Statistical Hadron Production from AGS to Collider Energies

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- Statistical Model
- Fixed Target Data
- RHIC Data
- Chemical Freeze-Out and the Phase Boundary
- Chemical Freeze-Out vis-a-vis Initial Condition and Thermal Freeze-Out
- Outlook and Open Questions
Choice of Statistical Ensemble

- **Grand Canonical Ensemble (GC):** in large system, with large number of produced particles, conservation of additive quantum numbers \((B, S, I_3)\) can be implemented on average by use of chemical potential \(\mu\); asymptotic realization of exact canonical approach.

- **Canonical Ensemble (C):** in small system, with small particle multiplicity, conservation laws must be implemented locally on event-by-event basis (Hagedorn 1971, Shuryak 1972, Rafelski/Danos 1980, Hagedorn/Redlich 1985).

  → severe phase space reduction for particle production "canonical suppression"

- **C relevant in**
  - low energy HI collisions (Cleymans/Redlich/Oeschler 1998/1999)
  - high energy hh or \(e^+e^-\) collisions (Becattini/Heinz 1996/1997)
  - peripheral HI collisions (Cleymans/Oeschler/Redlich 1998, Hamieh/Redlich/Tounsi 2000)
Connection C and GC: recently shown by Tounsi/Redlich hep-ph/0111159 and 0111261:

\[ n^C_k = n^{GC}_k F_s \]

where \( k \) stands for all hadrons with a given strangeness \( S \)

- for strangeness 1:

\[ F_s = \frac{I_1(x)}{I_0(x)} \]

\( x \) describes size of thermal phase space available for strange particles

\[ x = 2S_1 = 2 \sum_k Z^1_k \propto V = (V_0/2)N_{\text{part}} \]

ignoring multistrange hadrons, \( x \) is total of all particles with \( S=1 \) in GC

- for multistrange:

\[ F_s = \frac{I_s(x)}{I_0(x)} \]

- limiting cases:

\[ \lim_{x \to \infty} \frac{I_1(x)}{I_0(x)} = 1 \quad C \to GC \]

\[ \lim_{x \to 0} \frac{I_n(x)}{I_0(x)} \to (x/2)^n \propto V^n \]
Canonical Suppression Factor

Tounsi and Redlich, hep-ph/0211159

for $N_{\text{part}} \geq 60$ Grand Canonical ok to better 10%
Grand Canonical Ensemble

\[ \ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \exp[-(E_i - \mu_i)/T]] \]

\[ n_i = \frac{N}{V} = -\frac{T}{V} \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1} \]

\[ \mu_i = \mu_B B_i + \mu_S S_i + \mu_{I3} I^3_i \]

for every conserved quantum number there is a chemical potential \( \mu \) but can use conservation laws to constrain:

- Baryon number: \( V \sum_i n_i B_i = Z + N \rightarrow V \)
- Strangeness: \( V \sum_i n_i S_i = 0 \rightarrow \mu_S \)
- Charge: \( V \sum_i n_i I^3_i = \frac{Z - N}{2} \rightarrow \mu_{I3} \)

This leaves only \( \mu_b \) and \( T \) as free parameter when \( 4\pi \) considered for rapidity slice fix volume e.g. by \( dN_{ch}/dy \)
- finite volume correction: a la Balian-Bloch

\[ f = 1 - \frac{3\pi}{4pr} + \frac{1}{(pR)^2} \]

- interactions: van der Waals type via excluded volume correction

thermodynamically consistent form Rischke/Gorenstein/Stöcker/Greiner 1990

\[ p^{\text{excl.}}(T, \mu) = p^{\text{id.gas}}(T, \hat{\mu}); \quad \text{with } \hat{\mu} = \mu - v_{\text{eigen}} p^{\text{excl.}}(T, \mu) \]

use \( r = 0.3 \text{ fm} \) (hard core of nucleon-nucleon interaction)

- resonance width: e.g. Weinhold, Friman, Nörenberg, 1996

\[
\ln Z_R = N \frac{V dR}{2\pi^2} T \exp[\mu/T] \int_{s_{\text{min}}}^{s_{\text{max}}} ds s K_2(\sqrt{s}/T) \frac{1}{\pi} \frac{m_R \Gamma_R}{(s - m_R^2)^2 + m_R^2 \Gamma_R^2}
\]

not important for large enough \( T \)
Dependence of $\mu_S$ on T and $\mu_B$
free parameters:
\[ T = 0.170 \pm 0.005 \text{ GeV} \]
\[ \mu_b = 0.255 \pm 0.010 \text{ GeV} \]

fixed by conservation laws:
\[ \mu_s = 0.074 \text{ GeV from } \Delta S=0 \]
\[ \mu_{I_3}=0.005 \text{ GeV from } \Delta Q=0 \]

reduced \( \chi^2 \) (excluding \( \phi \) and d)

\( 2.0 \)

largest contribution:
\( \Lambda/\pi, \Lambda/h^-, \Lambda/K^0_s \)

weak feeding in

numerator/denominator same?
Strangeness Enhancement in 158 A GeV/c Pb + Pb Collisions

p_T > 0, \(|y-y_{cm}| < 0.5\)

\(\Omega^- + \Omega^+\)

\(\Lambda\)

\(\bar{\Lambda}\)

\(p_{Be}\) \(p_{Pb}\) \(Pb_{Pb}\)

\(<N_{\text{wound}}\rangle\)

\(<N_{\text{part}}\rangle\)

\(\Omega\) enhancement central ok. but doesn’t flatten at \(N_{part} = 100\)
central 40 A GeV/c Pb + Pb collisions - thermal model parameters: $T = 148\,\text{MeV}$, $\mu_b = 400\,\text{MeV}$

reduced $\chi^2 = 1.1$
Hadron Yields at AGS and Thermal Model


central 14.6 A GeV/c Si + Au collisions
thermal model parameters: $T = 125$ MeV, $\mu_b = 540$ MeV

D.Prorok: also $E_T$ and $N_{ch}$ described by same parameters
yields for 11.5 A GeV/c Au + Au are very similar
Yields of Light Nuclei at AGS and Thermal Model

Addition of every nucleon → penalty factor $R_p = 48$
but data are at very low $p_t$
$p_t$ int. with $A$-dependent slope → $R_p = 26$

Grand Canonical Ensemble:
$R_p \approx \exp[(m_n \pm \mu_b)/T]$
for $T=125$ MeV and $\mu_b = 540$ MeV
→ $R_p = 23$  good agreement!

also good for antideuterons
data: $R_p = 2 \pm 1 \times 10^5$ GC: $R_p = 1.3 \times 10^5$

P.Braun-Munzinger, J.Stachel

Note: AGS may be special here since
chemical and thermal freeze-out coincide

The Perennial Question of Use of $\gamma_s$

$n_i(S) = \gamma_s S \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 \, dp}{\exp[(E_i - \mu_i)/T] + 1}$

- Looks similar to canonical suppression but in that case dependence on $V_0$ and $N_{part}$ fixed.
- $\gamma_s$ as free fit parameter for every system and centrality has no thermodynamic justification, rather a measure of lack of equilibration.
- but if really no thermalization, need dynamical treatment, doubtful that this could be replaced by factor $\gamma_s$.
Strangeness Undersaturation?

central Au + Au and Pb + Pb collisions, from SIS to SPS


\[ \frac{\text{Strangeness Undersaturation?}}{} \]

Central Au + Au and Pb + Pb collisions, from SIS to SPS

Ratio Mid-rapidity to $4\pi$ Data

all data NA49, all measured or scaled to top 10 % centrality

158 A GeV/c Pb + Pb collisions

Fit of mid-rapidity data does not need strangeness suppression
Cenmary Dependence of Strangeness Saturation

Cleymans, Kämpfer, Steinberg, Wheaton, hep-ph/0212335

Fit $\mu_B$ and $\gamma_S$ to $\pi$, $K$, $p$ yields

$f_2$ fraction of $N_{\text{part}}$ with multiple collisions

- Central collisions reach strangeness saturation at mid-rapidity
- Constant $\phi/\pi$ from STAR does not support $\gamma_S \leq 1$ for more peripheral
- Centrality independence of $\Omega$ (NA57) not consistent with above
Strangeness Saturation - Light Systems and Peripheral Collisions

centrality dependence of PbPb data vs. CC, SiSi, SS – C. Höhne, NA49, nucl-ex/0209018

- no simple $N_{\text{part}}$ scaling as for canonical suppression
- scales with fraction of nucleons with multiple collisions or collision density
Choice of Rapidity Window

- **Bjorken expansion**: boost invariant
  → any rapidity window ok

- **AGS and lower**: no separation of central and fragmentation regions
  → choose 4 π

- **RHIC**: rapidity plateau $\Delta y \approx \pm 2$
  → within this window any cut should be okay

- **Full energy SPS**: evidence that some transparency sets in, distinction central region - fragmentation region becomes meaningful
  → assess width of plateau from HBT
  and get $\Delta y \approx \pm 1$

NA49: source rapidity vs pair rapidity

$0.1 < K_\perp < 0.2$ GeV/c
RHIC Data and Thermal Model


Central Au + Au collisions, data from all experiments combined

\( \chi_r^2 = 0.8 \)

\( \chi_r^2 = 1.1 \)

Model prediction for

\( T = 177 \text{ MeV}, \quad \mu_b = 41 \text{ MeV} \)

Model re-fit with all data

\( T = 176 \text{ MeV}, \quad \mu_b = 29 \text{ MeV} \)

Fit result confirmed by Becattini and Kaneta/Xu
\( \rho^0 \) and \( f_0 \) yield at RHIC

P. Fachini, QM2002, nucl-ex/0211001

\[
\frac{\rho^0/\pi}{f_0/\pi} (e^+ e^-, pp) \quad \ast \quad \frac{\rho^0/\pi}{f_0/\pi} (RHIC pp) \quad \ast \quad \frac{\rho^0/\pi}{f_0/\pi} (RHIC Au+Au)
\]

\[
\quad \frac{\rho^0/\pi}{f_0/\pi} (e^+ e^-, pp) \quad \ast \quad \frac{\rho^0/\pi}{f_0/\pi} (RHIC pp) \quad \ast \quad \frac{\rho^0/\pi}{f_0/\pi} (RHIC Au+Au)
\]

\( \sqrt{s_{NN}} \) (GeV)

\[
\text{Ratio} \quad 0.01 \quad 0.02 \quad 0.03 \quad 0.04 \quad 0.05 \quad 0.06 \quad 0.07 \quad 0.08 \quad 0.09 \quad 0.1 \quad 0.15 \quad 0.2 \quad 0.25 \quad 0.3 \quad 0.35 \quad 0.4
\]

\( \rho^0/\pi \) (e\(^+\) e\(^-\), pp) \quad \ast \quad \rho^0/\pi \) (RHIC pp) \quad \ast \quad \rho^0/\pi \) (RHIC Au+Au)

\( f_0/\pi \) (e\(^+\) e\(^-\), pp) \quad \ast \quad \ast \quad \ast \quad \ast \quad \ast

\( f_0/\pi \) (RHIC pp) \quad \ast \quad \ast \quad \ast \quad \ast

\( f_0/\pi \) (RHIC Au+Au)

statistical model \( T=177 \text{ MeV} \): \( \rho^0/\pi^- = 0.11 \) and \( T=120 \text{ MeV} \): \( 4 \times 10^{-4} \)
even with growth of \( \mu_\pi \) to nearly \( m_\pi \) and \( \Delta m(\rho) \approx 60 \text{ MeV} \) difficult!
Mass Changes Close to $T_c$?

repeat fit of RHIC data with several hypotheses:

- change all masses by constant factor $\rightarrow$ similar fit quality if variation $\leq 20\%$ (see also Michalec, Florkowski, Broniowski, nucl-th/0103029)
- reduce $m_{\phi}$ by 5\% $\rightarrow$ 3 $\sigma$ discrepancy with data
- reduce $m_{K^{0*}}$ by 10\% $\rightarrow$ 2.5 $\sigma$ discrepancy with data

no room for very significant changes
Evolution of Thermal Parameters with $\sqrt{s}$

Cleymans/Redlich PR C60 (1999) 054908

![Graph showing the evolution of thermal parameters](image)
Evolution of Thermal Parameters with $\sqrt{s}$

1. hadron yields equilibrated

2. for full SPS energy and above: hadron yields frozen at phase boundary

3. how is equilibrium achieved? at SPS and RHIC not with hadronic cross sections → QGP much more efficient equilibrator
What Characterizes Chemical Freeze-out Curve?

Cleymans/Redlich PRL 81 (1998) 5284: Constant energy per hadron $E/h = 1$ GeV
Braun-Munzinger/Stachel JP G28 (2002) 1971: Constant total baryon density $n_B = 0.12$ fm$^{-3}$

constant total baryon density could indicate $bb$ and $mb$ cross sections relevant for freeze-out full SPS energy and above freeze-out points below -- suggestive that actually phase boundary is met
Freeze-Out Density from Pion HBT

\[ \rho_f, \pi, \rho_f, N \]

HBT gives density at thermal freeze-out

\[ \rho_{f,\pi} \]  
\[ \rho_{f,N} \]

→ pion density at chemical freeze-out

→ nucleon density at chemical freeze-out

Volume appears to only grow 30% between chemical and thermal freeze-out!
Hadronic Phase after Chemical Freeze-out

- Densities from HBT as compared to chemical freeze-out:
  indicate growth $\Delta R_s \approx 10\%$ (at most $25\%$)
  at average expansion velocity $0.5c$ this takes $1-2$ fm/c

- Role of annihilation for $\bar{p}/p$ and $\bar{d}/d$; in UrQMD factor 2 change due to annihilation
  in data no room for that
  \[\rightarrow \text{no indication for a longlived hadronic phase}\]

- During hadronization $\# \text{dof}$ increases by factor 3.5
  volume has to grow accordingly
  \[\rightarrow \text{more time at } T_c \text{ than in hadronic phase}\]

CERES enhancement understood in this context?