Data vs. Theory

Failure & Success of GPD models

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• Exclusive Processes & GPDs
• The Double Distribution Model
• The Aligned Jet/Forward Model
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Exclusive processes and GPDs

All hard (large momentum scale), exclusive reactions are characterized by:

→ large rapidity gap

→ momentum transfer onto proton, \( t = (p_{in} - p_{out})^2 \)

rel. transverse position of probed structure. \( \uparrow \)

Further common characteristic: A Factorization Theorem

\[
\mathcal{T} = T \otimes H + \text{terms } O(m/Q)
\]

\( T(\tilde{T}) \): perturbatively calculable to all orders, infrared safe.

\( H(\tilde{F}) \): nonperturbative, large distance structure info in proton.
In DIS:

\[ \tilde{F} = \text{FT} \sum_X \langle p|\overline{\psi}(-z)\gamma^+|X\rangle \langle X|\psi(z)|p \rangle = \text{FT} \langle p|\overline{\psi}(-z)\gamma^+\psi(z)|p \rangle \]

(\text{Parton Distribution Function})

In exclusive scattering:

\[ H = \text{FT} \langle p|\overline{\psi}(-z)\gamma^+\psi(z)|p' \rangle \]

(Generalized Parton Distribution)

- **GPDs (proton):**
  - \( H(x, \xi, t, \mu^2) \) (unpol. spin-non-flip) \( \rightarrow \) like \( q, \bar{q}, g \)
  - \( \tilde{H}(x, \xi, t, \mu^2) \) (pol. spin-non-flip) \( \rightarrow \) like \( \Delta q, \Delta \bar{q}, \Delta g \)
  - \( E(x, \xi, t, \mu^2) \) (unpol. spin-flip) \( \rightarrow \) No incl. equivalent!
  - \( \tilde{E}(x, \xi, t, \mu^2) \) (pol. spin-flip) \( \rightarrow \) No incl. equivalent!

* \(-1 \leq x \leq 1\) (av. parton momentum \([ (p_{in} + p_{out})/2 ]\))
  - or \( 0 \leq X \leq 1 \) (with respect to \( p_{in} \)), \( x = \frac{X - \zeta/2}{1 - \zeta/2} \)
* \( \xi = \frac{x_{bj}}{2 - x_{bj}} \) (av. long. momentum transfer)
  - or \( \zeta = x_{bj} \) (with respect to \( p_{in} \)), \( \xi = \frac{\zeta}{2 - \zeta} \)
* \( \mu^2 \) = renormalization scale of operator product
• Symmetries & Constraints (polynomiality of moments in $\zeta$) known (Müller et. al ’94, Ji ’96, Radyushkin ’97)

• Forward limit: $\text{GPD} \to \text{PDF}$ for $\zeta \to 0, t \to 0$, $\text{GPD} \to \text{Form Factor}$ for $\zeta \to 0, t \neq 0$.

• GPDs are hybrids! In one region ($\text{DGLAP, } X \geq \zeta$), behave like inclusive PDFs, in other region ($\text{ERBL, } X \leq \zeta$) behave like (meson, etc.) distributional amplitudes!

• Twist-2 GPD evolution in LO & NLO for all $\zeta$ (AF et. al ’97, Belitsky, Müller et. al ’97,’98, Radyushkin, Musatov ’99 and AF, McDermott ’01)

• Region around $\zeta$ strongly enhanced through evolution compared to inclusive case. Gluon: about $15 – 40\%$, Quarks: $100 – 300\%$!! $\leftarrow$ Potential problem for “wrong” input! (see AF, McDermott ’01)

• Twist-3 GPDs: Expressible (in WW-approx.) through Twist-2 GPDs via spin rotation! $\leftarrow$ Twist-3 in WW pure kinematics! (Beltisky, Müller ’00, Radyushkin, Weiss ’00).
(a) $x > \xi$: DGLAP-type region for the quark distribution

(b) $-\xi < x < \xi$: ERBL-type probability amplitude

(c) $x < -\xi$: DGLAP-type region for the antiquark distribution
Models:

- **Chiral Quark Soliton Model (CQSM)** (Göcke, Weiss, Polyakov, Pobyliitsa, Petrov etc. ’97-’01) $\leftarrow$ only large $x_{bj} \Rightarrow$ no use in phenomenology!

- **Constituent Quark Models** (Scopetta et al. ’01 + more) $\leftarrow$ only DGLAP region and only large $x_{bj}$

- **Light Cone Distribution Model** (Diehl et al. ’00) $\leftarrow$ only DGLAP region and only large $x_{bj}$

- **Dual Parameterization** (Polyakov, Shuvaev ’02) *to valid for all $x_{bj} \leftarrow$ see upcoming paper by Guzey and Polyakov!

- **Double Distribution Model** (Radyushkin ’97,’98) $\leftarrow$ valid for all $x_{bj}$

- **The Aligned Jet/Forward Model** (Freund, McDermott, Strikman ’02) $\leftarrow$ also valid for all $x_{bj}$
The Double Distribution Model

- Ansatz:

\[ F_{DD}(x', y', t) = \pi^{q,g}(x', y') \ f(x') \ r^{q,g}(t) \]

\[ \pi(x', y') = \frac{\Gamma(2b + 2)}{2^{2b+1}\Gamma^2(b + 1)} \frac{[(1 - |x'|)^2 - y'^2]^b}{(1 - |x'|)^{2b+1}} \]

\( b = 1 \) maximal skewedness, \( b \to \infty \) GPD = forward PDF.

- GPDs through reduction formula:

\[ H^{q,a}(x, \xi, t) = \int dx' \int dy' \delta (x' + \xi y' - x) \ F_{DD}(x', y', t) \]

\[ H^q(x, \xi \to 0) = q(x) \ \text{← forward limit} \ \Rightarrow \ H(x, \xi) \]

generalization of usual PDF!

→ Do DDs capture all important non-pert. physics at low scale if forward PDF not known for all \( 0 \leq x' \leq 1 \) ?
Figure 1: $I(x') \cdot \zeta$ vs. $x'/\zeta$ for various $\zeta$ and $X - \zeta$ values

Figure 2: $\langle x' \rangle /\zeta$ vs. $\zeta$ for different distances of $X$ from $\zeta$
Figure 3: GPD/PDF at $\zeta = 0.0001$ (upper plot) and $\zeta = 0.1$ (lower plot) in DD model ($b = 1$), using LO/NLO MRST01 distributions at $Q_0 = 1$ GeV.
Figure 4: $\sigma(\gamma^*P \rightarrow \gamma P)$, vs. $Q^2$ at $W = 75$ GeV, and vs. $W$ at $Q^2 = 4.5$ GeV$^2$ from DD model (GRV98) vs. H1 data.
The Aligned Jet/Forward Model

In Aligned Jet Model (AJM):

\[ R = \frac{\text{Im} \mathcal{T}_{\text{DVCS}}}{\text{Im} \mathcal{T}_{\text{DIS}}} = \ln \left( \frac{1 + \frac{Q^2}{M_0^2}}{1 + \frac{Q'^2}{M_0^2}} \frac{1 + \frac{M_0^2}{Q^2}}{1 - \frac{Q'^2}{Q^2}} \right) \]

→ \( Q^2 \) incoming photon, \( Q'^2 \) outgoing photon, \( M_0 \) hadronic scale
\( \propto \) lowest allowed, excited intermediate state in \( s \)-channel.

→ Assume LO pert. QCD is ok at AJM scale \((1 - 3 \text{ GeV}^2)\).

\[ R(\lambda) = \frac{\text{Im} \mathcal{T}_{\text{DVCS}}}{\text{Im} \mathcal{T}_{\text{DIS}}} = \frac{(1 - \zeta/2) H^S(X = \zeta/(1 - \lambda), \zeta)}{q^S(X)} \]

\[ \lambda = \frac{X - \zeta}{X} = \frac{Q'^2}{Q^2} \text{ and } \zeta = x_{bj} \]

\[ \Rightarrow H^S (\zeta/(1 - \lambda), \zeta) = R(\lambda) \frac{q^S(X)}{(1 - \zeta/2)}. \]
Figure 5: $R(\lambda)$ for several values of $Q^2$ and $M_0^2 = 0.4$ GeV$^2$. 
→ DGLAP region: \( H^{S,NS,g}(X, \zeta) \equiv q^{S,NS,g} \left( \frac{X - \zeta/2}{1 - \zeta/2} \right) / (1 - \zeta/2) \)

→ ERBL region: Simple analytical form restoring polynomiality:

\[
H_{NS}^g(X, \zeta) = H_{NS}^g(\zeta) \left[ 1 + A_{NS}^g(\zeta) C_{NS}^g(X, \zeta) \right],
\]

\[
H^S(X, \zeta) = H^S(\zeta) \left( \frac{X - \zeta/2}{\zeta/2} \right) \left[ 1 + A^S(\zeta) C^S(X, \zeta) \right]
\]

\[
C_{NS}^g(X, \zeta) = \frac{3}{2} \left( \frac{2 - \zeta}{\zeta/2} \right)^2 \left( 1 - \left( \frac{X - \zeta/2}{\zeta/2} \right)^2 \right),
\]

\[
C^S(X, \zeta) = \frac{15}{2} \left( \frac{2 - \zeta}{\zeta} \right)^2 \left( 1 - \left( \frac{X - \zeta/2}{\zeta/2} \right)^2 \right)
\]

\( C \)'s vanish at \( X = \zeta \) to guarantee continuity of the GPDs. The \( A \)'s are polynomial in \( \zeta \) restoring polynomiality

Forward PDFs used from now on:

MRST2001 \((Q_0 = 1 \text{ GeV})\) and CTEQ6 \((Q_0 = 1.3 \text{ GeV})\)
Figure 6: GPD/PDF at $\zeta = 0.0001$ (LO/NLO MRST01 at $Q_0 = 1$ GeV).
Figure 7: The quark singlet and gluon GPDs in LO and NLO (MRST01 at $Q_0 = 1$ GeV) for $\zeta = 0.1$ (upper plot) and $\zeta = 0.001$ (lower plot).
**Figure 8:** Photon level cross section $\sigma(\gamma^* P \rightarrow \gamma P)$, vs. $W$ at $Q^2 = 4.5$ GeV$^2$ (H1), and at $Q^2 = 9.6$ GeV$^2$ (ZEUS) for $B = 6.5$ GeV$^{-2}$. 
Figure 9: Photon level cross section $\sigma(\gamma^* P \rightarrow \gamma P)$, vs. $Q^2$ at $W = 75$ GeV (H1), and at $W = 89$ GeV (ZEUS) for $B = 6.5$ GeV$^{-2}$. 
Figure 10: The effect on the DVCS cross section, in the average kinematics of the ZEUS data, of introducing a simple $Q^2$-dependent model for $B$. 
→ For HERMES kinematics (including twist-3)

\((\langle x_{bj} \rangle = 0.11, \langle Q^2 \rangle = 2.56 \text{ GeV}^2, \langle t \rangle = -0.265 \text{ GeV}^2)\)

SSA \((-0.21 \pm 0.08) -0.28 \text{ (LO)}, -0.23 \text{ (NLO)}.\)

CA \((0.11 \pm 0.07) 0.12 \text{ (LO)}, 0.09 \text{ (NLO)}.\)

→ For CLAS kinematics (including twist-3)

\((\langle x \rangle = 0.19, \langle Q^2 \rangle = 1.31 \text{ GeV}^2, \langle t \rangle = -0.19 \text{ GeV}^2)\)

SSA \((0.202 \pm 0.041) 0.2 \text{ (LO)}.\)
Improvements to the model:

- LO requires less skewing effect than NLO due to large LO gluon!

- Generate non-factorized $t$-dependence through perturbative evolution! → $e^{B_q t}$ for quarks ($B_q$ about 4 – 5), similar for gluons ($B_g$ about 1.5 – 2)! → Evolution will make $B_q > B_g$!

- Additional $t$-dependence of the regge-type $(X/X_0)^{-\alpha_{q,g}t}$ at $Q_0$ (has to reproduce Dirac and Pauli form factors for the first moment) → $\alpha_{q,g}$ will change under evolution similar to the $B$’s!
Summary & Conclusions

• The Double Distribution Model is still viable if a shape function can be found correctly suppressing the small $x$ bahaviour of PDFs.

• The Aligned Jet/Forward Model works for large and small $x_{bj}$! $\leftarrow$ describes all the data sets well, at least in NLO! However, there is room for improvements (see $t$ dependence at $Q_0$)!

• DVCS data both from collider and fixed-target experiments is already very restrictive on the leading GPD $H$!

• EIC DVCS data will be so precise over a large range in $x_{bj}$ and $Q^2$ that $H$ will be determined to better accuracy than PDFs!