Non-Supersymmetric
Gauge Gravity Dualities

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1. AdS/CFT Correspondence overview
2. Deforming $\mathcal{N} = 4$ SYM
3. Yang Mills* Geometry
4. Confinement, chiral symmetry breaking and glue balls

The AdS/CFT Correspondence

$\mathcal{N} = 4$ SYM

is dual to

IIB Strings (SUGRA) on $AdS_5 \times S^5$

Closed strings generate dynamical gravity background

Open Strings Generate surface gauge theory

$SO(6) \times SO(2, 4)$ symmetries match
Alternative Picture

5d SUGRA truncation - $S^5$ dropped

\[ ds^2 = \frac{du^2}{u^2} + u^2 \, dx_{\parallel}^2 \]

3+1d slices parallel to D3 on which field theory lives

What is $u$?

Dilatations in conformal $\mathcal{N} = 4$ SYM:

\[ \int d^4 x \, \partial^\mu \phi \partial_\mu \phi, \quad x \to e^{-\alpha} x, \quad \phi \to e^\alpha \phi \]

Become spacetime symmetry of AdS

\[ u \to e^\alpha u \]

$u$ is a continuous mass dimension

$\to$ RG Scale
The Magic

The SUGRA fields can be coupled in symmetry invariant ways to gauge theory fields at any 
$u = \text{const}$ slice

$$\int d^4x \; \Phi_{SUGRA}(u_0) \lambda \lambda$$

SUGRA field looks like a SOURCE

eg fermion mass

To be consistent with RG flow interpretation of $u$

$\Phi_{SUGRA}(u)$ must give RG flow of source...

CLAIM: The classical SUGRA EoM give non-perturbative RG flow of the field theory!

So to deform with masses or vevs must identify 5d SUGRA field and study solutions where it is non-zero
**Deformation Program**

- Identify by symmetries SUGRA field dual to interesting source
- Solve (numerically?) 5d SUGRA equations of motion
- Lift the solution to 10d (Pilch and Warner + work!)
- Brane probe the result to see physics encoded by metric

Probe brane samples geometry without disruption

**Dirac-Born-Infeld Action**

\[ S = -\tau_3 \int d^4\xi \sqrt{\text{det}G_{ab}} + \mu_3 \int d^4\xi \ C_{(4)} \]

eg the field theory scalar \((x^\mu)\) potential is

\[ V \sim G_{00}^2 - C_{(4)} \]
SUSY Example

Study scalar operator

\[ Tr \phi_i \phi_j = \text{diag}(1, 1, 1, 1, -2, -2) \]

Symmetries: \( 6 \times 6 = 20 + \ldots \) rep of SO(6)

Gravity is dynamical so when \( \rho(=e^\lambda) \neq 0 \) background can change

\[ ds^2 = e^{2A(r)} dx^\mu dx_\mu + dr^2 \]

Equations of Motion

\[ \lambda'' + 4A' \lambda' = \frac{\partial V}{\partial \lambda}, \quad 6A'^2 = \lambda' + 2V \]

Second order - describe operator vev and mass.

Vev only:
The lifted 10d metric
\[ ds^2 = \frac{X^{1/2}}{\rho} \left( e^{2A(r)} dx_{/\slash}^2 + dr^2 \right) + \frac{X^{1/2}}{\rho^2} \left( \frac{L^2}{\rho^2} \left[ d\theta^2 + \frac{\sin^2 \theta}{X} d\phi^2 + \frac{\rho^6 \cos^2 \theta}{X} d\Omega^2_3 \right] \right), \]
\[ X \equiv \cos^2 \theta + \rho^6 \sin^2 \theta \]
\[ C_4 = \frac{e^{4A} X}{\rho^2} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \]

(Slow moving) Probe action
\[ \int d^4x \left[ \frac{X e^{2A}}{\rho^2} \dot{r}^2 + \frac{L^2 e^{2A}}{\rho^4} (X \ddot{\theta}^2 + \sin^2 \theta \dot{\phi}^2 + \rho^6 \cos^2 \theta \dot{\Omega}_3^2) \right] \]

No potential - agrees with \( \mathcal{N} = 4 \) moduli space

After change of coordinates - this is of the familiar MULTI-CENTRE form
\[ ds^2 = H^{-1/2} dx_{/\slash}^2 + H^{1/2} \sum_{i=1}^6 du_i^2 \]
\[ C_4 = \frac{1}{H} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \]
\[ H(u) = \int d^6x \sigma(x) \frac{L^4}{|u-x|^4} \]
Simplist Non-Supersymmetric Deformation

First order EoM for $\rho$ are solution of second order EoM ($\lambda = \sqrt{6} \ln \rho$)

$$\lambda'' + 4A'\lambda' = \frac{\partial V}{\partial \lambda}$$

$$6A'^2 = \lambda'^2 - 2V$$

At large $r$ (AdS)

$$\lambda = Ae^{-2r} + Be^{-2r}$$

$A$ has conformal dimension 2 and is $tr\phi^2$ vev

$B$ also has conformal dimension 2 and is new

$$m^2(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 - 2\phi_5^2 - 2\phi_6^2)$$

Plots of $\rho$ vs $r$ for a variety of initial conditions on $\rho'$
10d Lift

Pilch and Warner’s metric ansatz is still good

\[ ds^2 = \frac{X^{1/2}}{\rho} e^{2A(r)} dx_{/\!/}^2 + \]
\[ \frac{X^{1/2}}{\rho} \left( dr^2 + \frac{L^2}{\rho^2} \left[ d\theta^2 + \frac{\sin^2 \theta}{X} d\phi^2 + \frac{\rho^6 \cos^2 \theta}{X} d\Omega_3^2 \right] \right) \]

\[ X \equiv \cos^2 \theta + \rho^6 \sin^2 \theta \]

but must recalculate 5-form using 2nd order EoM

\[ F_5 = \mathcal{F} + * \mathcal{F}, \quad \mathcal{F} = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dw(r, \theta) \]

\[ R_0^0 + R_r^r = \frac{1}{2} g^{00} g^{11} g^{22} g^{33} g^{rr} \left( \frac{\partial w}{\partial r} \right)^2 \]

\[ R_0^0 - R_r^r = \frac{1}{2} g^{00} g^{11} g^{22} g^{33} g^{\theta \theta} \left( \frac{\partial w}{\partial \theta} \right)^2 \]

\[ w(r, \theta) = \frac{e^{4A}}{\rho^2} - \frac{3 \sin^2 \theta \rho' e^{4A}}{\rho} - e^{4A} F(r) \]

\[ -2 - 2 \rho^6 = -4 \rho^2 A' + 4 \rho^4 F A' + \rho^4 F' + 2 \rho \rho' \]

The UV solution is

\[ F = \frac{1}{3} \left( \frac{1}{\rho} - \rho^5 \right) - \rho' + ... \]

can then solve numerically for \( F \) along flow
Brane Probing

A slow moving probe in the geometry sees

\[ V_{probe} = -e^{4A} \left[ \frac{X}{\rho^2} + \frac{3 \sin^2 \theta \rho'}{\rho} - \frac{1}{\rho^2} + F \right] \]

In the UV

\[ V = m^2 e^{2r} (2 - 6 \sin^2 \theta) + ... \]

vev does not appear - \( \mathcal{N} = 4 \) moduli space

mass term matches operator intended to include

The probe potential plotted over the \( r - \theta \) plane

for the mass only case (\( A = 0 \))

The duality seems to still hold!

Of course this case has an unbounded potential...
Towards QCD

\[ A^\mu \quad 4 \times \lambda \quad 6 \times \phi \]

\[
m(\lambda_1 \lambda_1 + \lambda_2 \lambda_2 + \lambda_3 \lambda_3 + \lambda_4 \lambda_4)\]

\[
m^2(\phi_1^2 + \ldots + \phi_6^2)\]

\[ A^\mu \text{ only - Yang Mills}^* \text{ Theory} \]

We introduce mass (or vev) for operator

\[ \lambda_1 \lambda_1 + \lambda_2 \lambda_2 + \lambda_3 \lambda_3 + \lambda_4 \lambda_4 \]
The Yang Mills* Geometry

\[ ds_{10}^2 = (\xi_+\xi_-)^{1/2} ds_{1,4}^2 + (\xi_+\xi_-)^{-3/2} ds_5^2 \]

\[ ds_{1,4}^2 = e^{2A} dx_{\perp/}^2 + dr^2 \]

\[ ds_5^2 = \xi_- \cos^2 \alpha \, d\Omega_+^2 + \xi_+ \sin^2 \alpha \, d\Omega_-^2 + \xi_+\xi_- \, d\alpha^2 \]

\[ d\Omega_{\pm}^2 = d\theta_{\pm}^2 + \cos^2 \theta_{\pm} \, d\phi_{\pm}^2 \]

\[ \xi_{\pm} = c^2 \pm s^2 \cos 2\alpha, \quad c = \cosh \lambda, \quad s = \sinh \lambda \]

\[ f = \frac{1}{\xi^{1/2}} \sqrt{\frac{\cosh^2 \lambda + (\xi_+\xi_-)^{1/2}}{2}}, \quad B = \frac{\sinh^2 \lambda \cos 2\alpha}{\cosh^2 \lambda + (\xi_+\xi_-)^{1/2}} \]

\[ A_{(2)} = iA_+(r, \alpha) \cos^3 \alpha \cos \theta_+ d\theta_+ \wedge d\phi_+ \]

\[ - A_-(r, \alpha) \sin^3 \alpha \cos \theta_- d\theta_- \wedge d\phi_- \]

\[ A_{\pm}(r, \alpha) = \sinh 2 \lambda / \xi_{\pm} \]

\[ F = \mathcal{F} + \ast \mathcal{F}, \quad \mathcal{F} = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\omega \]

\[ \omega(r) = e^{4A(r)} A'(r) \]
Probing

Substituting the geometry into DBI action gives

\[ V_{probe} = e^{4A} \left[ \xi - \xi' \right] \]

In UV with

\[ \lambda = \mathcal{M} e^{-r} + ..., \quad A = r + ... \]

\[ V = \mathcal{M}^2 |\phi|^2 + ... \]

all 6 scalars acquire mass radiatively

scalar potential is stable

IR plot

Gravity dual of Yang Mills*
Confinement and Glueballs

Identify SUGRA field that is source for
\[ \frac{1}{g^2} \quad \text{and} \quad \langle F_{\mu\nu} F_{\mu\nu} \rangle \]
\[ \rightarrow \Phi - \text{the dilaton} \]

\[ \delta \Phi = f(r) e^{ikx} \]
\[ k^2 = M^2 \]

5d dilaton EoM

\[ \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) \delta \Phi = 0 \]

Plus change of coords

\[ \frac{dz}{dr} = e^{2A} \quad f(r) \rightarrow e^{-3A/2} \psi \]

reduces to Schroedinger eigenvalue problem

\[ (-\partial_z^2 + V(z))\psi(z) = M^2 \psi(z) \]
Potential for flows fine tuned to mass only flow

If any condensate present then potential unstable

The mass only potential appears bounded though

Discrete glueballs → mass gap and confinement!

Can find regular solutions by “shooting” (at $z = 0$

$\psi \sim cz$, $A \sim -\log z$)

$M^2 = 36.7$

$M^2 = 21.7$

$M^2 = 10.3$
Comparison With Other Approaches

Witten’s Thermal Geometry: M5 branes wrapped on $S^2 (t&x_4)$ describes 5d gauge theory at strong coupling and finite temperature

Low energy theory is 4d pure glue

Csaki et al. have calculated glueball spectrum

$N = 3$ Lattice Results: Teper, Morningstar...

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<th>N=3 Lattice</th>
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Results surprisingly stable with errors at $\sim 10\%$ level!
Quarks and Chiral Symmetry Breaking

Quarks can be introduced via D7 branes in AdS

Extra scalars on D7 holographic to quark operators (Karch, Katz)

Currently studying EoM looking for

\[ \langle \bar{q}q \rangle \neq 0 \]
Overview

- Technology exists to deform AdS/CFT
  - 5d SUGRA
  - Lifting to 10d SUGRA
  - brane probes

- Non-supersymmetric deformations appear to make sense

- YM* solution
  - massive fermions by construction
  - scalar masses develop radiatively
  - stable
  - describes non-supersymmetric YM in IR

- Glueball spectrum under study

- Chiral symmetry breaking under study