Simulation of Supersymmetric Theories in 1+1 and 2+1 Dimensions

QCD and String Theory
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Outline

† SYM with a Chern–Simons (CS) term in 2+1 dimensions.
† Light cone quantization
† Discretization
† SDLCQ: an exactly supersymmetric numerical method.
† CS: a mass term without breaking SUSY
† Dimensional reduction to 1+1 dimensions
† Zero Mode Quantization
† Numerical Methods.
† SYM 1+1 dimensions: numerical results
† SYM / SUGRA Correspondence
† Massive 1+1 SYM numerical results.
† Approximate BPS states.
† Anomalously light states in 2+1 dimensions.
† SYM–CS with Fundamental matter in 1+1.
† Supersymmetric Transverse Lattice: winding states.
Light-cone coordinates in 2+1 dimensions:

\[(x^+;x^i;x^?)\]; \(A \cdot B = A^+ B^i + A^i B^+ \cdot A^? B^?\)

\(x^+ = (x^0 + x^3)\) = \(\frac{p}{2}\) is time coordinate

\(x^i = (x^0 - x^3)\) = \(\frac{p}{2}\) is space coordinate

\(x^2 = x^2\) is usual transverse coordinate.

\(\mathcal{P}^+\) is total longitudinal momentum

\(\mathcal{P}^i\) is light-cone energy (Hamiltonian)

\(\mathcal{P}^?\) is total transverse momentum

Free energy of a massive particle

\[\mathcal{P}^i = \frac{m^2 + (\mathcal{P}^?)^2}{2\mathcal{P}^+}\]

LC quantization \(\) we can always work in the sector:

\[(\mathcal{P}^?)_{total} = 0\]
Supersymmetric Chern-Simons gauge theory in $D = 2 + 1$

Lagrangian

$$S = \int d^3x \text{Tr} \left( \frac{1}{4} F'' F'' + i \gamma D'' + \frac{1}{2} \mathcal{L}_{CS} \right)$$

where

$$\mathcal{L}_{CS} = \frac{1}{3} \gamma [A'', A'''] + \frac{2i}{3} g [A'' A'''] + 2 \gamma D'$$

All fields are in the adjoint representation.

For convenience, we work in the large $N_c$ limit.
Supersymmetry

Supersymmetry transformations:

\[- \Omega = \frac{i}{4} \Omega \]

\[- \eta = \frac{i}{4} \eta \]

where

\[0 = 2; 1 = i 1; 2 = i 3;\]

Get supercurrent \( \Omega (\eta) \) [with Lorentz index (\( \eta \))] via

\[- L = \nabla \Omega (\eta) :\]

\[\int d^2 x Q (\eta) = \int d^2 x \hat{Q} (\eta) = i 2 \hat{Q} (\eta) + i g [\hat{A} (\eta) \hat{A} (\eta) + \hat{F} (\eta) \hat{F} (\eta) :\]

Explicitly the spinor components read

\[Q^\iota = i i 2^3 = 4 \int d^2 x \hat{Q} (\eta) = i \hat{Q} (\eta) + i g [\hat{A} (\eta) \hat{A} (\eta) :\]

\[Q^+ = i i 2^5 = 4 \int d^2 x \hat{Q} (\eta) = i \hat{Q} (\eta) + i g [\hat{A} (\eta) \hat{A} (\eta) :\]

Construction of the energy-momentum tensor \( T (\eta) \) establishes a proof of the SUSY Algebra

\[f Q^\iota ; Q^\iota g = 2 p \pm P^\iota ; f Q^+ ; Q^\iota g = i 4 P^\iota ;\]
Light-cone Quantization

Fields in the Lagrangian:

\[
\begin{pmatrix} A^+ ; A_i ; A^- \end{pmatrix} \quad \text{and} \quad \gamma = 2i \not{\!1} = \not{\!4} \not{\!A}
\]

To express supercharges in physical degrees of freedom, use the equations of motion, some of which are constraint equations.

In light-cone gauge

\[ A^+ = 0 \]

the constraint equations read

\[
A_i = \frac{1}{D_i^2} \left[ \not{\!1} \not{\!i} D_2 \not{\!D} A^- + 2g \not{\!\not{\!g}}_{\!+} + \gamma \right] ;
\]

\[
= \bigg[ \not{\!p} \frac{1}{2D_i} \left( D_2 \not{\!i} \not{\!1} \right) \bigg] ;
\]

\[ \ldots \]

\[ \implies \] Eliminate \( \gamma \) and \( A_i \) to quantize dynamical fields only

\[ \underline{Z_2 \text{ Symmetry}} \]

\[ S : a_{ij}(k) \not{\!1} a_{ji}(k) ; b_{ij}(k) \not{\!1} b_{ji}(k) \]

\[ S j i = S \not{\!j} \not{\!i} \]

\[ \ldots \]

\[ \] Label for resulting eigenfunctions
Discretization

Longitudinal Direction

Impose periodic boundary conditions: \( L_\text{j,j} < x^i < L_\text{j,j} \)

discrete momentum modes

\[
k^+ = \frac{n P^+}{K}; \quad k^+ , \quad 0 \quad n = 1; 2; 3; \ldots; K
\]

\( K \) is the harmonic resolution (maximal number of partons) Continuum limit: \( K \rightarrow 1 \).

transverse Directions (\( (P^\perp)_{\text{total}} = 0 \))

Impose periodic boundary conditions: \( L^\perp \cdot n^\perp \cdot L^\perp \)

\[
k^\perp = \frac{2 \cdots n^\perp}{L^\perp} \quad n = 0; 1; 2; 3; \ldots; T
\]

\( T = 0 \quad \) 2D theory \quad \( T = 1 \quad ) \) 3D theory
SDLCQ

(Supersymmetric Discretized Light-Cone Quantization)

$Q^i$, and $Q^+$ are constructed in this finite basic and preserve the super algebra and therefore the supersymmetry. The eigenvalue problem;

$$2p^+ p^i M_i = M_n^2 M_i$$

becomes,

$$p_- 2p^+ (Q^i)^2 M_i = p_- 2p^+ (gQ^i_{\text{SYM}} + Q^i_{\text{CS}})^2 M_i = M_n^2 M_i$$
Chern–Simons term in SDLCQ

$$Q_{CS}^{i} = \frac{\hat{P}}{L} \prod_{n} \frac{1}{2} \left[ A (n;n_\parallel ) B (n;n_\parallel ) - B (n;n_\parallel ) A (n;n_\parallel ) \right]$$

with rescaled discrete field operators $A_{ij}(n;n_\parallel )$ and $B_{ij}(n;n_\parallel )$.

The similar structure of the transverse momentum component of the supercharge in the full 2+1 dimensional theory is:

$$Q_{i}^{2} = i2^{7=4} \prod_{n} \frac{1}{2} \left[ A (n;n_\parallel ) B (n;n_\parallel ) - B (n;n_\parallel ) A (n;n_\parallel ) \right]$$

Therefore adding a CS term has the effect of

$$k^2 = k^2 + i\bullet$$

and

$$p^i = \frac{(k^2)^2}{2k^+} + \frac{(k^2)^2 + \bullet^2}{2k^+}$$
Dimensional reduction to $D = 1 + 1$

Sakai, Sakai and Matsumura 1995

Ignore all transverse variations: $@_2 ! 0$

Rename bosonic (gauge) fields: $`_{ij} \cdot A^{?}_{ij}$

With dynamical fermion $\hat{\gamma}$ get dimensionally reduced supercharge

$$Q^i = 2^{3=4} \int dx^i \left( ig \hat{\gamma};@_i \right) + 2g\hat{\gamma}^i \cdot @_i \left( \frac{1}{@_i} \right)^\gamma \hat{\gamma} \cdot gQ_{SYM}^i + p \hat{\gamma} \cdot Q_{\hat{c}S}^i :$$

Mode expansion of the fields in 1+1 dimensions:

$$`_{ij}(q; x^i) = \frac{1}{2} \int_{Z^1}^{Z^1} dk^+ \left( a_{ij}(k^+) e^{i k^+ x^i} + a_{ji}(k^+) e^{i k^+ x^i} \right)$$

$$\hat{\gamma}^{ij}(q; x^i) = \frac{1}{2} \int_{Z^1}^{Z^1} dk^+ \left( b_{ij}(k^+) e^{i k^+ x^i} + b_{ji}(k^+) e^{i k^+ x^i} \right) :$$
Mode expansion at $x^+ = 0$:

\[ i_{ij}(x) = \frac{1}{4\ldots} \hat{X} \sum_{n=0}^\infty A_{ij}(n) u_{ij}(n) e^{ik_n x^i} + h.c.; \]

\[ i_{ii}(x) = \frac{1}{4\ldots} \frac{1}{n} \hat{X} \sum_{n=1}^\infty A_{ii}(n) e^{ik_n x^i} + A_{ii}(n) e^{ik_n x^i}; \]

\[ i_{ij}(x) = \frac{1}{2\hat{X}} \frac{1}{2L} \hat{X} \sum_{n=0}^\infty B_{ij}(n) e^{ik_n x^i} + B_{ji}(n) e^{ik_n x^i}; \]

\[ k_n = \frac{n\pi}{L}; \quad u_{ij}(n) = jn; \quad z_i + z_j j^{1=2}; \quad z_i = \frac{Lg}{n} A_{ii}(x^+). \]

In the fundamental domain: $j_{ij} \cdot 1=2$.

Commutation relations:

\[ [A_{ij}(n); A_{kl}^y(m)] = \text{sgn}(n + z_j; z_i) - n; m (-ik-jl; 1/N -ij-kl); \]

\[ fB_{ij}(n); B_{kl}^y(m)g = -n; m (-ik-jl; 1/N -ij-kl). \]

In the fund. domain: $z_N < z_{N+1} < \cdots < z_1$, $A_{ij}(0)$ is a creation operator if $i < j$, choose the same for $B_{ij}(0)$.
Bound State Problem

\[ \mathcal{J}^i \left| \psi(z) \right> = \text{Fock State}_i \]

\[ \hat{H} = \hat{K}(z) + V(z; A; A^y; B; B^y) \]

\[ \hat{H} \mathcal{J}^i = E \mathcal{J}^i \]

Solving the BSP in two steps:

1. \[ V(z; A; A^y; B; B^y) \text{Fock State}_i = \mathcal{V}(z) \text{Fock State}_i, \]
2. \[ \hat{K}(z) + \mathcal{V}(z) \left| \psi(z) \right> = E \left| \psi(z) \right>. \]
Gauge Anomaly

Regularization of current operator by point splitting:

\[ \lim_{\tau \to 0} \mathcal{J}^+ \cdot (x; \tau) + \mathcal{J}^+ \cdot (x; \tau) \hat{=} \]

\[ \mathcal{J}^+ \cdot (x; \tau) = \frac{1}{i} \left( e^{i \frac{i}{2} \mathcal{L}^+ \mathcal{M}} \cdot (x^i; i; \tau) e^{i \frac{i}{2} \mathcal{L}^+ \mathcal{M}} \hat{D}_i \cdot (x^i) \right) \]

\[ \mathcal{J}^+ \cdot (x; \tau) = \frac{1}{i} \left( e^{i \frac{i}{2} \mathcal{L}^+ \mathcal{M}} \cdot (x^i; i; \tau) e^{i \frac{i}{2} \mathcal{L}^+ \mathcal{M}} \hat{\mathcal{D}}_i \cdot (x^i) \right) \]

\[ M = \text{diag}(z_1; \ldots; z_N) \]

\[ \lim_{\tau \to 0} \mathcal{J}^+ \cdot (x; \tau) = \mathcal{J}^+ \cdot (x) \hat{=} \frac{1}{4L} (z_1 \cdot (N + 1; i \cdot 2i))^{-1} \cdot (x^i) \]

\[ \lim_{\tau \to 0} \mathcal{J}^+ \cdot (x; \tau) = \mathcal{J}^+ \cdot (x) \hat{=} \frac{1}{4L} (z_1 \cdot (N + 1; i \cdot 2i))^{-1} \cdot (x^i) \]

Kalloniatis, Pinsky, 1996

Supersymmetric YM theory:

\[ \mathcal{J}^+_{ij} = : \mathcal{J}^+_{ij} : \]
Ground State

\[ \text{Fock State} i = \mathcal{D} i : \quad A \mathcal{D} i = B \mathcal{D} i = 0 \]

Vacuum Energy:

\[ P^i = K^0(z) + V_{JJ} + V_{\cdot \cdot} \]

\[ h_0 \mathcal{V}_{JJ} \mathcal{D} i = \quad h_0 \mathcal{V}_{\cdot \cdot} \mathcal{D} i = 0 \quad \Rightarrow \quad K^0(z) \mathcal{V}(z) = E^0(z) \]

Ground state: \( \mathcal{V} = \text{const}; \quad E = 0 \)

Supersymmetry transformation:

\[ -A_{ii}^+ = \begin{pmatrix} 1 \\ \frac{1}{2} \\ 0 \end{pmatrix}^T \cdot i \mathcal{D}_{ii} \]

\[ -\mathcal{D}_{ii} = \quad i \cdot 2@^+ + A_{ii}^+ + \]

Vacuum state: \( P^i \mathcal{D} i = P^+ \mathcal{D} i = 0 \quad \)

\[ [P^i ; \mathcal{D}_{ii}^0] \mathcal{D} i = 0; \quad [P^i ; \mathcal{D}_{ii}^0] = 0 \]

\[ \text{vac}_{\text{n}} = \quad \text{tr}^0 \quad \mathcal{D} i = \quad 0 \cdot n \cdot N \]

Witten, 1979
Numerics

Method

The spectrum is obtained by solving complex eigenvalue problem. Exploit structure of supercharge:

\[
Q^i = \begin{pmatrix}
0 & A + iB & \text{bosons} \\
A^T & iB^T & 0 & \text{femions}
\end{pmatrix}
\]

where \(A\) and \(B\) are real matrices. Decomposition of Hamiltonian:

\[
P^i_{\text{boson}} = AA^T + BB^T + iBA^T - A^TB^T; \\
P^i_{\text{fermion}} = A^TA + B^TB + iA^TB - B^TA^T; \\
\]

Degeneracy The smallest real representation of the super algebra is 4x4, therefore SYM is 4 fold degeneracy. The smallest complex representation is 2x2, therefore CS-SYM is 2 fold degenerate.

Hard/Software

† Linux workstation

† Pentium III at 733 MHz with 2 GB RAM

† Code improved: Mathematica! C++! large \(N_c\) code

† handles at least 2 million Fock states in full calculations (upper limit in Fock basis construction: 25 million states)

Here: limited to \(K = 9\) because of code structure (dedicated D=2+1 code)

But: sufficient resolution because of very good convergence
Pure N=(1,1) SYM

Antonuccio Lunin and Pinsky 1998

![Graph of spectrum of pure N=(1,1) SYM in 1+1 dimensions in units of g^2 N c =... as a function of the inverse of the resolution K]

**BPS States**

† In general: \( fQ_\dagger f \bar{Q}^i \bar{Q}^j_\dagger g = Z \bar{f}_i f \bar{f}_j \)

† BPS saturated states: \( M_{BPS} = Z \)

† pure SYM 1+1: \( Z = 0 \) \( M_{BPS} = 0 \)

† There are \( 2 (K \ i \ 1) \) BPS states.

† They are particle number eigenstates with \( 2; 3; \ldots; K \) particles, independent of the coupling.
SYM / SUGRA Correspondence

Antonuccio, Lunin, Hashimoto, Hiller, Trittmann and Pinsky 1999 2000

AdS/CFT correspondence: gauge theory on the boundary has the same degrees of freedom as gravity in the bulk (holography)

Maldacena, 1997

Diagonal fluctuations around D1–brane near horizon ) SUGRA correlator:

$$h O(x)O(0) = \frac{N^3=2}{g_{YM} x^5}$$

Dual theory on the boundary: SYM\(_{1+1}\) with \(N = (8,8)\) SUSY.

\[
\begin{array}{ccc}
N^2 = x^4 & N^3=2 = (g_{YM} x^5) & N = x^4 \\
0 & \text{UV} & \text{SUGRA} & \frac{p}{g_{YM}} & \text{IR} \\
\end{array}
\]

\[F(x^i ; x^+) = hO(x^i ; x^+)O(0) i \cdot H^{++}(x^i ; x^+)T^{++}(0) i\]
Correlation Function for SYM

\[ F^r (\mathbf{P}^+; \mathbf{x}^+) = \frac{1}{2L} \sum_{n} \mathcal{H}^{++} (\mathbf{P}^+; \mathbf{x}^+) T^{++} (\mathbf{P}^+; 0)i \]

\[ = \frac{1}{2L} \prod_{n} \mathcal{H}^{++} (\mathbf{P}^+; \mathbf{x}^+) \mathcal{H}^{++} \frac{2}{L^2} e^{i \mathbf{P}^+ \mathbf{x}^+} \]

\[ p_n^i = \frac{M^2}{2 \mathbf{P}^+}; \quad r^2 = 2 \mathbf{x}^+ \mathbf{x}^i \]

\[ \frac{x^i}{x^+} F (x^i; \mathbf{x}^+) = \mathcal{H}^{++} (\mathbf{x}) \frac{M^4}{8 \cdots k^3 K^4} \frac{M_n^3}{M_n^4} N M \]

For SYM with \((\mathcal{M}^{-}; \mathcal{M}^{+})\) SUSY:

\[ T^{++} (\mathbf{x}) = \text{tr} \left( \mathbf{X} \right)^2 + \frac{1}{2} \left( i u_i^i \mathbf{X} u_i^i \right) \left( i \mathbf{X} u_i^i \right) \]

In the limit \( r \rightarrow 0 \):

\[ \frac{x^i}{x^+} F (x^i; \mathbf{x}^+) \rightarrow \frac{3}{4 \cdots K^3 r^4} N^2 M^{2} \]

\[ i \frac{1}{K} \]
Correlation Function for (1,1) Model.

Log Derivative for (8,8) Model
Massive SYM in 1+1 Dimensions

Hiller, Trittmann and Pinsky 2002

Convergence of 1+1 Massive SYM

Masses at $\bullet = 1.0$ and $g=1.0$ as a function of the resolution $K$

Fit to $M^2 = M^2_1 + b(1=K)$ yields

$Z_2 = +1$

$Z_2 = i 1$

$M^2_{Z+} = 4.30; 18.33; 27.46; 43.20$  
$M^2_{Zi} = 10.06; 29.13; 32.52; 47.40$

Good numerical convergence
BPS-like states

Spectrum as a function of the Coupling

Note: there are states that appears to have masses independent of the coupling. A close inspections shows that there is a slight coupling dependence.
BPS-like states (cont’d)

We can understand these states at small $\frac{1}{g}$

$$Q^i = g \left( Q^i_{SYM} + \frac{1}{g} Q^i_{CS} \right)$$

We have “BPS-like” bound states with

$$\mathcal{BPS}_{\text{like}} = \mathcal{SYM} \oplus BPS + O \frac{1}{g}$$

and

$$M^2 = \mathcal{N}_{B}^{2} + \mathcal{N}_{F}^{2} + O \frac{1}{g}$$

We see these states that are a reflection of the BPS states in the pure SYM theory

As $g \to 1$ These are threshold bound states.

At $g = 0$ we have only a CS term (the number operator): Therefore we have a free particle spectrum. Namely,

$$M^2(K) = \frac{\mathcal{N}_{1}^{2}}{n_1} + \frac{\mathcal{N}_{2}^{2}}{n_2} + \cdots$$

=) Complete understanding of small and large coupling regimes!

New duality: $M_{\mathcal{BPS}_{\text{like}}} (g = 0) \cdot M_{\mathcal{BPS}_{\text{like}}} (g = 1)$

Example:

$g = 0$: $M^2 = 4$ are 2 free particles of unit mass

$g = 1$: $M^2 = 4$ single bound state of $M = 2$ (threshold)
Structure Functions

QCD-like features of the eigenstates motivate a look at the structure functions

\[ g_a(n) = \sum_{q=2}^{K^{q+1}} X^q - \sum_{i=1}^{n_1} X^{q-1} n_i K + \sum_{l=1}^{n} \frac{n_1 A}{A_1} \int (x_1; ...; x_q) dx : \]

Figure 2: Convergence of the structure function of the lightest state of the theory as the resolution \( K \) is increased to 9. The solid line is a spline interpolation to the data for \( K = 9 \) and the conjectured points at \( x = 0 \) and \( x = 1 \).
Figure 3: Structure functions of *bosonic* states at $K=9$. The top of the graphs is bosonic, the bottom is fermionic structure functions. Dashed lines are *sea* contributions, solid lines are the *two-parton* ($M_1^2 = 4.30$ and $M_1^2 = 27.46$), *three-parton* ($M_1^2 = 10.06$) and *four-parton* ($M_1^2 = 18.33$) sector contributions.

Note: Different scales! The fermionic contributions are suppressed.
SDLCQ in 2+1 Dimensions

SYM in 2+1 dimensions

† There are 2(K-1) exactly massless BPS states at resolution K.

† The average number of particles in BPS states grows with the coupling. Recall in 1+1 dimensions the average number of particles in the BPS states are independent of the coupling.

CS–SYM in 2+1 dimensions

† The massless BPS states of SYM acquire a mass.

† Since the average number of partons in the BPS state of SYM grow with coupling grow, the mass of these states also grow.

† They grow much slower that other states

† At strong coupling they are anomalously light.
Approximate BPS States in 2+1 Dimensions

Hiller, Trittmann and Pinsky 2002

Figure 4: The mass spectrum of CS–SYM theory as a function of the YM coupling in (a) 1+1 dimensions (b) in 2+1 dimensions at fixed K and T
Anomally Light State in SYM–CS in 2+1 Dimensions

Displayed in the table are the 10 lowest masses at $g = 0.1$ and $g = 0.5$. We see that at $g = 0.1$ the are more or less uniformly spaced from $M^2 = 4$ to $M^2 = 10$. At strong coupling, $g = 0.5$, we see that a large gap has developed between the state at $M^2 = 8.93$ and $M^2 = 13.14$.

The anomalously low mass state at strong coupling is a reflection of the massless BPS in the underlying SYM theory in 2+1 dimensions.

<table>
<thead>
<tr>
<th>$g$</th>
<th>$M^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>8.93</td>
</tr>
</tbody>
</table>
Structure Functions in 2+1 Dimensions

\[ \mathcal{G}_A (x; k^2) = \mathcal{G}^A_{1 \ldots q} \mathcal{G}^1_{1 \ldots 1} \]

\[ \mathcal{G}^A_{1 \ldots q} = \mathcal{G}^1_{1 \ldots 1} = \int dx_1 \cdots dx_q \int dk_1^2 \cdots dk_q^2 \]

Figure 5: Structure function of the lowest approximate BPS state at (a) \( g = 0.1 \) and (b) Unique structure function at \( g = 0.5 \)
SYM with Fundamental Matter in 1+1 Dimensions

Hiller, Trittmann and Pinsky 2003

The super charge

\[ Q^i = Q^i_s + Q^i_1 + Q^i_2 + Q^i_3 : \]

\( Q^i_s \) is pure adjoint matter.
\( A \) is adjoint gauge field and \( \partial \) is its superpartner.
\( \rightarrow \) is a fundamental scalar and \( \leftarrow \) is its superpartner.

\[ Q^i_1 = i \frac{\gamma}{2} \int_{-Z}^{Z} dx^i \partial \cdot \frac{1}{\partial_i} \cdot \left( \frac{1}{\partial_i} \right) ; \]

\[ Q^i_2 = i \frac{\gamma}{2} \int_{-Z}^{Z} dx^i \left( \frac{1}{\partial_i} \right) \cdot \left( \frac{1}{\partial_i} \right) ; \]

\[ Q^i_3 = i 2g \int_{-Z}^{Z} dx^i \rightarrow yA^2 \rightarrow + \left( \frac{1}{\partial_i} \right) . \]
“Meson" States

“Meson" states are a string-like systems of interacting gluons and gluinos terminated at each end with a dynamical (s)quarks and anti-(s)quarks. The Fock space construction for this states is as follows:

$$f_{i_1}^Y (k_1) a_{i_2}^Y (k_2) \cdots b_{i_{n+1}}^Y (k_{n+1}) \cdots f_{i_p}^Y (k_n) f_{j}^l$$

$$f_{i_1}^Y$$ and $$f_{j}^l$$ each create one of the fundamental partons and $$a_{i_2}^Y$$ and $$b_{i_{n+1}}^Y$$ create an adjoint partons.
**Spectrum “Meson” States**

We find that the spectrum of meson bound states divides into two bands of states as a function of Cern–Simons coupling $\bullet$. The oscillatory convergence is common to all the states in the low band while the states in the up band converge linearly.

![Graphs showing mass squared](image)

Figure 6: The mass squared in units of $g^2 N_c$... of (a) the spectrum as function of $\bullet$ at resolution of $K = 6$ (b) the lowest mass state at $\bullet = g \\frac{P}{N_c}$... as a function of $1/K$

The oscillatory behavior made this calculation particularly challenging numerically. We were forced to go to very high resolution, $K = 13$, to be certain that the spectrum really converged.
Some but not all of the super algebra is preserved.
The link variable can be written as;

\[ M_i(x) = \exp\left(ia\sum_{i+1=2}^{N} A_i(x)\right) \]

The super-charge is:

\[ Q^i = 2^{3/4}g \sum_{\text{sites}} \int dx^i \ tr(J_i^+ \partial_i \hat{A}_i) \]

\[ J_i^+ \cdot \frac{1}{2g^2a^2} \left( M_i \hat{L}_i M_i + M_i \hat{L}_{i+1} \hat{L}_i M_i + 2^{\hat{i}} \hat{i} \right) \]

The Hamiltonian is:

\[ P_{SD\perp Q} \cdot \frac{fQ^i \cdot Q^i}{2} \]
Preliminary

Winding States in $N = 1$ SYM in 2+1 Dimensions

Spectrum of a free massless winding state

$$M^2 = m^2 + k^2 = m^2 + \left(\frac{2n_i}{L}\right)^2$$

where $L = aW$. $W$ is the winding number and therefore,

$$M^2 = m^2 + \left(\frac{2n_i}{aW}\right)^2$$

Interacting theory $M^2 = a + bk_\phi + ck_\phi^2$

![Figure 7](a) Mass spectrum vs 1/resolution for winding number 1 to 5 (b) Mass spectrum at $K = 1$ vs 1/winding number

the fit in (b) is $M^2 = A + \frac{B}{W} + \frac{C}{W^2}$
Summary

† SQLCQ exactly preserve supersymmetry and therefore simulations in 1+1 and 2+1 require no renormalization.

† A CS term gives a masses to the constituents.

† Zero modes give a degenerate vacuum and additional degree of freedom.

† The states of SYM are “stringy”.

† N = (8;8) SYM in 1+1 Dim. appears to be dual to SUGRA in 2+1 Dim.

† Massive SYM in 1+1 has QCD–like bound states that are nearly independent of the coupling – approximate BPS states. with a new Duality:

† These bound states in 2+1 dimensions are anomalously light bound states.

† SYM-CS with Fundamental matter has two bands of states and the low mass states are below threshold and their convergence is oscillatory

† We can formulate a nearly supersymmetric transverse lattice method. The winding states in the transverse direction have $M^2 = a + b \bar{W} + c \bar{W}^2$. 