 \hat{d}^R = 0.041$</td>
</tr>
<tr>
<td>600</td>
<td>1.78</td>
<td>4.85</td>
<td>$0.097 - 0.031 \hat{d}^R = 0.042$</td>
</tr>
<tr>
<td>800</td>
<td>3.90</td>
<td>4.85</td>
<td>$0.129 - 0.022 \hat{d}^R = 0.042$</td>
</tr>
</tbody>
</table>

with $\hat{d}^R$-term, $\Lambda$-dependence has gone !!!

the astro $S$-factor (at threshold)

$S_{pp} = 3.94 \ (1 \pm 0.15 \% \ \pm 0.10 \%) \ 10^{-25} \text{ MeV-barn}$
Results: $M_\Lambda (pp)$
Results: $M_\Lambda (\text{hep})$
Results: $M_{\Lambda}$ (hep)

<table>
<thead>
<tr>
<th>$\Lambda$ (MeV)</th>
<th>&lt;1B&gt;</th>
<th>&lt;2B&gt;</th>
<th>&lt;1B+2B&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>−0.81</td>
<td>1.35 − 0.85 $\hat{d}^R$ = 0.49</td>
<td>−0.32</td>
</tr>
<tr>
<td>600</td>
<td>−0.81</td>
<td>1.76 − 1.22 $\hat{d}^R$ = 0.52</td>
<td>−0.29</td>
</tr>
<tr>
<td>800</td>
<td>−0.81</td>
<td>2.38 − 1.78 $\hat{d}^R$ = 0.59</td>
<td>−0.22</td>
</tr>
</tbody>
</table>

$\hat{d}^R$-term removes the major $\Lambda$-dependence. The small $\Lambda$-dependence in 2B is however amplified due to the cancellation between 1B & 2B.

Sizable but still reasonable $\Lambda$-dependence in net amplitude.
**hep** S-factor in $10^{-23}$ MeV-barn:

$$S_{hep}(\text{theory}) = (8.6 \pm 1.3)$$

**hep** neutrino flux in $10^3$ cm$^{-2}$ s$^{-1}$:

$$\phi_{hep}(\text{theory}) = (8.4 \pm 1.3)$$

$$\phi_{hep}(\text{experiment}) < 40$$

Super-Kamiokande data, hep-ex/0103033
The *hen* \((^3\text{He} + n \rightarrow ^4\text{He} + \gamma)\) process

- Both *pp* and *hep* process have **not been confirmed by experiments**.
- **Accurate experimental data** are available for the *hen*.
- The *hen* process has much in common with *hep*:
  - The leading order 1B contribution is strongly suppressed due to pseudo-orthogonality.
  - A cancellation mechanism between 1B and 2B occurs.
  - Trivial point: both are 4-body processes that involve \(^3\text{He} + \text{N}\) and \(^4\text{He}\).

**Q:** Can we test our *hep* MEEFT calculation by applying the same method to the *hen* process?
hen history

\[ \sigma(\text{exp}) = (55 \pm 3) \mu b, (54 \pm 6) \mu b \]

- 2-14 \mu b: ('81) Towner & Kanna
- 50 \mu b: ('91) Wervelman
- (112, 140) \mu b: ('90: VMC) Carlson et al
- (86, 112) \mu b: ('92: VMC) Schiavilla et al

\[ a(^3\text{He} - n) = (3.50, 3.25) \text{ fm} \]

- Accurate recent exp: \[ a(^3\text{He} - n) = 3.278(53) \text{ fm} \]
VMC wave functions with Av14 + Urbana VIII

- Predictions for the binding energy
  - $\text{BE}(^3\text{H})=8.21 \text{ MeV (exp}=8.48 \text{ MeV}$
  - $\text{BE}(^4\text{He})=27.23 \text{ MeV (exp}=28.30 \text{ MeV}$

- Prediction for the $^3\text{He}$-n scattering length:
  - Variational : $a_n=3.5 \text{ fm (exp}=3.278(53) \text{ fm}$
  - In our work, we have fit the Woods-Saxon potential parameters to reproduce $a_n=3.278 \text{ fm}$ and the low-E $^3\text{He}$-n phase shifts.
$^3$He-n phase shift [deg] wrt $E_{cm}$ [MeV]

solid line = Woods-Saxon potential
dots = $R$-matrix analysis by Fofmann & Hale, NPA613 ('97)
Remarks on the *hen* process

The *hen* process is governed by isoscalar and isovector M1 operators.

Contrary to GT, there is soft-OPE contribution to the isovector M1, which is NLO compared to 1B.

The N3LO of Isovector M1 corresponds to 1-loop.

At N^3LO, there appear two 4F contact counter-terms, \( g_{4S} \) and \( g_{4V} \), which we can fix by imposing the condition to reproduce the magnetic moments of \(^3\text{H}\) and \(^3\text{He}\).

\[
\vec{V}_{12}^{4F} = \frac{i}{2m_p} \vec{q} \times [g_{4S}(\vec{\sigma}_1 + \vec{\sigma}_2) + g_{4V}(\vec{r}_1 \times \vec{r}_2)z(\vec{\sigma}_1 \times \vec{\sigma}_2)]
\]
Results: $M_\Lambda (\text{hen})$
Results: $M_\Lambda$ (*hen*)

<table>
<thead>
<tr>
<th>$\Lambda$ (MeV)</th>
<th>$&lt;1B&gt;$</th>
<th>$&lt;2B&gt;$</th>
<th>$&lt;1B+2B&gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>−1.76</td>
<td>5.24 + 1.80 = 7.04</td>
<td>5.29</td>
</tr>
<tr>
<td>600</td>
<td>−1.76</td>
<td>6.79 + 0.35 = 7.14</td>
<td>5.39</td>
</tr>
<tr>
<td>800</td>
<td>−1.76</td>
<td>8.31 – 0.99 = 7.32</td>
<td>5.57</td>
</tr>
</tbody>
</table>

Contact terms remove the major $\Lambda$-dependence.

$\sigma$ (theory) = (60 ±3 ±1) $\mu$b, which is in reasonable agreement with the exp., (55 ±3) $\mu$b, (54 ± 6) $\mu$b.

A caveat: we have not included the so-called fixed-term contribution, which is expected-to-be small but hard-to-evaluate.
MEEFT in other processes

\(v-d\) scattering cross section: the \(\Lambda\) -dependence is less than 0.4 \%

Nakamura et al, NPA707(’02)561, Ando et al, PLB472(’03)49

\(\mu-d\) capture rate: Ando et al, PLB533(’02)25

<table>
<thead>
<tr>
<th>(\Lambda) (MeV)</th>
<th>(\hat{d}^R)</th>
<th>(\Gamma_{\mu d}^{L=0} [s^{-1}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>1.00 ± 0.07</td>
<td>(254.7 - 9.85 \hat{d}^R + 0.159 (\hat{d}^R)^2 = 245.0 ± 0.7)</td>
</tr>
<tr>
<td>600</td>
<td>1.78 ± 0.08</td>
<td>(261.1 - 9.09 \hat{d}^R + 0.132 (\hat{d}^R)^2 = 245.3 ± 0.7)</td>
</tr>
<tr>
<td>800</td>
<td>3.90 ± 0.10</td>
<td>(271.0 - 6.76 \hat{d}^R + 0.070 (\hat{d}^R)^2 = 245.7 ± 0.6)</td>
</tr>
</tbody>
</table>
Isoscalar M1 in np -> dγ

with respect to $r_C[fm]$: Park et al, PLB472(’00)232
Numerically, the results of MEEFT and the latest SNPA agree each other for the Gamow-Teller channel (\textit{pp} and \textit{hep}). But in the M1 channel, MEEFT can explain the \textit{hen} cross section, while SNPA could not yet.

MEEFT allows us to reduce theoretical uncertainties dramatically.

Other successful applications of MEEFT: isoscalar and isovector M1 in \textit{n + p $\rightarrow$ D + $\gamma$}, $\mu$-\textit{d} capture rate, $\nu$-\textit{d} scattering.

– The PDS scheme also has been successfully applied to 2B systems.

We can go up to \textit{N}^4\textit{LO} w/o having new parameters.
• Possibility to have pure-EFTs for the hep and hen in near future?
  – Low-energy amplitudes are very sensitive to the scattering length. To guarantee to reproduce the exp. value of it, we need 4-nucleon contact interaction, which is \( N^6\)LO (\( N^5\)LO in Epelbaum’s lang.)!

• Possibility to have MEEFT for more complicated systems?

• Thank you!