Abstract: While large-scale shell-model calculations may be useful, perhaps even essential, for reproducing experimental data, insight into the physical underpinnings of many-body phenomena requires a deeper appreciation of the underlying symmetries or near-symmetries of a system. Our program is focused on standard as well as novel algebraic approaches to nuclear structure ... many-body shell-model methods within a group theoretical framework.
postulate: $x(-2) + y(-12) = -12$
- Ongoing Research Projects -

- Pseudo-SU(3) calculations for heavy nuclei
- E2 / M1 (scissors) modes in deformed nuclei
  - Algebraic solution for non-degenerate pairing
  - Non-linear modes (surface solitons) of a drop
  - Boson and Fermion representations of sp(4)
- Mixed-mode (oblique) shell-model methods
- SU(3) calculations for lower fp-shell nuclei
E2 / M1 (scissors) modes in deformed nuclei ...

Study shell-model dynamics against the background of an extremely elegant algebraic/geometric picture

\[ \text{Rot}(3) \iff \text{SU}(3) \]

Macroscopic \quad Microscopic

Reproduce experimental M1 spectra including observed fragmentation and clustering in a microscopic model

Castaños / Draayer / Hirsch
Dirk Rompf (PhD-1998)
Thomas Beuschel (PhD-1999)
Carlos Vargas (PhD-2000)
Gabriela Popa (PhD-2001)

PRC 57 (1998) 1233; 1703
... PRC 61 (2000) 031301; 62 (2000) 064313 ...

Protons \quad Neutrons
Enhanced M1 strengths first predicted in 1978 by Lo Iudice and Palumbo within the framework of the so-called phenomenological two-rotor model (TRM) …
[N. Lo Iudice and F. Palumbo, PRL 41 (1978) 1532]

Enhanced M1 strengths first observed in 1984 in $^{156}$Gd …

Discussed within the context of the pseudo-SU(3) model …

Considered within the framework of the interacting boson (proton-neutron) model (IBM-2) …
[K. Heyde and C. De Coster, PRC 44 (1991) 2262]
Typical M1 Spectrum

M1 transition strength spectrum

$^{160}$ Gd (Experiment)

Clustering

Fragmentation*

*Adiabatic representation mixing ...

GS $\rightarrow$ $0^+$ $\rightarrow$ $1^+$

4 MeV

2 MeV
Basis States: Pseudo-SU(3)  
(spherical ↔ cartesian)

\[ |n[f]\alpha(\lambda, \mu)K(\text{LS})J\text{M}\rangle \]

\[ |n[f]\alpha(\lambda, \mu)\varepsilon\Lambda M\Lambda, S M\text{s}\rangle \]

spherical system

<table>
<thead>
<tr>
<th>particle number</th>
</tr>
</thead>
<tbody>
<tr>
<td>deformation [SU(3)]</td>
</tr>
<tr>
<td>multiplicity label (\lambda, \mu)</td>
</tr>
<tr>
<td>permutation symmetry</td>
</tr>
<tr>
<td>total orbital</td>
</tr>
<tr>
<td>total spin</td>
</tr>
<tr>
<td>a.m. proj.</td>
</tr>
</tbody>
</table>

transform between these yields a bit representation of basis states …

cartesian system

(Q0 \rightarrow z-deformation)

xy - orbital a.m.
xy - a.m. proj.

spin proj.

K-band [SU(3)]

\[ \text{xy - a.m. proj.} \]

\[ \text{xy - orbital a.m.} \]

\[ \text{total spin} \]

\[ \text{total a.m.} \]

\[ \text{a.m. proj.} \]
Tricks of the Trade
(config geometry ↔ algebraic shape)

Invariants ↔ Invariants

$\text{Rot}(3)$ ↔ $\text{SU}(3)$
$\text{Tr}(Q^2)$ ↔ $C_2$
$\text{Tr}(Q^3)$ ↔ $C_3$

$\beta^2 \sim \lambda^2 + \lambda\mu + \mu + 3(\lambda + \mu + 1)$

$\gamma = \tan^{-1} \left[ \sqrt{3} \mu / (2\lambda + \mu + 3) \right]$
Direct Product Coupling

Coupling proton and neutron irreps to total (coupled) SU(3):

\[(\lambda_\pi, \mu_\pi) \otimes (\lambda_\nu, \mu_\nu)\]

\[\rightarrow (\lambda_\pi + \lambda_\nu, \mu_\pi + \mu_\nu) \quad \text{GROUND STATE}\]

\[+ (\lambda_\pi + \lambda_\nu - 2, \mu_\pi + \mu_\nu + 1) \quad \text{SCISSORS}\]

\[+ (\lambda_\pi + \lambda_\nu + 1, \mu_\pi + \mu_\nu - 2) \quad \text{TWIST}\]

\[+ (\lambda_\pi + \lambda_\nu - 1, \mu_\pi + \mu_\nu - 1)^2 \quad \text{SCISSORS + TWIST}\]

\[+ \ldots\]

\[\rightarrow \sum_{m,l} \oplus (\lambda_\pi + \lambda_\nu - 2m + 1, \mu_\pi + \mu_\nu + m - 2l)^k \quad \rightarrow \ldots \text{multiplicity}\]

... Orientation of the \(\pi-\nu\) system is quantized with the multiplicity denoted by \(k = k(m,l)\)
Hamiltonian: Symmetry Preserving

SU(3) preserving Hamiltonian:

\[ H = c_1(Q_\pi \cdot Q_\pi + Q_\nu \cdot Q_\nu) + c_2 Q_\pi \cdot Q_\nu \]
\[ + c_3 L^2 + c_4 K^2 + c_5 (L_\pi^2 + L_\nu^2) \]

Using the identities ...

\[ Q_\sigma \cdot Q_\sigma = 4 C_{2\sigma} - 3 L_\sigma^2 \text{ with } \sigma = \pi \text{ or } \nu \]
\[ Q_\pi \cdot Q_\nu = [Q \cdot Q - Q_\pi \cdot Q_\pi - Q_\nu \cdot Q_\nu]/2 \]
\[ L = L_\pi + L_\nu \]
\[ \ell = L_\pi - L_\nu \]

the Hamiltonian becomes ...

\[ H = H_{\text{rot}} + H_{\text{int}} \]

with rotor & interactions terms ...

\[ H_{\text{rot}} = a L^2 + b K^2 \]
\[ H_{\text{int}} = c \ell^2 + d C_2 + e (C_{2\pi} + C_{2\nu}) \]
System Dynamics

\[ H_{\text{rot}} = a \ell^2 + b K^2 \]
\[ \rightarrow \text{quantum rotor} \]

\[ H_{\text{int}} = c \ell^2 + d \ell C_2 + e(C_{2\pi} + C_{2\nu}) \]
\[ \downarrow \quad \downarrow \]
\[ \text{kinetic} \quad \text{potential} \]

It can be shown that:

\[ H_{\text{int}} = c_\theta \ell_\theta^2 + d_\theta \theta^2 \]
\[ + c_\phi \ell_\phi^2 + d_\phi \phi^2 \]

where: \( \phi = \phi_{\pi} - \phi_{\nu} \)
so that:

\[ E_{\text{int}} = \hbar \omega_\theta (n_\theta + 1/2) \]
\[ + \hbar \omega_\phi (n_\phi + 1/2) \]

where: \( n_\theta = m, \quad n_\phi = l \)

Recall: dynamics driven by interplay between the interaction and statistics

...always present, axial and triaxial

Scissors Mode

Twist Mode

...requires triaxial shape distributions
M1 Operator

\[ \text{M1} = \left( \frac{3}{4} \right)^{1/2} \mu_N \sum_{\sigma} (g^{\text{orbit}}(\sigma) L^\sigma + g^{\text{spin}}(\sigma) S^\sigma) \]

- \( g^{\text{orbit}}(\text{proton}) = 1 \)
- \( g^{\text{orbit}}(\text{neutron}) = 0 \)
- \( g^{\text{spin}}(\text{proton}) = 5.5857 \)
- \( g^{\text{spin}}(\text{neutron}) = -3.8263 \)
Scissors and Twist Mode Examples

**M1 transition strength spectrum**

- $^{156}$Gd (Experiment)
  - Axial
  - Triaxial

- Conserved pseudo SU(3)
  - $^{(26,5)}$ scissors
  - $^{(27,3)}$ scissors + twist

**M1 transition strength spectrum**

- $^{160}$Gd (Experiment)
  - Triaxial
  - Triaxial

- Conserved pseudo SU(3)
  - Scissors
  - $^{(26,9)}$ scissors
  - $^{(27,7)}_{1,2}$ scissors + twist

- Twist
  - $^{(29,6)}$ twist
Example: M1 Modes in $^{156-160}\text{Gd}$

Valence space: $U(\Omega_\pi) \otimes U(\Omega_\nu)$

<table>
<thead>
<tr>
<th></th>
<th>total</th>
<th>normal</th>
<th>unique</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_\pi$</td>
<td>32</td>
<td>20</td>
<td>12 (h$_{11/2}$)</td>
</tr>
<tr>
<td>$\Omega_\nu$</td>
<td>44</td>
<td>30</td>
<td>14 (i$_{13/2}$)</td>
</tr>
</tbody>
</table>

Particle distributions:

<table>
<thead>
<tr>
<th></th>
<th>total</th>
<th>normal</th>
<th>unique</th>
<th>$($λ,μ$)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$:</td>
<td>$^{156-160}\text{Gd}$</td>
<td>14</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>$\nu$:</td>
<td>$^{156}\text{Gd}$</td>
<td>10</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>$^{158}\text{Gd}$</td>
<td>12</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>$^{160}\text{Gd}$</td>
<td>14</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>
Hamiltonian: Realistic (Version 1)

\[ H = H_0 \]

- \( a_2 C_2 \)
- \( a_3 C_3 \)
- \( b K^2 \)
- \( c L^2 \)
- \( d_\pi \sum l^2_\pi \)
- \( d_\nu \sum l^2_\nu \)
- \( g_\pi H_p^\pi \)
- \( g_\nu H_p^\nu \)

\[ \text{Oscillator} \quad \begin{array}{l}
\text{Q\cdot Q symmetric} \\
\text{Q\cdot(Q\times Q) asymmetric} \\
\text{K-Band splitting} \\
\text{Rotational bands} \\
\text{Proton single-particle } l^2 \\
\text{Neutron single-particle } l^2 \\
\text{Proton Pairing} \\
\text{Neutron Pairing} \\
\end{array} \]

\[ \text{Symmetry Preserving} \]
\[ \text{Symmetry Breaking} \]

\[ \text{mixed irrep shell-model basis with} \]
\[ \text{5 proton and 5 neutron irreps plus} \]
\[ \text{all products having } S = 0 \text{ and } J \leq 8 \]
Results for $^{156-160}$ Gd Nuclei

Energy [MeV]

$^{156}$Gd

Energy [MeV]

$^{158}$Gd

Energy [MeV]

$^{160}$Gd

Exp.  Theory

Exp.  Theory

Exp.  Theory

Exp.  Theory
$^{160}\text{Gd}: \text{ M1 Strength Distribution}$

![Graph showing M1 strength distribution for $^{160}\text{Gd}$](image)

- **Energy [MeV]**
- **$B(M1, 0^+ \rightarrow 1^+)$ [$\mu_n^2$]**

- **Exp.**
- **Theory**

The graph compares experimental and theoretical data for the M1 transition strength in $^{160}\text{Gd}$. The distribution peaks at specific energy levels, indicating the transition probability for various energy intervals.
Fragmentation of M1 Strength in $^{160}$Gd

M1 transition strength spectrum
(different Hamiltonians)
Results for $^{160-164}$ Dy Nuclei

\[ \begin{align*}
\text{Energy} [\text{MeV}] & \\
160\text{Dy} & \\
162\text{Dy} & \\
164\text{Dy} &
\end{align*} \]
M1 Sumrule vs E2 Strength
Triaxial Case: $^{196}\text{Pt}$

Sumrule ~ 1.5 $\mu^2$ or about one-half that of the strongly deformed case.
## Analytic Results

<table>
<thead>
<tr>
<th>Shape</th>
<th>Mode</th>
<th>B(M1) $[\mu^2_N]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a + a$</td>
<td>$(1,0)$</td>
<td>$s$</td>
</tr>
<tr>
<td>$a + t$</td>
<td>$(1,0)$</td>
<td>$s$</td>
</tr>
<tr>
<td>$t + t$</td>
<td>$(1,0)$</td>
<td>$s$</td>
</tr>
</tbody>
</table>

Note: $t \leftrightarrow s \quad (\lambda,\mu) \leftrightarrow (\mu,\lambda)$
Odd-A Case: $^{163}$Dy

Energy Spectrum $^{163}$Dy

Ground State Band

exp.  theo.  exp.  theo.  exp.  theo.

Energy [MeV]

M1 transition strength $^{163}$Dy

Recent Results: Beuschel/Hirsch
Hamiltonian: Realistic (Version 2)

\[ H = H_0 - \frac{\chi}{2} Q \cdot Q + a_3 C_3 + b K^2 + c L^2 + a_{\text{sym}} \Delta C_2 + d_\pi \sum l_\pi^2 + d_v \sum l_v^2 + g_\pi H_p^\pi + g_v H_p^v \]

- Oscillator
- Q\cdot Q symmetric
- Q\cdot(Q\times Q) asymmetric
- K-Band splitting
- Rotational bands
- Intrinsic symmetry
- Proton single-particle \( l^2 \)
- Neutron single-particle \( l^2 \)
- Proton Pairing
- Neutron Pairing

Symmetry Preserving

Symmetry Breaking

... mixed irrep shell-model basis with
~5 proton and ~5 neutron irreps &
all products having \( S = 0 \) and \( J \leq 8 \)
## Hamiltonian Parameters

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\chi$</th>
<th>$G_\pi$</th>
<th>$G_v$</th>
<th>HO</th>
<th>a</th>
<th>b</th>
<th>$a_3$</th>
<th>$a_{\text{sym}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{156}\text{Gd}$</td>
<td>0.0077</td>
<td>0.135</td>
<td>0.109</td>
<td>7.6163</td>
<td>0.003</td>
<td>0.165</td>
<td>9.5 $10^{-5}$</td>
<td>0.0</td>
</tr>
<tr>
<td>$^{158}\text{Gd}$</td>
<td>0.0076</td>
<td>0.109</td>
<td>0.108</td>
<td>7.5839</td>
<td>0.001</td>
<td>0.170</td>
<td>1.01 $10^{-4}$</td>
<td>0.81 $10^{-3}$</td>
</tr>
<tr>
<td>$^{160}\text{Gd}$</td>
<td>0.0074</td>
<td>0.106</td>
<td>0.106</td>
<td>7.5523</td>
<td>0.002</td>
<td>0.150</td>
<td>1.71 $10^{-4}$</td>
<td>1.91 $10^{-3}$</td>
</tr>
<tr>
<td>$^{160}\text{Dy}$</td>
<td>0.0074</td>
<td>0.106</td>
<td>0.106</td>
<td>7.5523</td>
<td>0.008</td>
<td>0.000</td>
<td>1.51 $10^{-5}$</td>
<td>0.81 $10^{-3}$</td>
</tr>
</tbody>
</table>

\[
- \frac{\chi}{2} \mathbf{Q} \cdot \mathbf{Q} \\
+ d_\pi \sum l^2_\pi \\
+ d_v \sum l^2_v \\
+ g_\pi H_\pi \\
+ g_v H_v
\]

\[
+ a_3 C_3 \\
+ b K^2 \\
+ c L^2 \\
+ a_{\text{sym}} \Delta C_2
\]
$^{158}$Gd

![Graph showing energy levels and $B(M1, 0^+ \rightarrow 1^+)$ transitions for $^{158}$Gd.](image)
$^{160}$Gd

Energy [MeV]

$B(M1, 0^+ \rightarrow 1^+)$ [$\mu_n^2$]

Energy [MeV]
Some Observations

- Scissors mode (proton-neutron) oscillation always exists in deformed systems.
- Extra “twist” mode if triaxial proton or neutron distributions are included.
- Fragmentation can be interpreted as effect of non-collective residual interactions.
- Gross structure reproduced by adding a new “twist” to the “scissors” system.
- Triaxial proton-neutron configuration gives a physical interpretation of multiplicity.
- Analytic results for scissors as well as twist modes and combination now available.
- Future research with S=1 mode for odd-A as well as even-even nuclei underway.
Conclusions - Part 1: E2 / M1 modes ...

- Near perfect excitation spectra …
- B(E2) values good (effective charge) …
- Scissors mode always present …
- Twist mode(s) requires triaxiality …
- Fragmentation via symmetry mixing …
Mixed-mode (oblique) shell-model methods ...

**Mixed-Mode SMC** ... a new shell-model code (SMC) that integrates the best shell model methods available:

- m-scheme spherical shell-model
- SU(3) symmetry based shell-model

----- Developers ----- 
Vesselin Gueorguiev  
Jerry Draayer  
Erich Ormand  
Calvin Johnson
The Challenge ...

Nuclei display unique and distinguishable characteristics:

- Single-particle Features
- Pairing Correlations
- Deformation/Rotations
Usual Shell-Model Approach

• Select a model space based on a simple approximation (e.g. Nilsson scheme) for the selection of basis states.

• Diagonalize a realistic Hamiltonian in the model space to obtain the energy spectrum and eigenstates.

• Evaluate electromagnetic transition strengths (E2, M1, etc.) and compare the results with experimental data.

Dual Basis “Mixed Mode” Scheme

• Select a dual (non-orthonormal) basis (e.g. pairing plus quadrupole scheme) for the interaction Hamiltonian.

• Diagonalize a realistic Hamiltonian in the model space to obtain the energy spectrum and eigenstates.

• Evaluate electromagnetic transition strengths (E2, M1, etc.) and compare the results with experimental data.
Hamiltonian Matrix

standard jj-coupling
shell-model scheme

single-particle
pairing driven

overlap

overlap

SU(3)
Shell-Model Hamiltonian

\[ H = \sum_i \varepsilon_i a_i^+ a_i + \sum_{i,j,k,l} V_{ijkl} a_i^+ a_j^+ a_k a_l \]

where \( a_i^+ \) and \( a_i \) are fermion creation and annihilation operators, \( \varepsilon_i \) and \( V_{ijkl} \) are real and \( V_{ijkl} = V_{klij} = -V_{jikl} = -V_{ijlk} \)

- Spherical shell-model basis states are eigenstates of the one-body part of the Hamiltonian - **single-particle states**.

- The two-body part of the Hamiltonian \( H \) is dominated by the **quadrupole-quadrupole interaction** \( Q \cdot Q \sim C_2 \) of SU(3).

- SU(3) basis states - **collective states** - are eigenstates of \( H \) for degenerate single particle energies \( \varepsilon \) and a pure \( Q \cdot Q \) interaction.
Eigenvalue Problem in an Oblique Basis

- Spherical basis states $e_i$
- SU(3) basis states $E_\alpha$

- Overlap matrix

$$\mathbf{g} = \begin{pmatrix}
\langle e_i | e_j \rangle & \langle e_i | E_\beta \rangle \\
\langle E_\alpha | e_j \rangle & \langle E_\alpha | E_\beta \rangle
\end{pmatrix} = \begin{pmatrix}
1 & \mu \\
\mu^* & 1
\end{pmatrix}$$

- The eigenvalue problem

$$H\psi = E\psi \quad \Rightarrow \quad \mathbf{g} \cdot \psi = E\mathbf{g} \cdot \psi$$
Current Evaluation Steps

Matrix elements

(\mathbf{H} \text{ and } \mathbf{g})

\begin{align*}
\mathbf{H} \text{ and } \mathbf{g} \\
\text{SM basis} \\
\text{(spherical)} \\
\text{SU(3) basis} \\
\text{(cylindrical)} \\
m\text{-scheme}
\end{align*}

\begin{align*}
\varepsilon_i \text{ and } \langle j_1 j_2 J^I | \mathbf{M} | j_3 j_4 J^I \rangle
\end{align*}

\begin{align*}
g = U U^T \text{ (Cholesky)} \\
(\mathbf{U}^{-1})^T \mathbf{H} \mathbf{U}^{-1} (\mathbf{U} \Psi) = \mathbf{E} (\mathbf{U} \Psi) \\
\text{Eigenstates (Lanczos)}
\end{align*}

\begin{align*}
\langle \mathbf{O} \rangle \text{ and } \langle \mathbf{E}_1 | \mathbf{O} | \mathbf{E}_2 \rangle
\end{align*}
Example of an Oblique Basis Calculation: $^{24}\text{Mg}$

We use the **Wildenthal USD interaction** and denote the **spherical basis** by SM(#) where # is the number of nucleons outside the $d_{5/2}$ shell, the **SU(3) basis** consists of the leading irrep $(8,4)$ and the next to the leading irrep, $(9,2)$.

<table>
<thead>
<tr>
<th>Model Space</th>
<th>SU3 (8,4)</th>
<th>SU3+ (8,4) &amp; (9,2)</th>
<th>GT100</th>
<th>SM(0)</th>
<th>SM(1)</th>
<th>SM(2)</th>
<th>SM(4)</th>
<th>Full</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension</td>
<td>23</td>
<td>128</td>
<td>500</td>
<td>29</td>
<td>449</td>
<td>2829</td>
<td>18290</td>
<td>28503</td>
</tr>
<tr>
<td>(m-scheme)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>%</td>
<td>0.08</td>
<td>0.45</td>
<td>1.75</td>
<td>0.10</td>
<td>1.57</td>
<td>9.92</td>
<td>64.17</td>
<td>100</td>
</tr>
</tbody>
</table>

**Visualizing** the SU(3) space with respect to the SM space using the naturally induced basis in the SU(3) space.
Better Dimensional Convergence!

Ground State Convergence for 24Mg

Ground State Energy (MeV)

Number of Basis States

-94 -91 -88 -85 -82 -79 -76 -79 -82 -85 -88 -91

-2000 4000 10000 16000 22000 28000

-3.3 MeV (0.5%) -4.2 MeV (54%)

SM(0) SM(1) SM(2) SM(3) SM(4) SM(5) SM(6) FULL

- SM ground state
- SM+1 SU(3) irrep
- SM+2 SU(3) irreps
Level Structure

Energy (MeV)

SM(0)  SM(1)  SM(2)  SM(4)  FULL

(8,4)  (8,4)  (9,2) More irreps

0  2  3  4  4  2  2
Oblique Basis Spectral Results
Oblique basis calculations have the levels in the correct order!

- **Spherical basis** needs 64.2% of the total space - SM(4)!
- **SU(3) basis** needs 0.4% of the total space - (8,4)&(9,2)!
- **Oblique basis** 1.6% of the total space for SM(1)+(8,4)!
- **SM(0)+(8,4)** is almost right… 0.2%
Overlaps With The Exact Eigenvectors For 24Mg

<table>
<thead>
<tr>
<th>Eigenvectors</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM(2)</td>
<td>57.77</td>
<td>53.02</td>
<td>39.78</td>
<td>42.50</td>
<td>42.99</td>
<td>35.92</td>
</tr>
<tr>
<td>SU(3)+</td>
<td>63.02</td>
<td>63.77</td>
<td>71.49</td>
<td>59.46</td>
<td>70.15</td>
<td>54.14</td>
</tr>
<tr>
<td>SM(2)+SU(3)+</td>
<td>91.58</td>
<td>90.95</td>
<td>87.72</td>
<td>89.06</td>
<td>87.35</td>
<td>82.23</td>
</tr>
<tr>
<td>SM(4)</td>
<td>93.25</td>
<td>92.81</td>
<td>89.98</td>
<td>92.47</td>
<td>91.10</td>
<td>88.33</td>
</tr>
<tr>
<td>SM(4)+SU(3)+</td>
<td>98.57</td>
<td>98.73</td>
<td>97.92</td>
<td>98.41</td>
<td>98.55</td>
<td>96.59</td>
</tr>
</tbody>
</table>
Oblique basis calculations have better overlap with the exact states!

- Using 10% of the total space in oblique basis $\text{SM}(2)+(8,4)+(9,2)$

  as good as

- using 64% of the total space in spherical basis $\text{SM}(4)$
Conclusions - Part 2: Mixed modes ...

Mixed mode (oblique basis) calculations lead to:

- better dimensional convergence
- correct level order of the low-lying states
- significant overlap with the exact states
Further Considerations #1

Traditional configuration shell-model scheme(s) versus SU(3) and symplectic shell-model approaches

- Common “bit arithmetic” (Slater determinants)
- Both use harmonic oscillator basis functions ...
- Translation invariance okay for both approaches
- Both use effective reduced matrix element logic
- One single-particle the other quadrupole driven

Complementary ... ???
Further Considerations #2

Configuration Shell-Model (multi-$h\omega$)

Symplectic Shell-Model (multi-$h\omega$)

$4h\omega$, $2h\omega$, $0h\omega$

horizontal slices

$6h\omega$ symplectic slice

SU(3) limit ($0h\omega$)

vertical slices

translation invariance okay for both
Further Considerations #3

... mixed-mode shell-model ...

Q Space

\[ x\hbar \omega \]

\[ y\hbar \omega \]

... symplectic ...

P Space

postulate: \( x(\sim 2) + y(\sim 12) = \sim 12 \)
Abstract: … consider standard as well as novel algebraic approaches to nuclear structure, including use of the Bethe ansatz and quantum groups, that are being used to explore special features of nuclei: pairing correlations, quadrupole collectivity, scissors modes, etc. In each case the underlying physics is linked to symmetries of the system and their group theoretical representation.

- E2 / M1 (scissors) modes in deformed nuclei
- Mixed-mode shell-model methods feasible
- Possible configuration + symplectic extension