Symmetry Tests in Positronium Decay

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\[
\begin{array}{c}
e^- \\
\vdots \\
e^+
\end{array}
\quad \begin{array}{c}
\gamma_1 \\
\gamma_2 \\
\gamma_3 \\
\gamma_4 \\
\gamma_5
\end{array}
\]
Positronium People

Weak Interactions People
Paul Vetter
Stuart Freedman
Michelle Cyrier (now Stanford student)

Gammasphere People
Rod Clark
Paul Fallon
Augusto Macchiavelli, Mario Cromaz, etc.
States in Positronium

The spectroscopic structure is the same as the Bohr atom (same Hamiltonian) except that the reduced mass is less:

\[
H\psi = \left( -\frac{\hbar^2}{2m} \nabla^2 - \frac{e^2}{r} \right)\psi
\]

\[
M = \frac{(m_{e-})(m_{e+})}{m_{e-} + m_{e+}} = \frac{1}{2} m_e
\]

Relativistic corrections are the same (except for factors of 2)
New field retardation effects (“nucleus” moves)

Ground state has \( n = 1, \ell = 0 \) and total spin = 0 or 1

\[
\psi_{S_{tot}, s_z} = \begin{cases} 
\psi(1,+1) = e^+ (+\frac{1}{2}) e^- (+\frac{1}{2}) \\
\psi(1,0) = \frac{1}{\sqrt{2}} (e^+ (+\frac{1}{2}) e^- (-\frac{1}{2}) + e^+ (-\frac{1}{2}) e^- (+\frac{1}{2})) \\
\psi(1,-1) = e^+ (-\frac{1}{2}) e^- (-\frac{1}{2}) \\
\psi(0,0) = \frac{1}{\sqrt{2}} (e^+ (+\frac{1}{2}) e^- (-\frac{1}{2}) - e^+ (-\frac{1}{2}) e^- (+\frac{1}{2}))
\end{cases}
\]

States \( \psi(1,x) \) are \( ^3S_1 = \text{Orthopositronium (O-Ps)} \) mean lifetime \( \sim 140 \text{ ns} \)
State \( \psi(0,0) \) is \( ^1S_0 = \text{Parapositronium (P-Ps)} \) mean lifetime \( \sim 120 \text{ ps} \)
Charge Conjugation Symmetry in Positronium

The charge symmetry operator changes particle to antiparticle:

\[ \mathbf{C}|p\rangle = |\bar{p}\rangle \]

\( \mathbf{C} \) reverses the sign of the “internal” quantum numbers of a particle (charge, baryon number, lepton number, strangeness, beauty…)

\( \mathbf{C} \) does not change mass, energy, momentum, spin...

Obviously, \( \mathbf{C}^2 = \mathbf{I} \) so eigenvalues of \( \mathbf{C} \) are +1, -1.

Particles, in general, are not eigenstates of \( \mathbf{C} \). That would mean

\[ \mathbf{C}|p\rangle = |\bar{p}\rangle = \pm |p\rangle \]

True for photons, uncharged mesons, (\( \pi^0, \rho^0, \eta, \phi, \ldots \)), and uncharged bound states – like positronium.

Positronium is an eigenstate of \( \mathbf{C} \) and transforms like:

\[ \mathbf{C}|\text{Ps}\left(^2S+1L_J\right)\rangle = (-1)^{\ell+S}|\text{Ps}\left(^2S+1L_J\right)\rangle \]

The photon is intrinsically \( \mathbf{C} \)-odd, so for a state of \( n \) photons:

\[ \mathbf{C}|n\gamma\rangle = (-1)^n|n\gamma\rangle \]

So! if a given state of Ps annihilates by a \( \mathbf{C} \) conserving process, the number of photons, \( n \), is strictly odd or even by \( \mathbf{C} \) symmetry:

\[ |^1S_0\rangle \rightarrow 2\gamma, 4\gamma, \ldots \quad |^3S_1\rangle \rightarrow 3\gamma, 5\gamma, \ldots \]
Fundamental Physics Tests in Ps Decay

QED tests at high orders of $\alpha$ at tree level:
Four photon decay of P-Ps:

$$\Gamma(P - Ps \rightarrow 4\gamma) = 0.0138957(4) \, m\alpha^7 \left( 1 - 14.5(6) \, \alpha / \pi + O(\alpha^2) \right) = 1.43 \times 10^{-6}$$


Five photon decay of O-Ps:

$$\Gamma(O - Ps \rightarrow 5\gamma) = \Gamma_{LO} \left( 0.0189(11) \, \alpha^2 \right)$$

$$= (2 / 9\pi) \left( \pi^2 - 9 \right) m\alpha^6 \left( 0.0189(11) \, \alpha^2 \right) \times \alpha^8 = 0.959 \times 10^{-6}$$


Search for $C$ violating, forbidden decay modes:

$$R^C_{4} = \frac{\Gamma(O - Ps \rightarrow 4\gamma)}{\Gamma(O - Ps \rightarrow 3\gamma)}$$

O-Ps ($^3S_1$) $\rightarrow$ 4 $\gamma$

$$R^C_{3,5} = \frac{\Gamma(P - Ps \rightarrow 3,5\gamma)}{\Gamma(P - Ps \rightarrow 2\gamma)}$$

P-Ps ($^1S_0$) $\rightarrow$ 3 $\gamma$, 5$\gamma$
Outline of the Experiment

Goals

- Search for O-Ps $\rightarrow 4\gamma$ (C violation)
- Search for O-Ps $\rightarrow 5\gamma$ (QED allowed)
- Search for P-Ps $\rightarrow 5\gamma$ (C violation)
- Measure P-Ps $\rightarrow 4\gamma$ (QED allowed)

Make as much O-Ps as possible
Minimize backgrounds

- $^{68}$Ge source
- BGO Compton suppression
- Beta detector timing

Count as long as possible

Sort data

- Identify unambiguous annihilations from scattering
- Count $N(2\gamma)$, $N(3\gamma)$, $N(4\gamma)$, $N(5\gamma)$

Estimate detection efficiencies

- Monte Carlo simulation of Gammasphere
- Input Ps annihilation photons ...
- Derive $\varepsilon(2\gamma)$, $\varepsilon(3\gamma)$, $\varepsilon(4\gamma)$, $\varepsilon(5\gamma)$

Estimate backgrounds

- Monte Carlo different background mechanisms? Too slow!
- Event mixing, use raw data

Calculate $N/\varepsilon$ for all desired ratios for O-Ps and P-Ps populations
Making Positronium

- Long-lived positron source -- low decay energy $^{22}\text{Na}$, $^{68}\text{Ge}$
- Clean (no daughter $\gamma$'s)
- Stop positrons in fine-grained insulator material
- If $E(e^+) <$ ionization potential of stopping material and $E(e^+) >$ ionization potential - binding energy of Ps (6.8 eV) Ps can be formed -- “Ore gap theory”
- Pickoff in material (O-Ps $\rightarrow$ e+e- $\rightarrow$ 2 x 511 keV )
- Timing trigger when e+ is emitted

The $^{68}\text{Ge}$ aerogel Positronium source

MARK I

Positronium Source inside Gammasphere

MARK II

Polarized O-Ps source (upward e+)
in Gammasphere target chamber

MARK II
Three Photon O-Ps Decay Kinematics

Energy in (O-Ps -> 3γ) decay derived from two angles included between photon momenta in a plane. Probability for angles $\alpha_1$ and $\alpha_2$ calculable in closed form

$$P(\alpha_1, \alpha_2, \alpha_3) = \sum_i (1 - \cos \alpha_i)^2 \frac{\prod_i \sin \alpha_i}{(\sum_i \sin \alpha_i)^3}$$

![Graph showing relative probability per energy interval for O-Ps 3 photon decay](image)

Angular distribution probability for O-Ps 3 photon decay (Probability to have angles $\alpha_1$ and $\alpha_2$)
4 and 5 photon decays:
No closed form for spectra.
Event generating software
GRACE/BASES/SPRING (KEK)
QED allowed diagrams
calculate allowed phase space
Generate Random events (phase space weighted)
Feed to GEANT simulation of Gammasphere
Determine detection efficiency

4 Photon Decay Kinematics

Five Photon Decay Kinematics
Valid Ps annihilation events passing all cuts
O-Ps decays after 20 ns (late), P-Ps decays early (< 20ns)

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Counts per channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 $\gamma$</td>
<td>3.0101 x 10$^9$</td>
</tr>
<tr>
<td>3 $\gamma$</td>
<td>1.0907 x 10$^7$</td>
</tr>
<tr>
<td>4 $\gamma$</td>
<td>16</td>
</tr>
<tr>
<td>5 $\gamma$</td>
<td>1</td>
</tr>
</tbody>
</table>

Energy spectrum of valid 3$\gamma$ annihilation events. Angular resolution of Gammasphere for correlated photons.

Annihilation Event Selection criteria:
1. Sum Energy = 1022 keV +/- 5 keV
2. Prompt photons
3. Vector Momentum Sum < 100 keV/c
4. Distance of decay plane to origin
5. Colinearity

Sorted Data

Live Time: 0.98 x 10$^6$ seconds
Data: 0.8 TB
Raw events: 1 x 10$^{11}$
Background contributions to the 4 and 5 photon counts estimated with an “event-mixing” technique on raw data. Events with two or three hits are combined.
Simulated bremsstrahlung or 1077 keV photons.
-- Run synthetic data through sort code.

### Table III: Sources of background counts for 4γ and 5γ Ps decays.

| Event type              | Absolute  | $|t| \leq 20\text{ms}$ | $t > 20\text{ms}$ |
|-------------------------|-----------|------------------------|-------------------|
| Four-photon decays      |           |                        |                   |
| $2\gamma + \gamma_{\text{brems}} + \gamma_{\text{brems}}$ | $1.33 \times 10^4$ | 0.016                  | 0.003             |
| $3\gamma + \gamma_{\text{brems}} + \gamma_{\text{brems}}$ | 198       | $< 0.05$               | $< 0.05$          |
| $2\gamma + 2\gamma$    | $2.25 \times 10^6$ | 0.079                  | 0.014             |
| $2\gamma + 3\gamma$    | $1.44 \times 10^6$ | 0.066                  | 0.24              |
| $3\gamma + 3\gamma$    | $9.14 \times 10^5$ | 0.074                  | 0.27              |
| $3\gamma + \gamma_{\text{brems}}$ | $2.23 \times 10^5$ | 1.3                     | 0.01              |
| $3\gamma + \gamma_{1077}$ | $9.34 \times 10^3$ | 0.34                   | 0.04              |
| $2\gamma + 2\gamma + \gamma_{\text{brems}}$ | $9.44 \times 10^4$ | 0.08                   | 0.02              |
| $2\gamma + 2\gamma + \gamma_{1077}$ | $3.95 \times 10^3$ | 0.015                  | 0.02              |
| $\sum 4\gamma$        |           |                        | 2.1               | 0.65              |
| Five-photon decays      |           |                        |                   |
| $3\gamma + \gamma_{\text{brems}} + \gamma_{\text{brems}}$ | 10       | 0.01                   | 0.01              |
| $2\gamma + 3\gamma$    | $7.99 \times 10^4$ | $< 0.03$               | $< 0.03$          |
| $3\gamma + 3\gamma$    | $1.35 \times 10^5$ | 0.04                   | 0.1               |
| $2\gamma + 2\gamma + \gamma_{\text{brems}}$ | $1.88 \times 10^4$ | 0.004                  | 0.004             |
| $2\gamma + 2\gamma + \gamma_{1077}$ | 787     | 0.001                  | 0.001             |
| $\sum 5\gamma$        |           |                        | 0.25              | 0.1               |

$E_C = 1.5\%$
Four and Five Photon Branching Ratio Results

C Symmetry Tests
(90% Confidence level limits)

\[
\frac{(O-Ps \rightarrow 4\gamma)/(O-Ps \rightarrow 3\gamma)}{} < 3.7 \times 10^{-6} \text{ (90\% C.L.)}
\]
Previous limit (1996) < 2.6 \times 10^{-6} (90\% C.L.)

\[
\frac{(P-Ps \rightarrow 5\gamma)/(P-Ps \rightarrow 2\gamma)}{} < 2.7 \times 10^{-7} \text{ (90\% C.L.)}
\]
(no previous limit)

QED Tests (1 \sigma errors)

\[
\frac{(O-Ps \rightarrow 5\gamma)/(O-Ps \rightarrow 3\gamma)}{} = 1.67(99)(37) \times 10^{-6}
\]
QED value(tree) = 0.9591 \times 10^{-6}
Previous mmt.(1 event, '95) = 2.2(2.2) \times 10^{-6}

\[
\frac{(P-Ps \rightarrow 4\gamma)/(O-Ps \rightarrow 2\gamma)}{} = 1.14(33)(21) \times 10^{-6}
\]
QED value(one-loop) = 1.4388 \times 10^{-6}
Previous mmt. (1994) = 1.50(11) \times 10^{-6}
$\chi^2 = 0.507$

P-Ps Four Photon Decay Measurements

Crystal Ball

QED value

Gammasphere

World Average
Time Reversal Symmetry in O-Ps Decay

In the decay (O-Ps -> 3γ), the photon momenta all lie in a plane.
Define the normal vector to the plane as \( \mathbf{r}_1 \times \mathbf{r}_2 \), where 1 and 2 are the highest energy photons.

\[
\mathbf{r}_1 \times \mathbf{r}_2 (\mathbf{k}_1 \times \mathbf{k}_2)
\]

Under (naive) Time Reversal, the photon momenta and spin all reverse. \( \mathbf{r}_1 \times \mathbf{r}_2 \) does not reverse.

\[ \mathbf{s} \cdot (\mathbf{k}_1 \times \mathbf{k}_2) \]

is a \( \mathbf{T} \)-odd observable.
O-Ps is an eigenstate of \( \mathbf{C} \) and \( \mathbf{P} \), so \[ \mathbf{s} \cdot (\mathbf{k}_1 \times \mathbf{k}_2) \] is \( \mathbf{CPT} \)-odd
Polarized Postronium – easier than it sounds

Polarization of positrons from $\beta^+$ decay is $P = +v/c$

Calculate average polarization of $e^+$ emitted from a source:

- Use upward betas only, average over emission direction to Ps moderator

\[
\langle \overline{P} \rangle = \frac{\langle \overline{v} \rangle}{c} = 0.686 \cdot \frac{\langle v \rangle}{c}
\]

Polarized $e^+$ and unpolarized $e^-$ => polarized Ps formed (spin statistics of O-Ps, P-Ps)

$P(\text{Ps}) = \frac{2}{3} P(e^+) =$

- 41% ($^{68}\text{Ge}$)
- 30% ($^{22}\text{Na}$)
Search for Time Reversal Symmetry Violation

CPT Violation Data -- June/July, 2002
(analysis in progress)

$^{68}$Ge CPT data:
Direction of observed normal vectors
for 3 photon events passing cuts:

Annihilation Event Selection criteria:
1. Sum Energy = 1022 keV +/- 5 keV
2. Prompt photons
3. Vector Momentum Sum < 100 keV/c
4. Distance of decay plane to origin < 2 cm
5. Colinearity (no two detectors colinear w/ center of GS)

Live time:
$2.6 \times 10^6$ seconds
Some kind of record (for masochism)
$1.7 \times 10^7$ counted clean O-Ps decays
QuickTime™ and a Motion JPEG A decompressor are needed to see this picture.
Testing CPT in Ps Annihilation

Polarized O-Ps has a $T$-even distribution of decay planes with respect to $\vec{s}$.

$$\frac{d\Gamma}{d\Omega_n} = \Gamma_0 \frac{3}{16\pi} \left(1 + |\hat{n} \cdot \vec{s}|^2\right)$$

Search for a $\cos(\theta)$ distribution of decay planes with respect to spin. $\vec{s} \cdot (\hat{k}_1 \times \hat{k}_2)$

Determine polarization from $\cos^2(\theta)$ distribution of decay planes

Data Analysis:
• Simulate GS angular response to $\cos^2(\theta)$, $\cos(\theta)$, isotropic distributions
• Sort to find good annihilations
• Fit angular data to simulated angular dependence
• Extract polarization
• Extract $T$-odd limit.

Current Limits on this $T$ violating signature: $C_n \cdot [\vec{s} \cdot (\hat{k}_1 \times \hat{k}_2)]$

$C_n = +0.020 \pm 0.023$


$C_n = +0.014 \pm 0.019$


Factor of 10 improvement with Gammasphere data set
Analysis in progress
Constraining T-Odd New Physics


A CP-odd interaction would cause mixing between S and P states in Ps.

Mixing parameter $\delta$:
\[
\delta_n = \frac{\langle M P,nS \rangle}{E_{nS} - E_P}
\]

Scale of CP odd physics

1. $r_{CP} \ll 1/m_e$ (Heavy New Physics, short distance scale)
   If so, write an effective lagrangian
   \[
   L = L_{4e} + L_{ee\gamma} + L_{ee2\gamma} + L_{ee3\gamma} + \ldots
   \]
   - Constrained by high energy $e^+ e^-$ scattering:
     \[
     L_{4e} = i \cdot f \cdot (\bar{e}e)(\bar{\sigma}_5 e)
     \]
     Optimistic (direct data applies):
     \[
     |f| < 1/(1.8 \text{ TeV})^2 \Rightarrow \delta \approx 10^{-15}
     \]
     Pessimistic (slipping thru cracks):
     \[
     |f| < 1/(100 \text{ MeV})^2 \Rightarrow \delta \approx 10^{-6}
     \]
   - Related to EDM of $e^-$
     \[
     L_{ee\gamma} = -i \cdot \frac{d_e}{2} \cdot (\bar{\sigma}_5 \sigma^{\mu\nu} e F_{\mu\nu}) \Rightarrow \delta \approx 10^{-27}
     \]
     \[
     \delta \approx \frac{(d_e m_e)^2}{e^2}
     \]

Supersymmetry, Leptoquarks, Left-Right Symm. -- too massive

2. $r_{CP} \sim 1/m_e$
   CP violation in neutrino mixing matrix?
   \[
   L \propto \bar{N}_L \gamma_\mu V_{lep} e_L W_{\mu}^+
   \]
   - No CP violation in Ps to second order
   Complicated Higgs sector:
   \[
   L = (x_s \bar{e}e + i x_p \bar{\sigma}_5 e)\phi
   \]
   - Constrained somewhat by $(g-2)_e$, other atomic limits
   Fine-tuning could evade $d_e$ but have
   \[
   \delta \approx 10^{-5}
   \]
Final State T-Odd effects in Ps -> 3γ


QED effects: “light by light” scattering $\langle \hat{n} \rangle = 8.66(45) \times 10^{-10}$ $\bar{s}$
Gammasphere Ps CPT Data, partial analysis

15\% of data set, one spin angle

\((O - Ps \rightarrow 3\gamma)\) sorted events

Dot product \(\bar{s} \cdot (\vec{k}_1 \times \vec{k}_2)\)

(energy weighting)

\[ \cos(\theta) \ (\text{spin} -- \text{normal angle}) \]

\[ C_w < 0.008 \]

Scaled counts
Conclusions

- Charge conjugation symmetry: still conserved
- Improved measurement of five (!) photon decay of Ps
- Agreement with QED
- Anyone want to do 1st order corrections?
- \( (\text{Ps} \rightarrow 6\gamma) \) looks impossible to detect
- Time reversal symmetry/CPT: data look good so far.
- Sun to rise in east tomorrow.

- Gammasphere: a really nice instrument.
- Ps: an interesting little system, still useful.