Neutrino Mixing and Cosmology

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What are the consequences of neutrino oscillations in the early universe?

It depends whether the lepton number (asymmetry) is large.

This is true for both active-active and active-sterile neutrino mixing.
Unlike the CMB photons, the relic neutrino background has never been directly detected – so we have to infer its properties through indirect means.

Today, $T_v \sim 2K$, so:

If $m > 5 \times 10^{-4}$ eV, the relic neutrinos are non-relativistic today.

Given the solar and atmospheric mass squared differences:

$$\sqrt{\delta m^2_{atm}} > 0.04 \text{ eV}$$
$$\sqrt{\delta m^2_{solar(LMA)}} > 0.004 \text{ eV}$$

at least 2 of the 3 neutrinos states are non-relativistic today.
Neutrino contribution to the matter density:

Energy density = mass x number density

\[ \rho_\nu = m_\nu n_\nu \]

Large scale structure can tell us about \( \rho \).

Manoj Kaplinghat’s talk

So, provided we have a good understanding of \( n \), we can “weigh” neutrinos with cosmology

2dF + WMAP \( \rightarrow \) sum of the 3 neutrino masses < 0.7 eV

This assumes \( n \) has the standard value (negligible lepton number)
Baryon & Lepton Asymmetries

Baryon asymmetry: \[ B = \frac{n_b - n_{\bar{b}}}{n_\gamma} \sim 10^{-10} \]

Lepton asymmetry: \[ \rightarrow \text{only very weak constraints.} \]

Charge neutrality of the universe prevents a large asymmetry in the charged leptons, but a large lepton number could reside in the neutrino sector.

How would we know?
Relic neutrino background

In thermal equilibrium, the neutrinos have Fermi-Dirac distributions:

\[ f(p, \xi) = \frac{1}{1 + \exp(p/T - \xi)} \]

Lepton asymmetries imply chemical potentials: \( \xi \)

Lepton asymmetry:

\[ L_\alpha = \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_\gamma} = \frac{\pi^2}{12\zeta(3)} \left( \xi_\alpha + \frac{\xi_\alpha^3}{\pi^2} \right) \]

Asymmetries increase the effective number of species in equilibrium:

\[ \Delta N_\nu = \frac{30}{7} \left( \frac{\xi}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi}{\pi} \right)^4 \]
Constraints on relic neutrino asymmetries…

…in the absence of neutrino mixing

Big Bang Nucleosynthesis (BBN) + CMB set weak bounds on the lepton numbers:

\[ |\xi_{\mu,\tau}| < 2.6 \]

Very weak bound for \( \nu_{\mu,\tau} \):

\[ -0.01 < \xi_e < 0.22 \]

Stronger bound for \( \nu_e \): 

(See also Barger, Kneller, Langacker, Marfatia and Steigman, hep-ph/0306061)
Neutrinos and Big Bang Nucleosynthesis

Temperature $\sim$ MeV

Neutron to proton ratio set by the processes:

\[ n + \nu_e \leftrightarrow p + e^- \]
\[ n + e^+ \leftrightarrow p + \bar{\nu}_e \]

Practically all the neutrons eventually end up in Helium nuclei. All the leftover protons form Hydrogen.
\[ \frac{n}{p} \approx \exp\left[-\frac{(m_n - m_p)}{T}\right] \]

These processes “freeze out” when:

Interaction rate < Expansion rate

Expansion rate \( \propto \) energy density

\[ H = \frac{\dot{R}}{R} \propto \rho_{\text{radiation}} \]

expansion rate \( \rightarrow \) contribution from \( \gamma, e, \nu_e, \nu_\mu, \nu_\tau + \text{antiparticles} \)
If there were extra neutrinos (or any other relativistic particles)

• The universe would expand faster

• Weak interaction rates would freeze out earlier

• Larger n/p ratio and hence more Helium

Successful nucleosynthesis puts constraints on the expansion rate... and therefore tells us how many relativistic particles species were in thermal equilibrium.
\( \nu_e \) directly affects neutron-proton equilibrium:

\[
\begin{align*}
n + \nu_e & \leftrightarrow p + e^- \\
n + e^+ & \leftrightarrow p + \bar{\nu}_e
\end{align*}
\]

\[
n/p \approx \exp\left[-(m_n - m_p)/T - \xi_e\right]
\]

If there were an electron neutrino asymmetry:

eg. \( n(\nu_e) > n(\bar{\nu}_e) \)

\[
\Rightarrow \quad n/p \downarrow
\]

\[
Y_P \downarrow
\]
BBN (+CMB) constraint:

\[ |\xi_{\mu,\tau}| < 2.6 \]
\[ -0.01 < \xi_e < 0.22 \]

Note that the upper limits can only be obtained in tandem.

-- this is the degenerate BBN scenario, in which effects of the asymmetry in \( v_e \) is compensated by a faster expansion rate due to extra energy density.

Without this compensation the limit for \( v_e \) would be:

\[ |\xi_e| \sim 0.04 \]
How do neutrino oscillations change things?

• Active-sterile mixing

→ LSND inspired mixing schemes

• Active-active mixing

→ Oscillations with the solar and atmospheric parameters.
Sterile neutrinos?

Active -sterile oscillations in the early universe:

\[ \nu_{\text{active}} \leftrightarrow \nu_{\text{sterile}} \]

would thermalise the sterile neutrinos....

…and we know that BBN works more or less OK with just three neutrinos

In fact, successful BBN sets stringent bounds on active-sterile oscillation parameters:

eg. Assuming a BBN bound of \( N_\nu < 3.4 \)

Naive constraint on \( \nu_\mu \leftrightarrow \nu_s \) or \( \nu_\tau \leftrightarrow \nu_s \) mixing:

\[ |\delta m^2 (\sin^2 2\theta)^{1.6}| < 10^{-7} \text{ eV}^2 \]

e.g. Dolgov; Enqvist, Kainulainen & Thomson.
BBN says $N < 4$ → Sterile neutrinos are cosmologically disfavoured if they mix significantly with active neutrinos.

ALL “3+1” and “2+2” models which accommodate LSND are problematic for BBN. See di Bari (2001); Abazajian (2002) for recent analyses.

HOWEVER… there are ways out…

• Equilibration of the sterile is avoided if a lepton asymmetry is present → the mixing angle is suppressed due to the refractive index
  
  Foot & Volkas (1995)

• Some other much more exotic scenarios…
  ...low reheating scenarios, coherent majoron fields, etc ...

If MiniBooNE were to confirm LSND, it would be of enormous cosmological significance.
Mixing angle suppression:

The matter effected mixing angle is:

\[
\sin^2 2\theta_m = \frac{\sin^2 2\theta_0}{\sin^2 2\theta_0 + (2EV / \delta m^2 - \cos^2 2\theta_0)}
\]

\(V=\) effective matter potential

\[V(p) = \pm A(p) + B(p)\] \(+/\) for neutrinos/antineutrinos

Where: \[A(p) = \sqrt{2}G_F n_\gamma L^{(\alpha)}\] \(L=\) lepton number
Lepton numbers $> 10^{-3}$ will suppress the population of steriles with mixing parameters in the LSND range (until after neutrino decoupling)

However, there are further consequences of active sterile mixing:

Low temperature MSW resonances can destroy lepton number...this can also have consequences for BBN.
At low temperature, \( \sim O(\text{MeV}) \), an MSW resonance condition is fulfilled (for either neutrinos or antineutrinos, but not both.)

This will destroy lepton number. The active neutrino spectrum will thus be left distorted, since this depletion occurs after decoupling.

\[ \rightarrow \text{If the initial asymmetry were large, this could affect the BBN Helium yield.} \]

\( \text{(Abazajian, Bell, Fuller and Wong, work in progress.)} \)
Cosmological mass limits and sterile neutrinos

Cosmological neutrino mass limits are \( \sim \mathcal{O}(eV) \).

Q: Does this constrain LSND?  
A: Not necessarily.

So long as the sterile neutrino isn’t brought into thermal equilibrium in the early universe, the mass limits only to the three active neutrinos.

BBN considerations already suggest that the sterile cannot brought into thermal equilibrium.
Models with a sterile neutrino to accommodate LSND:

“3+1” scheme

“2+2” scheme

\( \delta m^2_{\text{LSND}} \)

\( \delta m^2_{\text{atmospheric}} \)

\( \delta m^2_{\text{solar}} \)
We now know that the Large Mixing Angle (LMA) solution is the correct resolution of the solar neutrino anomaly.

Early universe implication: Large-angle mixing could potentially equilibrate the flavours.


Matter effects are quite significant and must be included to determine if equilibration would take place before weak freeze-out.

Dolgov et al. (2002); Abazajian, Beacom and Bell (2002); Wong (2002).
MSW transition

\[ v_2 = \sin \theta v_e + \cos \theta v_\mu \]

\[ v_1 = \cos \theta v_e - \sin \theta v_\mu \]
The neutrino-neutrino forward scattering term makes the problem highly non-linear.

Note that this term includes both diagonal and off-diagonal refractive indices, the off-diagonal contributions coming from forward scattering processes of the type:

\[
\nu_\alpha (p) + \nu_\beta (k) \rightarrow \nu_\alpha (k) + \nu_\beta (p)
\]


Also, Friedland and Lunardini (2003)

The non-linear term dominates in size as long as the initial asymmetry is larger than about:

\[
L > 10^{-5}
\]

…that is, it dominates for all initial asymmetries of interest.
Evolution equations:

\[
\begin{align*}
\partial_t P_p &= (A_p + \alpha I) \times P_p \\
\partial_t \bar{P}_p &= (-A_p + \alpha I) \times \bar{P}_p
\end{align*}
\]

A = vacuum mixing term + non-neutrino background

I = neutrino-neutrino forward scattering term

\[
A_p = \frac{\delta m_0^2}{2p} (\sin 2\theta_0 \hat{x} - \cos 2\theta_0 \hat{z}) + V_B \hat{z}
\]

\[
I = \int \frac{d^3(p/T)}{(2\pi)^3} [P_p - \bar{P}_p]
\]
Without neutrino-neutrino forward scattering

With neutrino-neutrino forward scattering

Abazajian, Beacom & Bell
New Constraints:

\[ \xi_e^f \approx \left( \frac{1 - \cos 2\theta_0}{2} \right) \xi_\mu^i \]

Using the best fit value of the LMA mixing angle \( \sin^2 2\theta_0 \approx 0.8 \)

\[ \xi_e^f < 0.04 \quad \Rightarrow \quad \xi_\mu^i < 0.3 \]

Collisional processes will help make the equilibration more complete, as does non-zero \( U_{e3} \).

Degenerate BBN is eliminated since chemical potentials in any flavour will effectively impact neutron-proton equilibrium.
What does this mean for the relativistic energy density?

LMA solar neutrino solution \( \rightarrow \) close to complete flavour equilibration just before BBN, which sets the best limit on the lepton number of the universe:

Taking, very conservatively: \( \xi_\mu < 0.3 \)

the new limit (with nu mixing) is:

HUGE improvement over the old limit: (without mixing)

\[ \Delta N_\nu < 0.04 \]

\[ \Delta N_\nu < \text{a few} \]

Dolgov et al.; Wong; Abazajian, Beacom & Bell.

Implication: no uncertainty on \( n \) in cosmological determinations of mass via:

\[ \rho_\nu = m_\nu n_\nu \]
The end of the road for Degenerate Big Bang Nucleosynthesis?

Strictly speaking, we could still resurrect “degenerate” BBN with the presence of energy density in additional particle species.

i.e. it is possible that: 
\[ \xi_e \sim \xi_\mu \sim \xi_\tau \sim 0.2 \]

Provided another light particle species contributes the extra energy density required to compensate for the large electron-neutrino chemical potential.

This extra energy density can no longer consist of active-neutrinos….it would have to be something more exotic.

Such a non-standard contribution to the relativistic energy density would eventually be detectable via the CMB.

Abazajian, Beacom and Bell (2002);
Summary

• Cosmological limits on neutrino mass are competitive with the best laboratory limits … but they require that N(ν) is well constrained.

• Sterile neutrinos are cosmologically disfavoured by BBN… though constraints can be avoided if a neutrino asymmetry exists.

• The LMA (large mixing angle) solar neutrino solution
  → equilibration of neutrino flavours just before weak freezeout

• The stringent constraints on ν_e apply to all three flavours.
  → DBBN eliminated
  → Sets the tightest limit on the lepton number of the universe.