Generalized Parton Distributions, Orbital Angular Momentum and recent results from Lattice QCD

Philipp Hägler, MIT

In collaboration with

John Negele, MIT
Dru Renner, MIT
Wolfram Schroers, MIT
LHPC, SESAM and MILC-collaborations

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GPDs: Short overview

off-forward matrix elements of bilocal operators

\[ \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}(-\frac{\lambda n}{2}) U \psi(\frac{\lambda n}{2}) | P \rangle = h_\Gamma \{ H, \tilde{H} \}(x, \xi, \Delta^2) + e_\Gamma \{ E, \tilde{E} \}(x, \xi, \Delta^2) \]

forward limit

"local" limit, Mellin transformation

longitudinal momentum transfer \( \xi \to 0 \)

momentum transfer squared

(transverse momentum transfer) \( \Delta^2 \to 0 \)

\[ H(x,0,0) = q(x) \]

\[ \tilde{H}(x,0,0) = \Delta q(x) = \Delta \Sigma(x) \]

\[ \langle P' | \bar{\psi}(0) \Gamma D^{\mu_1} D^{\mu_2} \ldots \psi(0) | P \rangle \]

\[ \propto \sum_i \left( a_i^{\mu_1 \mu_2} \ldots A_i0(\Delta^2) + b_i^{\mu_1 \mu_2} \ldots B_i0(\Delta^2) \right) + c^{\mu_1 \mu_2} \ldots C(\Delta^2) \]

\[ n = 1 \]

\[ F_1(\Delta^2), F_2(\Delta^2), \ldots \]
GPDs and basic experimental probes

- GPD, DVCS
- Forward
- PD, DIS
- Local
- FF, elastic scattering
GPDs and (O)AM

(orbital) angular momentum and off-forward distributions

\[ \langle P' | T_{q+g}^{\mu_1 \mu_2} | P \rangle = a^{\mu_1 \mu_2} A(\Delta^2) + b^{\mu_1 \mu_2} B(\Delta^2) + c^{\mu_1 \mu_2} C(\Delta^2) \]

Noether

related sumrules

nucleon spin sumrule

\[ \frac{1}{2} = J_N = \frac{1}{2} (A(0) + B(0)) = \frac{1}{2} \left( H_{n=2}^{n=2}(0) + E_{n=2}^{n=2}(0) \right) = \frac{1}{2} (A_{20}(0) + B_{20}(0)) \]

Is the relation GPDs↔nucleon spin trivial?

spin and orbital decomposition

\[ \frac{1}{2} = S + L = S + (J - S) \]

flavour and gluon decomposition

\[ \frac{1}{2} = \sum_q J_q + J_g \]
Quark OAM and twist-3 GPDs

Parametrization including transverse parts in terms of twist-3-distributions

\[ \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P|\bar{\psi}(-\frac{\lambda n}{2})\gamma_{\mu} \gamma_{\nu}(-\frac{\lambda n}{2})\gamma_{\nu} P \rangle = \langle \langle \gamma_{\mu} \rangle \rangle \{H + E\} + \langle \langle \gamma_{\mu} \rangle \rangle G_2 + \langle \langle n\gamma_5 \rangle \rangle i\epsilon_{\mu\nu}\Delta_{\perp} G_4 + (\epsilon_{\Delta_{\perp}}) \]

Kiptily/Polyakov, hep-ph/0212372

Use EOF

derive an integrated sumrule for the twist three distributions and OAM

\[ \int dx [G_2(x,0,0) + 2G_4(x,0,0)] = -\frac{1}{2} \int dx [H(x,0,0) + E(x,0,0)] + \frac{1}{2} \int dx \tilde{H}(x,0,0) = -J_q + \frac{1}{2} \Delta \Sigma = -L_q \]

Pentinnenn et al, PLB 491, 2000

Sheds some new light on quark OAM compared to standard definition \( \{DVCS \rightarrow J_q, DIS \rightarrow \Delta \Sigma \} \rightarrow L_q = J_q - \frac{1}{2} \Delta \Sigma \)

Is a generalization to distributions \( L_q(x), \ldots \) possible?
start with the definition of OAM-distribution

\[ L_q(x) \equiv \frac{1}{NV} \int dx^- e^{ixx^-P^+} / 2 \langle P \int d^2 x_\perp \psi_\perp^\dagger (x_\perp)[ix_1D_2 - ix_2D_1] \psi_+ (x_\perp + x^-) | P \rangle \]

Bashinsky/Jaffe, NPB 536, 1998

define new off-forward function

\[ f_{L_q}(x, \Delta_\perp) \equiv \frac{1}{4\pi} \int dx^- e^{ix(x^-P^+)} / 2 \langle P' | \int d^2 x_\perp \psi_\perp^\dagger (x_\perp)[ix_1\partial_2 - ix_2\partial_1] \psi_+ (x_\perp + x^-) | P \rangle \]

with a test function T we have

\[ f_{L_q}(x) \equiv \frac{\int d^2 \Delta_\perp T(\Delta_\perp) f_{L_q}(x, \Delta_\perp)}{(2\pi)^2 T(0)} \]

if \( T(\Delta_\perp) \rightarrow \delta^2(\Delta_\perp) \Rightarrow f_{L_q}(x) = L_q(x) \)

now rewrite \( x_\perp \) as \( \partial_{\Delta_\perp} \)

\[ f_{L_q}(x) \equiv \frac{1}{4\pi} \int dx^- e^{ix(x^-P^+)} / 2 \varepsilon_{jk} \partial_{\Delta_\perp} \int \{ \langle P' | \psi_\perp^\dagger (0) \partial_{x_\perp j} \psi_+ (x^-) | P \rangle \}_{\Delta=0} \]
Consider a general off-forward correlator

\[
\int \frac{d\lambda d^2 x_{\perp}}{(2\pi)^3} e^{i\lambda x - i x_{\perp} \cdot k_{\perp}} \langle P' | \bar{\psi} \left( - \frac{\lambda n}{2} - \frac{x_{\perp}}{2} \right) \eta \psi \left( \frac{\lambda n}{2} + \frac{x_{\perp}}{2} \right) | P \rangle
\]

Take one unit of intrinsic quark \( k^{\perp} \) into account

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi} \left( - \frac{\lambda n}{2} \right) \eta \psi \left( \frac{\lambda n}{2} \right) | P \rangle = a^{\perp} A(x, \Delta^{2}_{\perp}) + \ldots
\]

Comparing with our expression for \( f_{Lq}(x) \),
it's straightforward to see that

\[
\epsilon_{\mu \nu} \partial^{\mu}_{\Delta} \left( a^{\perp} A(x, \Delta^{2}_{\perp}) + \ldots \right)_{\Delta = 0} = 2f_{Lq}(x)
\]

Use \( \text{EOF} \) and (dropping gluons)

\[
\bar{\psi}(x_2) \gamma^{\{ \mu} \leftrightarrow \nu \} \psi(x_1) = i\epsilon^{\nu \mu \alpha \beta} \bar{\psi}(x_2) \gamma^{\{ \alpha} \leftrightarrow \beta \} \gamma_5 \psi(x_1)
\]

Finally, replace

\[
\begin{align*}
n \cdot \partial & \rightarrow -i x \\
\partial \beta_{\perp} & \rightarrow i \Delta \beta_{\perp} / 2 \\
n \cdot \partial & \rightarrow i n \cdot \Delta / 2
\end{align*}
\]

and use the standard twist-2 and 3 parameterization in the WW-approximation

\[
L_q(x) = x(H(x) + E(x) + G_2(x) + 2G_4(x)) - \Delta q(x)
\]

\[
= 2J_q(x) - \Delta q(x) + x(G_2(x) + 2G_4(x)) = 2L_q(x) + x(G_2(x) + 2G_4(x))
\]

\[\Rightarrow \quad L_q(x) = -x(G_2(x) + 2G_4(x))\]
GPDs as transverse-coordinate space distributions

\[ \xi \to 0 \]

infinite momentum frame \(\rightarrow\) Galilean structure in transverse plane

Fourier-transform in transverse plane

\[ \int d^2\Delta_\perp e^{i\Delta_\perp \cdot b_\perp} H(x,\xi = 0,\Delta_\perp^2) = H(x,b_\perp^2) \]

distributions in \(x\) and \(b_\perp\) of quarks in the nucleon

- charge \(F_1(b_\perp), F_2(b_\perp)\)
- momentum \(q(b_\perp)\)
- spin \(\Delta \Sigma(b_\perp)\)
- total angular momentum \(J_q(b_\perp)(?)\)

distributions in \(b_\perp\) of quarks in the nucleon

\[ \xi \neq 0 \]

Burkardt, 2000

Diehl, 2002
GPDs in terms of light-cone wave functions

Brodsky et al, NPB596, 2001

\[ H(x, \xi \to 0, \Delta^2) \propto \sum_n \prod_{i=1..n} dx_i d^2 k_{\perp i} \delta(1 - \sum_j x_j) \delta^2 \left( \sum_j k_{\perp j} \right) \Psi^*_n(x'_m, k'_{\perp m}) \Psi_n(x_p, k_{\perp p}) \]

momentum space

\[ \Delta_{\perp} \]

\[ x = x_1 \]
\[ k'_{\perp 1} = k_{\perp 1} - (1 - x) \Delta_{\perp} \]
\[ x'_j = x_j \]
\[ k'_{\perp j > 1} = k_{\perp j} + x_j \Delta_{\perp} \]

in the limit \( x \to 1 \), apparently
\( x_{j > 1} \to 0 \), and therefore \( k'_{\perp j} = k_{\perp j} \), i.e. \( H(x \to 1, \xi \to 0, t = \Delta^2) = \text{const.} \)

how does this translate to coordinate (impact parameter) space?
GPDs in terms of light-cone wave functions 2

\[ k'_{\perp 1} = k_{\perp 1} - (1 - x)\Delta_{\perp} \]

\[ k'_{\perp j > 1} = k_{\perp j} + x j \Delta_{\perp} \]

\[ \int d^2k'_{\perp 1} \delta^2 (k'_{\perp 1} - (k_{\perp 1} - (1 - x)\Delta_{\perp})) \]
\[ \int d^2k'_{\perp j > 1} \delta^2 (k'_{\perp j > 1} - k_{\perp j > 1} - x j \Delta_{\perp}) \]

Fourier transform

\[ H(x,0,b_{\perp}) = \int d^2\Delta_{\perp} e^{-ib_{\perp} \cdot \Delta_{\perp}} H(x,0,\Delta_{\perp}) \]

\[ H(x,0,b_{\perp}) \propto \sum \prod_{n=1}^{\infty} \int dx_{m}d^2r_{\perp m}d^2r'_{\perp m} \delta^2 (b_{\perp} + (1 - x)r'_{\perp 1} - \sum_{i} x_{i}r'_{\perp i}) \]
\[ \times \delta(1 - \sum x) \prod_{j} \delta^2 (r'_{\perp 1} - r_{\perp 1} + r'_{\perp j} - r_{\perp j}) \Psi_{n}^{*}(x,r'_{\perp})\Psi_{n}(x,r_{\perp}) \]

i.e. \[ H(x \to 1,0,b_{\perp}) \propto \delta^2 (b_{\perp}) \]

distribution in impact parameter space
is in fact a delta-function in the limit \( x \to 1 \)
GPDs in Lattice QCD

- Discretization, change of language
- Infinite continuous space-time $\rightarrow$ finite space-time lattice

\[
\psi(x) \rightarrow \psi(n), \quad n = (n_1, n_2, n_3, n_4), \quad "sites"
\]
\[
A^\mu(x) \rightarrow U_{n,n+\mu} = U^\mu(n), \quad "links"
\]
\[
F^{\mu\nu}(x) \rightarrow U_{\mu\nu}(n), \quad "plaquettes"
\]
\[
\int dx \rightarrow \sum_n
\]

Lattice QCD-action is not unique

\[
S_{QCD} = S_G[U] + S_F[U, \psi, \bar{\psi}],
\]
\[
S_G[U] \propto \sum_P \left[ 1 - \frac{1}{6} \text{Tr}(U_P + U_P^\dagger) \right]
\]

General correlation functions

\[
\langle \psi_\alpha(n) \ldots U_\beta(m) \ldots \bar{\psi}_\gamma(j) \ldots \rangle
\]
\[
\propto \int [DU][D\psi][D\bar{\psi}] [\psi_\alpha(n) \ldots U_\beta(m) \ldots \bar{\psi}_\gamma(j) \ldots] e^{-S_{QCD}}
\]

Path integrals are numerically evaluated using Monte Carlo techniques
GPDs in Lattice QCD…continued

\[ \langle N(P') | \bar{\psi}(0) \Gamma D^{\mu_1} D^{\mu_2} \ldots \psi(0) | N(P) \rangle \]

\[ C_{3pt}(\tau, P', P) = \text{Tr} \left\{ \Gamma_{pol} \langle 0 | N(\tau_{\text{src}}, P') O_{\Gamma}(\tau) \bar{N}(\tau_{\text{src}}, P) | 0 \rangle \right\} \]

= nucleon source/sink
= operator insertion
= quark-path through lattice

connected, unquenched/full QCD

Horsley, PhD thesis, 2000
GPDs in Lattice QCD…continued

in the limit $\tau - \tau_{src} \gg \frac{1}{E}$ and $\tau_{snk} - \tau \gg \frac{1}{E}$, excited states drop out, and we have

$$\frac{\langle N(P')|O_{\Gamma}|N(P) \rangle}{\sqrt{E(P')E(P)}} = \frac{C_{3pt}(\tau,P,P')}{C_{2pt}(\tau_{snk},P')} \left( \frac{C_{2pt}(\tau_{snk} - \tau + \tau_{src},P)C_{2pt}(\tau,P')C_{2pt}(\tau_{snk},P')}{C_{2pt}(\tau_{snk} - \tau + \tau_{src},P')C_{2pt}(\tau,P)C_{2pt}(\tau_{snk},P)} \right)^{\frac{1}{2}} \equiv R(\tau,P',P)$$

finally, we equate lattice result and continuum parametrization for different momenta and indices

$$\bar{R}_{\mu_1 \mu_2 \cdots} (P', P) = \langle P'|\bar{\psi}(0)\Gamma D_{\mu_1} D_{\mu_2} \cdots \psi(0)|P \rangle$$

$$= \sum_i \left( a_i^{\mu_1 \mu_2 \cdots} A_{i0}(\Delta^2) + b_i^{\mu_1 \mu_2 \cdots} B_{i0}(\Delta^2) \right) + C^{\mu_1 \mu_2 \cdots} C(\Delta^2)$$

this gives an (overdetermined) set of linear equations which is solved to get the GFFs

plateau
Numerical results for heavy and „light“ pions

SESAM - lattice - parameters:
- unimproved Wilson action
- unquenched calculation, only connected contributions
- lattice - size is $16^3 \times 32$
- lattice - spacing is roughly $a^{-1} = 2 \text{ GeV}$ (i.e. the scale is $\mu^2 \approx 4 \text{ GeV}^2$)
- quarks, pions, nucleons are quite heavy, $m_\pi \approx 700 \ldots 900 \text{ MeV}$
- roughly 200 configurations / $\kappa$
- 3 $\kappa$ available, corresponding to $m_\pi \approx 700, 800$ and 900 MeV

MILC - lattice - parameters:
- domain - wall (chiral) fermions with staggered kernel
- improved gauge action with HYP - smearing
- unquenched calculation, only connected contributions
- lattice - size is $20^3 \times 32$
- lattice - spacing is roughly $a^{-1} = 1.5 \text{ GeV}$ (i.e. the scale is $\mu^2 \approx 2.3 \text{ GeV}^2$)
- quarks, pions, nucleons are closer to chiral limit, $m_\pi \approx 313$ (and 580) MeV
- roughly 100 (eventually 200) configurations / $m_\pi$

See also QCDSF (Göckeler et al.), hep-ph/030424

**off-forward matrix element**

**only forward matrix element so far**
Results for the GFF A(t), B(t), C(t)

concentrate on u-d in order to cancel disconnected pieces

dipole - fits work well \( GFF(t) = \frac{a}{(1 - t/m_D^2)^2} \)

\[ \int_{-1}^{1} dx x^0 H(x,0,t=0) = \frac{1}{2} \int_{-1}^{1} dq(x) = A_{10}(t=0) \]

\[ \int_{-1}^{1} dx x^1 H(x,0,t=0) = \frac{1}{2} \int_{-1}^{1} dx x q(x) = A_{20}(t=0) \]
Flattening of t-slope for higher moments of H

\[ \int dxx^{n-1}H(x, \xi = 0, t) = A_{n0}(t) \]

A, n=(1,2,3), up-down

\( n \rightarrow \infty \implies x \approx 1 \)

\( H(x \approx 1, t) \propto \text{const.} \)

\( H(x \approx 1, b) \propto \delta(b) \) for factorized ansätze like

\( H(x,t) = q(x)f(t) \)

can be ruled out for these pion masses
( Orbital ) Angular Momentum

\( \text{n}=2, \text{up+down, disconnected pieces missing} \)

\[ J_q = \frac{1}{2} \int_{-1}^{1} dxx(q(x) + E(x, 0, t=0)) = \frac{1}{2} (A_{20}(0) + B_{20}(0)) \]

\[ C_{20}^{u+d}(t) = 0 \]

\[ C_{20}^{u+d}(t) \text{ is small and negative} \rightarrow \]

extrapolated \( C_{20}^{u+d}(t = 0) = ? \)

seems to be at least compatible with the chiral quark soliton model (Kivel et al., 2000)

<table>
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<th>( \kappa )</th>
<th>0.1570</th>
<th>0.1565</th>
<th>0.1560</th>
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<td>( \Delta \Sigma )</td>
<td>0.666 ± 0.033</td>
<td>0.727 ± 0.028</td>
<td>0.684 ± 0.018</td>
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<tr>
<td>( 2J_q )</td>
<td>0.730 ± 0.035</td>
<td>0.688 ± 0.024</td>
<td>0.682 ± 0.029</td>
</tr>
<tr>
<td>( 2L_q )</td>
<td>0.064 ± 0.048</td>
<td>-0.039 ± 0.037</td>
<td>-0.002 ± 0.034</td>
</tr>
</tbody>
</table>
Towards the chiral region

\[ m_{\pi, \text{exp}} \]

\[ m_{\pi, \text{latt}} [\text{GeV}^2] \]

observable

\[ \chiPT(?) \]

lattice calculations of hadronic observables

see e.g.
Göckeler et al., hep-lat/0303019
Towards the chiral region…continued

axial coupling/charge//form factor and the quark spin contribution

should be a clean observable… „delocalization of axial charge“?

looks not bad, probably accidentally: disconnected pieces are missing!

Jaffe PLB 529, 2002

chiral logs for (O)AM Chen/Ji, PRL88,2002

results for the lowest two pion masses are not renormalized, I expect $Z\approx 1\pm 0.05$ for the FF
Towards the chiral region… continued

quark longitudinal momentum fraction, u-d

**n=3, u-d**

error pretty large, inconclusive
Outlook

- better statistics for MILC is on the way
- extract GPDs from the recent MILC-configurations
- extract the spin-dependent GFFs for n=3
- (perturbative) renormalization with domain wall fermions + HYP-smeared links
- chiral extrapolation, if possible
- what is going on with the axial coupling?
- non-factorized models for H,E
- disconnected diagrams
- lower pion masses/larger lattices