Do we need a three-nucleon force?
Nuclear structure with modern two- & three-body interactions

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● Why we think we need a three-nucleon force
● Ab initio no-core shell model (NCSM)
  ▷ Calculations with realistic two-nucleon interactions for p-shell nuclei
● Inclusion of a realistic three-nucleon interaction in the NCSM
  ▷ NCSM results for p-shell nuclei using two- plus three-nucleon interaction
● Do we really need a three-nucleon interaction?
  ▷ Calculations with realistic coordinate-space non-local two-body interaction
● Conclusions

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Why we think we need a three-nucleon interaction

- Realistic two-nucleon potentials highly accurate
  - Configuration space potentials
    - Local: AV18, AV8', Nijmegen II
    - Non-local: Nijmegen I
  - Momentum space non-local potentials
    - Boson-exchange potentials: CD-Bonn
    - Effective field theory potentials: N^3LO

- Exact few-body calculations with these potentials show that \(^3\)H, \(^3\)He, \(^4\)He underbound by 5-10%

- Problems in A=3 scattering: p+d, n+d \(A_y\) puzzle

- Nuclear structure calculations with realistic two-nucleon potentials in the \(p\)-shell
  - GFMC, NCSM, CCM
  - Underbinding
  - Level spacing not quite right
  - Ordering of lowest states not correct for some nuclei

Regardless of which current realistic two-nucleon potential used

problems remain:

Need to include a three-nucleon interaction
No-Core Shell-Model Approach

**Goal:** Solution of nuclear structure problem for light nuclei

- Many-body Schroedinger equation
  - $A$-nucleon wave function
  
  $$ H |\Psi\rangle = E |\Psi\rangle $$

- Hamiltonian
  $$ H^\Omega = \sum_{i=1}^{A} \left[ \frac{\vec{p}_i^2}{2m} + \frac{1}{2} m\Omega^2 \vec{r}_i^2 \right] + \sum_{i<j}^{A} \left[ V_{NN} (\vec{r}_i - \vec{r}_j) - \frac{m\Omega^2}{2A} (\vec{r}_i - \vec{r}_j)^2 \right] + \sum_{i<j<k}^{A} V_{ijk}^{3b} $$

  - Realistic nucleon-nucleon (NN) potentials
    - Configuration space - Argonne V18, AV8', (three-body chiral-sym.-based Tucson-Melbourne’)
    - Momentum space - CD-Bonn, EFT chiral-perturbation theory N³LO
  - Modification by center-of-mass HO potential

- Finite harmonic-oscillator basis
  - Complete $N_{\text{max}} \hbar \Omega$ basis space $\leftrightarrow$ defines the model space $P$, $Q\equiv 1-P$

- Truncated basis + nature of NN potentials $\rightarrow$ Effective interaction must be derived
  - Unitary transformation of $H^\Omega$ such that
  - Effective Hamiltonian then

  $$ QXHX^{-1}P = 0 $$

  $$ H_{\text{eff}} = PHXHX^{-1}P $$

  - $A$-nucleon solutions needed to find $X$
    $\Leftrightarrow$ Applied in $n$-body cluster approximation, $2 \leq n \leq A$
  - Need $n$-nucleon solutions to find $H_{\text{eff}}^{(n)}$
Test of convergence

$^{3}\text{He}$ with Argonne V18

Effective interaction speeds up convergence dramatically.
Reasonable approximation even for small basis sizes

$^{6}\text{Li}$ with Minnesota NN potential

Weaker dependence on $\Omega$, convergence to the SVM result as the basis size increases

$E_{gs}$ [MeV] vs $N_{\text{max}}$ for $^{6}\text{Li}$ MN

$h\Omega$ [MeV] vs $1/b^{2}_{\text{HO}}$
p-shell nuclei with realistic NN forces

The best established exact method for A>4

$A=6-10$ binding energies by GFMC $\rightarrow$ NCSM $\sim 2\%$

$^{13}$C
NCSM - no parameters to adjust
6 lowest levels in correct order
Problems remain
NCSM calculations with two-nucleon interactions in the $p$-shell

$^6$Li with accurate NN potential at fourth order of chiral-perturbation theory ($N^3$LO)

<table>
<thead>
<tr>
<th></th>
<th>$N^3$LO</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3$H</td>
<td>7.85 MeV</td>
<td>8.48 MeV</td>
</tr>
<tr>
<td>$^4$He</td>
<td>25.35(5) MeV</td>
<td>28.30 MeV</td>
</tr>
<tr>
<td>$^6$Li</td>
<td>28.5(5) MeV</td>
<td>31.99 MeV</td>
</tr>
</tbody>
</table>

Converged excitation energies
Correct level ordering, level spacing not right
$^{10}\text{B}$ ground-state spin

Experiment $3^+ 0$, realistic NN potentials $1^+ 0$

$\hbar \Omega=14 \text{ MeV}$

Need for the three-body interaction
Calculations performed in four steps

1) Three-nucleon solutions for all relevant $n=3$ $J^\pi T$ channels with and without $V^{3b}_{\text{eff}}$

\[
H^\Omega = \sum_{i=1}^{3} \left[ \frac{\vec{p}_i^2}{2m} + \frac{1}{2} m\Omega^2 \vec{r}_i^2 \right] + \sum_{i<j}^{3} \left[ V_{XN}(\vec{r}_i - \vec{r}_j) - \frac{m\Omega^2}{2A} (\vec{r}_i - \vec{r}_j)^2 \right] + V^{3b}_{123}
\]

- Jacobi coordinate HO basis up to $N_{3\text{max}}=30$

2) Three-body effective interaction by unitary transformation method

\[
\rightarrow X_3 \rightarrow H_{\text{3eff}}^{\Omega}
\]

- Must separate contributions from two-nucleon and three-nucleon interaction

3) Effective interaction in Jacobi coordinate HO basis, $p$-shell nuclei calculations more efficient in Cartesian coordinate Slater determinant basis

\[
\rightarrow \text{transformation must be performed}
\]

4) $A$-nucleon calculation performed by a shell model code with a three-body capability

- MFD, REDSTICK

\[
h_1 + h_2 + h_3 + V_{12} + V_{13} + V_{23} + V_{123} \rightarrow X_3 \rightarrow P_3 \left[ h_1 + h_2 + h_3 + V^{2b+3b}_{\text{eff},123} \right] P_3 \rightarrow \begin{pmatrix} \sum_{i=1}^{A} h_i + \frac{1}{A-2} \sum_{i<j<k}^{A} V^{2b}_{\text{eff},ijk} + \sum_{i<j<k}^{A} V^{3b}_{\text{eff},ijk} \end{pmatrix} P
\]
$^6$Li with the Tucson-Melbourne force

Importance of the real three-body interaction

Binding energy

$E \text{[MeV]}$

-31.04 MeV

$^6$Li

AV8' + TM'(99)

AV8'

-29.3 MeV

-31.99 MeV

$E \text{[MeV]}$

1+ 0

2+ 1

1+ 0

-2+ 1

2+ 0

-2+ 0

0+ 1

-0+ 1

3+ 0

-3+ 0

d+α

$V_{3\text{eff}}$

$\hbar \Omega = 14 \text{ MeV}$

$V_{3\text{eff}}$

$\hbar \Omega = 14 \text{ MeV}$

Dim=197 822; # non-zero matrix elements: $V_{3\text{eff}} \rightarrow 981 142 479$

$V_{2\text{eff}} \rightarrow 60 236 339$
**10^B and 11^B with the Tucson-Melbourne force**

**Importance of the real three-body interaction**

**Calculated spectrum very sensitive to V_{3b} $\Rightarrow$ hope to constrain V_{3b}**
$^{12}\text{C}$ with the Tucson-Melbourne force

- Binding energy increase by 6 MeV
- Improved level ordering
  - $1^+ 0 \leftrightarrow 4^+ 0$
  - $T=1$ states

$^{12}\text{N}$ and $^{12}\text{B}$ correct ground-state spin only when the three-nucleon interaction included
Neutrino scattering on $^{12}$C

Exclusive $0^+ 0 \rightarrow 1^+ 1$ cross section & transitions

- Extremely sensitive to the spin-orbit interaction strength
  - $B(GT) \langle B(M1) \rangle \langle \sigma \tau \rangle$
    - No spin-orbit $\rightarrow 0^+ 0$ and $1^+ 1$ in different SU(4) irreps
      $\rightarrow$ no transition
    - $^{12}$C ground state 8 nucleons in $p3/2$
      $\rightarrow$ Transition overestimated by a factor of six

- NCSM - no fit, no free parameters
  - $V^{2b}$ up to $6\hbar\Omega$ - saturation
    - Underestimates by a factor of 2-3
  - $V^{2b} + V^{3b}$ up to $4\hbar\Omega$
    - Significant improvement
  - Different processes dominated by different $Q$
    - Correlation with M1 transverse form factor

\[
\begin{array}{ccc}
AV8' & AV8'+TM'(99) & \text{Exp} \\
B(GT) & 0.26 & 0.67 & 0.88 \\
CD-Bonn & AV8'+TM'(99) & \text{Exp} \\
(v_e,e^-) & 3.69 & 6.8 & 8.9\pm0.3\pm0.9 \\
(v_\mu,\mu^-) & 0.312 & 0.537 & 0.56\pm0.08\pm0.1 \\
\mu\text{-capture} & 2.38 & 4.43 & 6.0\pm0.4 \\
\end{array}
\]

$V^{3b}$ increases spin-orbit interaction strength
Two-nucleon interaction in coordinate space
- Local at long ranges: Yukawa tail
- Non-local at shorter distances (up to 3 fm)

Published version: Non-local $^1S_0$, $^3S_1$-$^3D_1$ channels
- Remaining channels taken from AV18
  - Denoted as IS S

Two more (not yet published) versions:

i) Non-local in all channels containing $S$, $P$ and $D$ waves
  - Remaining channels taken from AV18
    - Denoted as IS SPD

ii) Non-local in all channels containing $S$, $P$ and $D$ waves
  - Triplet $P$ waves modified to reproduce three-nucleon analysing powers
  - Remaining channels taken from AV18
    - Denoted as IS SmodPD

Straightforward to use in the NCSM and apply to $p$-shell nuclei
NCSM binding energy calculations with IS NN potentials

<table>
<thead>
<tr>
<th>IS S</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^3)H</td>
<td>8.47(1) MeV</td>
</tr>
<tr>
<td>(^3)He</td>
<td>7.71(1) MeV</td>
</tr>
<tr>
<td>(^4)He</td>
<td>29.1(2) MeV</td>
</tr>
<tr>
<td>(^6)Li</td>
<td>32.3(3) MeV</td>
</tr>
<tr>
<td>(^6)He</td>
<td>29.1(5) MeV</td>
</tr>
<tr>
<td>(^7)Li</td>
<td>38.7(1.5) MeV</td>
</tr>
<tr>
<td>(^{10})B</td>
<td>62.5(2.0) MeV</td>
</tr>
<tr>
<td>(^{12})C</td>
<td>93.5(2.5) MeV</td>
</tr>
</tbody>
</table>

\(^6\)Li IS S-wave non-local NN potential
\(^6\text{Li} \) with the IS non-local \(NN\) potentials

**IS S**
- Binding energy close to experiment
- Excitation spectrum similar as with standard \(NN\) potentials:
  - \(3^+ 0\) too high
  - \(3^+ 0 \leftrightarrow 2^+ 0\) splitting underestimated

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**IS S, mod-P, D-wave non-local**
- Binding energy close to experiment
- Excitation spectrum improves compared to standard \(NN\) potentials:
  - \(3^+ 0\) lower
  - \(3^+ 0 \leftrightarrow 2^+ 0\) splitting larger
$^6\text{Li}$ spectrum sensitive to the $NN$ potential

$^6\text{Li}$

NN potentials

3$^+$ 0 state converged

2$^+$ 0
0$^+$ 1
3$^+$ 0
d$+\alpha$

1$^+$ 0

$E$ [MeV]

Exp S,mod-P,D S,P,D S CD-Bonn $N^3$LO AV8$'$

IS non-local $NN$ potential with modified triplet P-waves
⇒ best agreement with experiment
⇒ very similar effect as adding the three-nucleon interaction to standard $NN$ potentials
$^{10}$B with the IS SmodPD non-local NN potential

IS S-, mod-P-, D-wave non-local

$\hbar \Omega = 16$ MeV

10$^{10}$B

E [MeV]

0 1 2 3 4 5 6 7 8

0 1 2 3 4 5 6 7 8

Exp 10$\hbar \Omega$ 8$\hbar \Omega$ 6$\hbar \Omega$ 4$\hbar \Omega$ 2$\hbar \Omega$ 0$\hbar \Omega$ 2$^+$ 1 4$^+$ 0 2$^+$ 1 3$^+$ 0 2$^+$ 0 0$^+$ 1 1$^+$ 0 3$^+$ 0

$^{10}$B, non-local NN potential with modified triplet P-waves

better agreement with experiment than with any standard NN potentials
Spin-orbit interaction strength sensitive transitions in $^{12}$C

- IS SmodPD excitation spectrum more spread out than with standard $NN$ potentials
- $B(M1;0^+ 0 \rightarrow 1^+ 1)$ stronger than with standard $NN$ potentials but still weaker than in experiment
- AV8'+TM'(99) seems to do better for $^{12}$C
- However: Convergence of $^{12}$C NCSM calculations incomplete
**Conclusions**

- *Ab initio* no-core shell model is now capable to include realistic three-nucleon interactions
- First application for the $p$-shell nuclei: Chiral-symmetry-based Tucson-Melbourne-prime
  - Increase of binding energies
  - Increased strength of spin-orbit interaction
    - In general an improvement for the low-lying levels
      → Correct ground-state spin in $^{10,11,12}$B, $^{12}$N
    - Significant improvement of $0^+ 0 \rightarrow 1^+ 1$ transitions and neutrino cross sections on $^{12}$C
- New non-local $NN$ potentials that fit $NN$ data and $A=3$ binding energies
  - *Ab initio* no-core shell model calculations with these potentials straightforward
  - Increase of binding energies - close to experiment for $p$-shell nuclei
  - IS SmodPD version with modified triplet $P$-wave channels to fit $A=3$ analysing powers
    - Increase of spin-orbit interaction strength: Improvement of nuclear spectra and transitions
      → Correct ground-state spin in $^{10}$B, improvement of $0^+ 0 \rightarrow 1^+ 1$ transitions in $^{12}$C
- Do we need a three nucleon interaction? *Yes*
  - However, at least a part of the three-nucleon interaction effects can be built into non-local $NN$ interaction
    - Such interaction much easier to use in many-body *ab initio* calculations
- A thing to do: Use transformations discussed by B. Desplanques *et al.*, W.N. Polyzou and W. Gloeckle to develop a non-local $NN$ interaction from the two- plus three-nucleon interaction derived by the chiral perturbation theory

Possibility to learn about the form and parametrization of the three-nucleon interaction and/or non-local $NN$ interaction from nuclear structure calculations