What can we learn about spin-dependent GPDs from the Lattice?

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LHPC, MILC & SESAM Collaborations
QCD

- Pert. Theory
- Effect. models
- Lattice

High energy

Low energy

OPE

- Form factors, Structure functions
- Hadron masses, decay constants

Field configs.

Measurements
Overview

• Parameterizing soft physics
• Using the Lattice for soft physics
• Recovering “known” quantities
• Latest news from the chiral regime
• Summary and Outlook
e.g.: \( e + p \rightarrow e + p + \gamma \)
Factorization ansatz

\[ t \equiv \Delta^2 = (p' - p)^2 \]
Forward limit

$\Rightarrow$ Recover forward parton distributions
Local limit

$\vec{p}' \quad \gamma^* \quad \vec{p}$

$\Rightarrow$ Recover form factors
QCD matrix elements

Matrix element:

\[ \bar{p}^+ \int \frac{dz^-}{2\pi} e^{ip^+ z^-} \langle p' | \bar{\psi} (-z^-/2) \gamma_5 \gamma^+ \psi (z^-/2) | p \rangle = \tilde{H}(x, \xi, t) \langle \gamma_5 \gamma^+ \rangle - \tilde{E}(x, \xi, t) \frac{\Delta^+}{2m} \langle \gamma_5 \rangle \]
Interpreting GPDs

- quark emitted and absorbed with l.m.f. $x + \xi$ and $x!\xi$

- quark/antiquark pair is emitted with l.m.f. $x + \xi$ and $\xi!x$
Types of GPDs

• Three fermion GPDs:

\[ \langle p' | \bar{\psi} \gamma^{\mu} \psi | p \rangle \Rightarrow H(x, \xi, t) & E(x, \xi, t) \]

\[ \langle p' | \bar{\psi} \gamma^{\mu} \gamma^{5} \psi | p \rangle \Rightarrow \tilde{H}(x, \xi, t) & \tilde{E}(x, \xi, t) \]

\[ \langle p' | \bar{\psi} \sigma^{\mu \alpha} \psi | p \rangle \Rightarrow H_{T_q}(x, \xi, t) & E_{T_q}(x, \xi, t) \]

• Plus three gluon GPDs
GPDs on the Lattice

GPDs are non local objects
On the Lattice: we can only measure local matrix elements

⇒ use light cone OPE
reexpress GPDs in terms of generalized local currents

\[ \langle p | O_{q}^{\{ \mu_{1} \ldots \mu_{n} \}} | p \rangle \]

\[ O_{q}^{\{ \mu_{1} \ldots \mu_{n} \}} = \bar{\psi}_{q} \gamma^{\{ \mu_{1} iD^{\mu_{2}} \ldots iD^{\mu_{n}} \}} \psi_{q} \]
GPDs from GFFs

Similarly:

$$\langle p' | O_q \{ \mu_1 \cdots \mu_n \} | p \rangle$$

contains information on the 1st moment of non-forward GPD via expansion in terms of Generalized Form Factors.
In practice: Matrix elements

Get matrix element $\langle P'|\mathcal{O}|P\rangle$ from ratio

$$R_{\mathcal{O}}(\tau, P', P) = \frac{C^3_{\mathcal{O}}(\tau, P', P)}{C^2_{pt}(\tau_{snk}, P')} \times \left[ \frac{C^2_{pt}(\tau_{snk} - \tau + \tau_{src}, P)}{C^2_{pt}(\tau_{snk} - \tau + \tau_{src}, P')} \frac{C^2_{pt}(\tau, P')}{C^2_{pt}(\tau, P)} \frac{C^2_{pt}(\tau_{snk}, P')}{C^2_{pt}(\tau_{snk}, P)} \right]^{1/2}$$

This ratio ensures the correct cancellation of wave function normalization and exponential factors for smeared sources and sinks.
In practice: GFFs

Get generalized form factors from continuum expression at fixed virtuality but not necessarily at fixed external momenta! e.g.

\[
\langle P' | \bar{\psi}_q \gamma^\{\mu \gamma_5 i D^n\} \psi_q | P \rangle = \\
\tilde{A}_2^q(t) \langle \gamma^\{\mu \gamma_5\} \bar{p}^\nu \rangle + \tilde{B}_2^q(t) \frac{i}{2m} \langle \gamma_5 \rangle \bar{p}^\{\mu \Delta \nu\}
\]

Then: Use all available index combinations and external momenta at fixed \( t \) and compute the GFFs
Merits of our approach

• $x$-dependence similar to forward parton dist. reconstruct via inverse Mellin transform

• polynomiality condition: $\xi$-dependence fully under control

• $t$-dependence under good control by using different known virtualities

• Lattice method allows for model-independent and assumption-free assessment of GPDs
Problems and shortcomings

• **Primary concern:** Fermions still far from chiral regime

• **Renormalization** continuum limit and lattice artifacts

• **Theoretical understanding** of partially quenching

• **We will see:** Problems can be hoped to be resolved in the near future
What are sea and valence quarks?

FIG. 1. Connected (upper row) and disconnected (lower row) diagrams contributing to hadron matrix elements. The left column shows typical contributions of quarks and the right column shows contributions of antiquarks.

Figure taken from PRD66, 034506 2002
Partially quenched $\chi$PT

Unquenched Theories:
$m_{sea} = m_{val}$

QCD

$\frac{m_{valence}}{m_{strange}}$ vs. $\frac{m_{sea}}{m_{strange}}$

PQ Chiral Pert. Theory

Chiral Pert. Theory

Lattice Simulations

Figure taken from PRD62, 094503 2000
Current status of PQ\(\chi\)PT

- Expressions available for spectroscopy
- finite volumes
- finite and separate lattice spacings
- finite within range of PQ\(\chi\)PT quark masses
- different species of sea/valence quarks
  Wilson/GW and Clover/GW

- **To do:** Matrix elements with Staggered/GW
Fermion discretizations

- **Wilson fermions**
- Staggered fermions
- Clover! improvement
- Ginsparg!Wilson fermions
  - Domain!wall
  - Overlap

Simple, well understood

Practically very important question!
Fermion discretizations

- Wilson fermions
- **Staggered fermions**
- Clover!improvement
- **Ginsparg!Wilson fermions**
  - **Domain!wall**
  - Overlap

**Cheap**

Practically very important question!

**Chiral, \( O a^2 \), but expensive**
Our calculation

- Wilson fermions for sea+valence quarks
- Moderate/cheap price
- Renormalization and continuum limit understood
- Difficult for light quark masses
- $O(a)$ cut-off effects
Exploratory study

- Staggered sea and Domain!Wall valence quarks
- Moderate/expansive price
- Renormalization/continuum limit not yet understood
- Conceptual question!mark for staggered quarks
- Light quarks possible until finite size effects show up
- $O \, a^2$ cut-off effects
Numerical Results

- Unquenched calculation, SESAM & MILC lattices
- Two & Three sea quarks
- Five quark masses
- GFFs for n=1,2

**SESAM lattices**

\[ \Omega = 32 \times 16^3, \beta = 5.6, \]

\[ N_f = 2, \text{ Wilson ferm. } \kappa_{\text{sea}} = \kappa_{\text{val}}, \]

\[ \kappa_{\text{val}}^1 = 0.1560, \]

\[ \kappa_{\text{val}}^2 = 0.1565, \]

\[ \kappa_{\text{val}}^3 = 0.1570, \]

\( O(200) \) configs each
Numerical Results

- Unquenched calculation, SESAM & MILC lattices
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MILC lattices
\[ \Omega = 32 \times 20^3, \beta = 6.85, \]
HYP-smearing,
\[ N_f = 3, \text{Staggered/DWF}, \]
\[ a m_s = 0.05, \]
\[ a m_{u+d} = 0.05, \]
\[ \mathcal{O}(100) \text{ configs} \]
Numerical Results

- Unquenched calculation, SESAM & MILC lattices
- Two&Three sea!quarks
- Five quark masses
- GFFs for $n=1,2$

MILC lattices
\[ \Omega = 32 \times 20^3, \beta = 6.76, \]
HYP-smearing,
\[ N_f = 2 + 1, \text{Staggered/DWF}, \]
\[ a m_s = 0.05, \]
\[ a m_{u+d} = 0.01, \]
\[ O(100) \text{ configs} \]
Sample plateau of ratios
Axial form factor
Axial GPD

\[ A_2^{ud}(t) \]
Moment!dependence

\[ A_n^{u-d}(-t) \]

\[ A_1^{u-d}(-t) \]

\[ A_2^{u-d}(-t) \]
Axial $ff$ dipole masses

![Graph showing experimental $\tilde{A}_1^{u-d}(-t)$ and $\tilde{A}_1^{u-d}(-t)$ data points.](image)
Axial $u+d$ form factor

\[ \hat{A}_{1}^{u+d}(-t) = D S \]
Pseudoscalar form factor

\[ \tilde{B}_{1}^{u-d}(-t) \] vs. \[-t \/ \text{GeV}^2\]
Pseudoscalar coupling

\[ \hat{B}^{d-u}(0) = \hat{g}_P \left[ M^u = 0 \right] \]

\[ \text{LHPC/SESAM (Wilson)} \]

\[ \text{Experiment} \]
First moment of GPD

\[ \tilde{A}_2^{u-d}(0) = \langle x\rangle_{u-d} \]

- LHPC/SESAM (Wilson)
- LHPC/MILC (DWF, prel. unren.)
- Experiment
Axial coupling

$A_{1}^{u-d}(0) = g_{A}$

- LHPC/SESAM (Wilson)
- LHPC/MILC (DWF, prel. unren.)
- Experiment

$m_{\pi}^{2} / \text{GeV}^{2}$
Summary

- GPDs are accessible primarily by Lattice Simulations
- Primary concern is the chiral extrapolation
- First attempt to address the primary concern BUT: results not conclusive so far!
Outlook

- Need to improve statistics and compute more data points
- Need to compute renormalization constants
- Need to improve theoretical understanding of staggered sea quarks and domain wall valence quarks