Determination of Parton Distribution Functions

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Generalized Parton Distributions and Hard Exclusive Processes

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Contents

• Introduction

• Polarized PDFs in the nucleon
  AAC analysis (2000, new)
  Comments on \( v \) factory

• PDFs in nuclei

• Summary
Why PDFs?

(1) basic interest to understand hadron structure
   - perturbative & non-perturbative QCD
     e. g. spin is a fundamental quantity

(2) practical purpose: to describe hadron cross sections precisely
   For hadron reactions with $Q^2 > 1 \text{ GeV}^2$, accurate PDFs are needed.
   For example
   - exotic events at large $Q^2$: physics “beyond QCD”
   - heavy-ion reactions: quark-gluon plasma signature
   - neutrino reactions: $\nu + \text{O}$ (neutrino properties)
   -...
Situation of PDFs?

(1) **unpolarized PDFs in the nucleon**
   3 major groups (CTEQ, GRV, MRST)
   → well established from small $x$ to large $x$

(2) **polarized PDFs in the nucleon**
   several groups
   → not established

(3) **PDFs in nuclei**
   only a few papers
Parton distributions are determined by fitting various experimental data.

- electron/muon \( \mu + p \rightarrow \mu + X \)
- neutrino \( \nu_\mu + p \rightarrow \mu + X \)
- Drell-Yan \( p + p \rightarrow \mu^+\mu^- + X \)
- direct photon \( \mu/p + p \rightarrow \gamma + X \)
- ...

1. Assume parton distributions at \( Q_0^2 (\sim 1 \text{ GeV}^2) \)
   
   \[ f_i(x, Q_0^2) = A_i x^{\alpha_i} (1 - x)^{\beta_i} (1 + \gamma_i x) \]
   
   where \( i = u, d, \bar{u}, \bar{d}, s, g \)

2. Calculate structure functions at experimental \( Q^2 \) points

3. Then, \( A_i, \alpha_i, \beta_i, \gamma_i \) are determined in comparison with data
Recent unpolarized distributions

see http://durpdg.dur.ac.uk/hepdata/pdf.html


Determination of Polarized Parton Distribution Functions

AAC (Asymmetry Analysis Collaboration) studies on the polarized PDFs

M. Hirai et al., to be submitted for publication.

http://spin.riken.bnl.gov/aac/
(Polarized PDF codes could be obtained from this site.)
• proton-spin issue

polarized e/µ-proton scattering
→ measurement of \( g_1^{LO} \)
\[
g_1^{LO} = \frac{1}{2} \sum_i e_i^2 (\Delta q_i + \Delta \bar{q}_i)
\]
proton, deuteron, \(^3\text{He}\) \( g_1 \) data
with isospin symmetry
→ valence and sea polarization
\( \Delta u_v, \Delta d_v, \Delta \bar{q} \)
quark spin content
\( \Delta \Sigma = \Delta u_v + \Delta d_v + 6 \cdot \Delta \bar{q} \)
experimentally
\[
\int_0^1 dx \Delta \Sigma(x) \approx 0.1 - 0.3
\]
rest of the spin ????


### Experimental data

<table>
<thead>
<tr>
<th>Target</th>
<th>Exp.</th>
<th>x</th>
<th>Q^2 GeV^2</th>
<th>Data #</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>EMC</td>
<td>0.015-0.466</td>
<td>3.5~29.5</td>
<td>10</td>
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<tr>
<td></td>
<td>SMC</td>
<td>0.005-0.480</td>
<td>0.25~72.07</td>
<td>59</td>
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<tr>
<td></td>
<td>E130</td>
<td>0.18-0.70</td>
<td>3.5~10.0</td>
<td>8</td>
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<tr>
<td></td>
<td>E143</td>
<td>0.022-0.847</td>
<td>0.28~9.53</td>
<td>81</td>
</tr>
<tr>
<td></td>
<td>HERMES</td>
<td>0.028-0.66</td>
<td>1.01~7.36</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>SMC</td>
<td>0.005-0.480</td>
<td>1.3~54.4</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>E143</td>
<td>0.022-0.847</td>
<td>0.28~9.53</td>
<td>81</td>
</tr>
<tr>
<td></td>
<td>E155</td>
<td>0.015-0.75</td>
<td>1.22~34.79</td>
<td>24</td>
</tr>
<tr>
<td>Neutron</td>
<td>E142</td>
<td>0.035-0.466</td>
<td>1.1~5.5</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>E154</td>
<td>0.0174-0.5643</td>
<td>1.21~15.0</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>HERMES</td>
<td>0.033-0.464</td>
<td>1.22~5.25</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td>375</td>
</tr>
</tbody>
</table>

The graph shows a scatter plot of Q^2 (GeV^2) vs. x for various experiments and targets, indicating the data points for different experiments (E130, E143, EMC, SMC, HERMES) and targets (protons, deuterons, neutrons). The data points are color-coded to distinguish between different experiments and targets.
Initial distributions

\[ \Delta f_i(x,Q_0^2) = A_i x^{\alpha_i} (1 + \gamma_i x^{\lambda_i}) f_i(x,Q_0^2) \]

\[ i = u_v, d_v, \bar{q}, g \quad A_i, \alpha_i, \gamma_i, \lambda_i: \text{parameters} \]

\[ \chi^2 \text{ fit to the data [p, n (}^3\text{He)}, d] \]

\[ \chi^2 = \sum_i \frac{(A_{1i}^{\text{data}} - A_{1i}^{\text{calc}})^2}{(\sigma_{A_{1i}}^{\text{data}})^2} \]

\[ A_1 \approx \frac{g_1}{F_1} = g_1 \frac{2 x (1 + R)}{F_2} \]

\[ R = \frac{F_L}{2 x F_1} = \frac{F_2 - 2 x F_1}{2 x F_1} \]

We analyzed with the following conditions.

- unpolarized PDF \( \text{GRV98} \)
- initial \( Q^2 \) \( Q_0^2 = 1 \text{ GeV}^2 \)
- number of flavor \( N_f = 3 \)
- positivity \( |\Delta f(x)| \leq f(x) \) (to be precise, \( |\Delta \sigma| \leq \sigma \))
- antiquark flavor: \( \Delta u = \Delta d = \Delta s \)
Results

Total $\chi^2$ LO $\chi^2$/d.o.f. = 0.896
NLO $\chi^2$/d.o.f. = 0.834

Total data 375

Spin asymmetry $A_{1p}$

$Q^2 = 5 \text{ GeV}^2$

Diagram showing $A_{1p}$ as a function of $x$. Data points from various experiments are plotted, including LO, NLO, E130, E143, EMC, SMC, and HERMES.
Neutron ($^3$He)

$Q^2 = 5 \text{ GeV}^2$

Deuteron

$Q^2 = 5 \text{ GeV}^2$
Q² dependence of $A_1^p$

Parton distributions (Q²=1 GeV²)
First moments \((Q^2 = 1 \text{ GeV}^2)\)

<table>
<thead>
<tr>
<th></th>
<th>(\Delta u_v)</th>
<th>(\Delta d_v)</th>
<th>(\Delta \bar{q})</th>
<th>(\Delta g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>0.926</td>
<td>-0.341</td>
<td>-0.064</td>
<td>0.831</td>
</tr>
<tr>
<td>NLO</td>
<td>0.926</td>
<td>-0.341</td>
<td>-0.089</td>
<td>0.532</td>
</tr>
</tbody>
</table>

Spin content \(\Delta \Sigma\)
LO : 0.201  
NLO : 0.051  

rather small spin content in the NLO, \(\Delta \Sigma = 0.1\sim 0.3\) ?

→ check the antiquark distribution
"Spin content" $\Delta \Sigma$

$$\Delta \Sigma(x_{min}) = \int_{x_{min}}^{1} \Delta \Sigma(x) \, dx$$

$$\frac{\Delta q}{q} \propto x^{\alpha_q} \quad (x \to 0)$$
AAC studies in progress

(1) re-analysis
   with SLAC-E155 (proton)

(2) errors of the polarized PDFs

by M. Hirai et. al.
Results

• Total $\chi^2$  
  New : $\chi^2$/d.o.f. = 346.33 (0.900)  
  $\Delta g(x)=0$ : $\chi^2$/d.o.f. = 355.01 (0.922)

• First moments  
  ($Q^2 = 1$ GeV$^2$, $\overline{\text{MS}}$ scheme)

<table>
<thead>
<tr>
<th></th>
<th>$\Delta u_v$</th>
<th>$\Delta d_v$</th>
<th>$\Delta q$</th>
<th>$\Delta g$</th>
<th>$\Delta \Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>0.926</td>
<td>-0.341</td>
<td>-0.062 ± 0.023</td>
<td>0.499 ± 1.268</td>
<td>0.213 ± 0.138</td>
</tr>
<tr>
<td>$\Delta g(x)=0$</td>
<td>(fixed)</td>
<td>(fixed)</td>
<td>-0.054 ± 0.011</td>
<td>0.00</td>
<td>0.259 ± 0.063</td>
</tr>
<tr>
<td>AAC00 (NLO-2)</td>
<td></td>
<td></td>
<td>-0.057 ± 0.038</td>
<td>0.532 ± 1.949</td>
<td>0.241 ± 0.228</td>
</tr>
</tbody>
</table>

LSS01 (MS) : $\Delta g = 0.680$, $\Delta \Sigma = 0.210$
GRSV01 : 0.427, 0.204
BB02 (SET4) : 0.931, 0.150
Proton spin asymmetry $A_1^P$

preliminary!
New results vs. AAC2000

- $\Delta d_v(x)$ is almost the same as AAC2000
- $\Delta u_v(x)$, $\Delta q(x)$ and $\Delta g(x)$ are slightly changed by the E155 proton data

preliminary!
Errors of the PDFs

reduction of the error band due to the E155 data

NEW AAC00

E155 data

correlation with $\Delta q(x)$

preliminary!
Analysis with $\Delta g(x) = 0$

preliminary!

The error band shrinks due to the correlation with $\Delta g(x)$.
Summary: AAC determination of the polarized PDFs

(1) 2000 version

- $Q^2$ dependence of $A_1$ especially at small $Q^2$
- positivity condition is taken into account (unless, unphysical result: $|\Delta \sigma| >$
- issue of $\Delta \bar{q}(x)$ at small and large $x$
  - $\Delta \bar{q}(x \to 0)$ issue $\to$ the quark spin content $\Delta \Sigma$
- The obtained PDFs are available from http://spin.riken.bnl.gov/aac/.

(2) new analysis (2003)

- include E155 (p) data, errors of the polarized PDFs
  - Errors of $\Delta \bar{q}$ and $\Delta g$ become smaller; however,
    - $\Delta \bar{q}$ and $\Delta g$ are not well determined (especially $\Delta g$).
    - $\Delta g$ error is correlated with $\Delta \bar{q}$ error, $\Delta \Sigma = 0.213 \pm 0.136$, $\Delta g = 0.468 \pm 1$
- analysis with RHIC $\gamma$ pseudo-data
  - Including the pseudo-data in our $\chi^2$ analysis,
Prospects

(1) new data are needed for the PDF determination

- fortunately, experiments are going on
  JLab, RHIC-Spin, COMPASS, HERMES, ...
- these new data should lead to accurate determination
  of the polarized PDFs (bright prospects!)

(2) possibilities in Japan

- J-PARC (Japan Proton Accelerator Research Complex
  primary proton beam: large-x physics
- Neutrino Factory (also in Europe / US)
  valence polarization, spin content, strange,
Comments on polarized PDFs in $\nu$ scattering
Polarized neutrino-proton scattering (CC)

\[ W_{\mu \nu} = (-g_{\mu \nu} + \frac{q_{\mu}q_{\nu}}{q^2}) F_1 + \frac{\hat{p}_\mu \hat{p}_\nu}{p \cdot q} F_2 - i \varepsilon_{\mu \nu \lambda \sigma} \frac{q^\lambda p^\sigma}{2p \cdot q} F_3 \]

where \( \hat{p}_\mu = p_\mu - \frac{p \cdot q}{q^2} q_\mu \)

\[ + i \varepsilon_{\mu \nu \lambda \sigma} \frac{q^\lambda s^\sigma}{p \cdot q} g_1 + i \varepsilon_{\mu \nu \lambda \sigma} \frac{q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma)}{(p \cdot q)^2} g_2 \]

\[ + \left[ \frac{\hat{p}_\mu s_\nu + \hat{s}_\mu \hat{p}_\nu - s \cdot q \hat{p}_\mu \hat{p}_\nu}{2p \cdot q} \right] g_3 + \frac{s \cdot q \hat{p}_\mu \hat{p}_\nu}{(p \cdot q)^2} g_4 + (-g_{\mu \nu} + \frac{q_{\mu}q_{\nu}}{q^2}) \frac{s \cdot q}{p \cdot q} g_5 \]

new structure functions \( g_3, g_4, g_5 \)

be careful about “various” definitions of \( g_3, g_4, g_5 \)!

\[ \frac{d(\sigma_{\lambda_{\mu}=-1}^{CC} - \sigma_{\lambda_{\mu}=+1}^{CC})}{dx \, dy} = \frac{G_F^2 Q^2}{\pi (1 + Q^2/M_W^2)^2} \left\{ [\lambda_{\ell} y(2-y)x g_1^{CC} - (1-y)g_4^{CC} - y^2 x g_5^{CC}] \right\} \]

\[ + 2xy \frac{M^2}{Q^2} \left[ \lambda_{\ell} x^2 y^2 g_1^{CC} + \lambda_{\ell} 2x^2 y g_2^{CC} + \left(1 - y - x^2 y^2 \frac{M^2}{Q^2} \right) x g_3^{CC} \right] \]

\[ - x \left(1 - \frac{3}{2} y - x^2 y^2 \frac{M^2}{Q^2} \right) g_4^{CC} - x^2 y^2 g_5^{CC} \}

0 at \( Q^2 \gg M^2 \)
### g₁, g₄, g₅ in parton model (CC)

\[ g_4 = 2 \times g_5 \]

\[ g_{vp} = +\Delta d + \Delta s + \Delta \bar{u} + \Delta \bar{c}, \quad g_{i vp} = +\Delta u + \Delta c + \Delta \bar{d} + \Delta \bar{s} \]

\[ g_{vp} = -\Delta d - \Delta s + \Delta \bar{u} + \Delta \bar{c}, \quad g_{i vp} = -\Delta u - \Delta c + \Delta \bar{d} + \Delta \bar{s} \]

\[ g_{5v}^v + g_{5\bar{v}}^\bar{v} = -(\Delta u_v + \Delta d_v) - (\Delta s - \Delta \bar{s}) - (\Delta c - \Delta \bar{c}) \]

**determination of valence polarization**

\[ g_{v(p+n)/2}^v - g_{5(v(p+n)/2)}^\bar{v} = - (\Delta s + \Delta \bar{s}) + (\Delta c + \Delta \bar{c}) \]
Possibilities at $\nu$ factory

$$x(g_1^{\overline{V}p} - g_5^{\overline{V}p})/2$$

$$x(g_1^{\nu p} + g_5^{\nu p})/2$$

S. Forte, M. L. Mangano, G. Ridolfi
**Quark spin content**

$e/\mu$ scattering $\rightarrow \Delta \Sigma = 0 \sim 30\%$

It is not uniquely determined.

**v scattering**

$$g_1^{vp} + g_1^{\bar{v}p} = (\Delta u + \Delta \bar{u}) + (\Delta d + \Delta \bar{d})$$

$$+ (\Delta s + \Delta \bar{s}) + (\Delta c + \Delta \bar{c})$$

in LO $\int dx (g_1^{vp} + g_1^{\bar{v}p}) = \Delta \Sigma$

**independent determination of quark spin content $\Delta \Sigma$**!
Determination of
Nuclear Parton Distribution Functions

http://hs.phys.saga-u.ac.jp/nuclp.html

(Nuclear PDF codes could be obtained from this site.)

Refs. (1) M. Hirai, SK, M. Miyama,
(2) to be submitted for publication.
Today’s talk on
• $\chi^2$ analysis method, used data
• results

Purposes
• nuclear mechanisms in the high-energy region
• heavy-ion reactions: quark-gluon plasma signature
• neutrino physics: nuclear effects in $\nu + ^{16}\text{O}$
Nuclear modification

Nuclear modification of $F_2^A / F_2^D$ is well known in electron/muon scattering.

\[ F_2^A = \sum_i e_i^2 \times \left[ q_i(x) + \bar{q}_i(x) \right]_A \]

The graph shows the ratio of $F_2^A$ to $F_2^D$ for different experiments (EMC, NMC, E139, E665). The data points indicate shadowing effects and Fermi motion. The x-axis represents the momentum transfer $x$, with sea quarks and valence quarks indicated. The EMC finding is highlighted with an arrow.
Nuclear parton distributions (per nucleon) if there were no modification

\[ A \ u^A = Z \ u^p + N \ u^n, \quad A \ d^A = Z \ d^p + N \ d^n \]

Isospin symmetry: \( u^n = d^p \equiv d \), \( d^n = u^p \equiv u \)

\[ \rightarrow u^A = \frac{Z \ u + N \ d}{A}, \quad d^A = \frac{Z \ d + N \ u}{A} \]

Take into accout the nuclear modification by the factors \( w_i(x,A) \)

\[ u^A_v(x) = w_{u_v}(x,A) \frac{Z \ u_v(x) + N \ d_v(x)}{A} \]
\[ d^A_v(x) = w_{d_v}(x,A) \frac{Z \ d_v(x) + N \ u_v(x)}{A} \]
\[ \bar{q}^A(x) = w_{\bar{q}}(x,A) \ \bar{q}(x) \]
\[ g^A(x) = w_g(x,A) \ g(x) \]
A dependence


\[ R = r_0 A^{1/3} \]

"volume" roughly speaking \[ \sigma_A = A \sigma_v + A^{2/3} \sigma_s \]

\[ \frac{\sigma_A}{A} = \sigma_v + \frac{1}{A^{1/3}} \sigma_s \]

\[ \sim \frac{1}{A^{1/3}} \] dependence

\[ \begin{array}{c|cccccc}
F_A/F_D \\
\hline
F_2 & H & e & e & e & e & e \\
0.3 & 0.96 & 0.95 & 0.94 & 0.93 & 0.92 & 0.91 \\
0.4 & 0.96 & 0.95 & 0.94 & 0.93 & 0.92 & 0.91 \\
0.5 & 0.96 & 0.95 & 0.94 & 0.93 & 0.92 & 0.91 \\
0.6 & 0.96 & 0.95 & 0.94 & 0.93 & 0.92 & 0.91 \\
0.7 & 0.96 & 0.95 & 0.94 & 0.93 & 0.92 & 0.91 \\
0.8 & 0.96 & 0.95 & 0.94 & 0.93 & 0.92 & 0.91 \\
0.9 & 0.96 & 0.95 & 0.94 & 0.93 & 0.92 & 0.91 \\
\end{array} \]

\[ x = 0.5, Q^2 = 5 \text{ GeV}^2 \]
Functional form of $w_i(x,A)$

$$f_i^A(x) = w_i(x,A) \ f_i(x), \quad i = u, v, d, q, g$$

first, assume the $A$ dependence as $1/A^{1/3}$

then, use

$$w_i(x,A) = 1 + (1 – 1/A^{1/3}) \frac{a_i+b_i x+c_i x^2+d_i x^3}{(1 – x)^{\beta_i}}$$

$a_i, b_i, c_i, d_i, \beta_i$: parameters to be determined by $\chi^2$ analysis

Fermi motion: $\frac{1}{(1 – x)^{\beta_i}} \to \infty$ as $x \to 1$ if $\beta_i > 0$

Shadowing: $w_i(x \to 0, A) = 1 + (1 – 1/A^{1/3}) a_i < 1$

Fine tuning: $b_i, c_i, d_i$
Constraints

- **Nuclear charge**

\[
Z = A \int dx \left[ \frac{2}{3} (u^A - \bar{u}^A) - \frac{1}{3} (d^A - \bar{d}^A) - \frac{1}{3} (s^A - \bar{s}^A) \right]
\]

\[
= A \int dx \left( \frac{2}{3} u_v^A - \frac{1}{3} d_v^A \right)
\]

- **Baryon number**

\[
A = A \int dx \frac{1}{3} (u_v^A + d_v^A)
\]

- **Momentum**

\[
A = A \int dx x (u_v^A + d_v^A + 6 \bar{q}^A + g^A)
\]

Three parameters can be determined by these conditions.
Experimental data

(1) $F_2^A / F_2^D$
NMC: $^4$He, Li, C, Ca
SLAC: $^4$He, Be, C, Al, Ca, Fe, Ag, Au
EMC: C, Ca, Cu, Sn
E665: C, Ca, Xe, Pb
BCDMS: N, Fe
HERMÈSES: $^3$He, N, Kr

(2) $F_2^A / F_2^{A'}$
NMC: Be / C, Al / C, Ca / C, Fe / C, Sn / C, Pb / C, C / Li, Ca / Li

(3) $\sigma_{DY}^A / \sigma_{DY}^{A'}$
E772: C / D, Ca / D, Fe / D, W / D
E866: Fe / Be, W / Be
Analysis conditions

- parton distributions in the nucleon
  
  \( \text{MRST01 - LO (}\Lambda_{\text{QCD}}=220 \text{ MeV)} \)

- \( Q^2 \) point at which the parametrized PDFs are defined: \( Q^2 \geq 1 \text{ GeV}^2 \)

- used experimental data: \( Q^2 \geq 1 \text{ GeV}^2 \)

- total number of data: \( 1106 \)

  \[ 761 \left( F_2^A/F_2^D \right) + 293 \left( F_2^A/F_2^{A'} \right) + 52 \text{ (Drell-Yan)} \]

- subroutine for the \( \chi^2 \) analysis: \text{CERN - Minuit}

\[
\chi^2 = \sum_i \frac{\left( R_i^{\text{data}} - R_i^{\text{calc}} \right)^2}{\left( \sigma_i^{\text{data}} \right)^2}
\]

\[
R = \frac{F_2^A}{F_2^D}, \quad \frac{F_2^A}{F_2^{A'}}, \quad \frac{\sigma_{\text{DY}}^{PA}}{\sigma_{\text{DY}}^{PA'}}, \quad \sigma_i^{\text{data}} = \sqrt{\left( \sigma_i^{\text{sys}} \right)^2 + \left( \sigma_i^{\text{stat}} \right)^2}
\]
Analysis results

small nuclei

Be/D

C/D

Preliminary

$F_2^{Be} / F_2^{D}$

$F_2^{C} / F_2^{D}$

$Q^2 = 5 \text{ GeV}^2$

$\times$

NMC

EMC

NMC

E139

E665

preliminary
medium-size nuclei

Ca/D

Fe/D

preliminary

EMC
NMC
E87
E139
E140
E665

Q² = 5 GeV²

preliminary

x

x
large nuclei

**Au/D**

**Pb/D**

---

$p_{E139}$

$q^2 = 5$ GeV$^2$

---

$p$E665

preliminary
\[ \frac{F_2^A}{F_2^{A'}} \]

Ca/C

Pb/C

\[ Q^2 = 5 \text{ GeV}^2 \]
Drell-Yan

Fe/D

Fe/Be

preliminary

$Q^2 = 50 \text{ GeV}^2$

$\frac{\sigma_{DY}^{Fe}}{\sigma_{DY}^{D}}$

$\frac{\sigma_{DY}^{Be}}{\sigma_{DY}^{Be}}$

$\bar{q}^{Fe}/\bar{q}^{D}$

$\sigma_{DY}^{Fe}/\sigma_{DY}^{D}$
Nuclear corrections for Ca

\[ w(Ca, x) \]

\[ Q^2 = 1 \text{ GeV}^2 \]
Comments on Future Experimental Studies of Nuclear PDFs
Valence quark \( \frac{1}{2} [F_3^{\nu N} + F_3^{\nu N}]_{CC} \approx u_\nu + d_\nu \)

- test of shadowing models
  - \( F_3 \) (valence) vs. \( F_2 \) (sea) shadowing
- accurate determination of nuclear PDFs
Studies at hadron facilities

e.g. Drell–Yan: \[ x_1 x_2 = \frac{m_{\mu\mu}^2}{s} \]

\[ x \approx \frac{\sqrt{m_{\mu\mu}^2}}{\sqrt{s}} \]

- \( s = (p_1 + p_2)^2 \)

RHIC: \( \sqrt{s} = 0.2 \text{ TeV} \)
LHC: \( \sqrt{s} = 5.5 \text{ TeV} \)

- pQCD: \( Q^2 \geq \text{a few GeV}^2 \)

\[ x \approx \frac{\sqrt{m_{\mu\mu}^2}}{\sqrt{s}} \geq \frac{1}{200} = 0.005 \text{ RHIC} \]
\[ \geq \frac{1}{5500} = 0.0002 \text{ LHC} \]
Antiquark distributions

J-PARC / Fermilab

\( \frac{\sigma_{DY_{Fe}}}{\sigma_{DY_{D}}} \)

\( x \)
Gluon distributions

$w(Ca, x)$

$\begin{array}{c}
gluon \\
eRHIC \\
LHC \\
RHIC 
\end{array}$
Summary

(1) $\chi^2$ analysis for the nuclear PDFs

Computer codes could be obtained from http://hs.phys.saga-u.ac.jp/nuclp.html.

(2) Nuclear PDF studies are still premature.

→ analysis refinements
→ experimental efforts:

RHIC, LHC, eRHIC, JPARC, $\nu$ factory, …