Light Front Quantum Mechanics and the Form Factor

OR: Waiting for Mathematica

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Light Front Quantum Mechanics and the Form Factor
The Light Front Variables were "invented" by Dirac in 1949.

\[
\frac{+d}{\zeta u + \zeta d} = -d
\]

which gives a really cool equation:

\[
\zeta d - _d + d = \zeta d - \zeta^d \zeta d - \zeta d = \eta d^d d = \zeta u
\]

With momentum variables defined similarly:

\[
\zeta \eta \zeta + xx = \zeta x
\]

\[
z - t = _x
\]

\[
z + t = +x
\]

The Light Front (LF) variables (in natural units, where \( c = 1 \))
What Do the LF Variables Mean?

QM commutation relations are:
\\( h^x_j p^k_i = i \delta^{jk} \delta_{ij} \)

Lightfront commutators are:
\\( +d^i -x \)
\\( _-d^i +x \)
\\( +d^i +x \)
\\( -d^i -x \)

So we think of the third spatial variable as the third momentum variable, while the third momentum variable is likened to time and +d to energy.
The Two-Particle Problem

The process we care about:

For the sake of simplicity we work with the 2 particle problem.

I can do yet.

We’re thinking about the proton, which is a 3 particle prob.
Now what?

\[ \frac{d}{\sqrt{d}} = x \]

The Bjorken variable, \( x \), gives the fraction of the total plus momentum that particle one has.

Another question?

\[ \frac{(x - 1)}{\frac{Z}{2} \mu + \frac{\Sigma}{2} d} + \frac{x}{\frac{1}{2} \mu + \frac{1}{2} d} = s \]

To find relativistically correct wave functions we need the energy squared operator, which we call \( s \):

\[ \frac{1}{\Gamma} d + \frac{1}{\Gamma} d = \Gamma d \]

Letters denote the sum of both particles, e.g., \( \Gamma d \) denotes the sum of the two particles. Variables of particle (or 2) are denoted by a subscript. Big
\[
\begin{align*}
\varepsilon u + \varepsilon d &= \mathcal{H} \\
\varepsilon (\varepsilon d) + \frac{\tau}{\varepsilon} d &= d \\
\frac{(x - \mathbb{I}) x}{\varepsilon u + \frac{\tau}{\varepsilon} d} (\varepsilon - x) &= \varepsilon d
\end{align*}
\]

Now define \( \varepsilon \) using a new variable \( \varepsilon \). We can rewrite
\[
\frac{(x - \mathbb{I}) x}{\varepsilon u + \frac{\tau}{\varepsilon} d} = s
\]

means \( \mathbb{I} d \equiv \mathbb{I} d = \tau d \) and \( u \equiv \varepsilon u = \mathbb{I} u \).

More Useful Variables
A relativistic equation for a starting equation for relativistic QM is

\[ \Phi(M + \gamma^d) = \Phi(\gamma m - \frac{\Phi}{\gamma}) \]

where \( W \) is the interaction,

\[ \Phi W + \Phi^s = \Phi \gamma^d W \]

\( \Phi \) eignevalue equation – doesn’t it look a lot like the Schrodinger equation?

Letting \( M = \frac{\gamma m}{e} \) we can rewrite this, using our nifty variables, as:

A relativistic equation for \( \Phi \)
Plan of Attack

- Model quark interactions as simple central potentials studied in elementary QM
- Use $\Psi(x)$ solution to SE to find $u(x)$
- Set constants in $\Psi$ so that we get $\langle \Psi | \Psi \rangle = \frac{1}{2}$ fm

We now have a relativistic $\Psi$. (d)

$$\begin{align*}
\frac{d^2}{d^2} - \left( \frac{\mathbf{x}}{\Psi} - \mathbf{I} \right)^4 \mathbf{v} + \frac{1}{2} \mathbf{d} &= \mathbf{v}^2 \left( \varepsilon_d \right) + \frac{1}{2} \mathbf{d} \left( \varepsilon_d \right) \\
\text{(d)} \Psi &\text{ Make substitution in } \Psi \text{ to find } \mathbf{v} \text{ solution to SE}
\end{align*}$$
The Form Factor-What and Why

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\[ \mathcal{F}(\bar{p}) \frac{d\Sigma p}{dp} = \frac{\Sigma p}{dp} \]

Perimenetally since:

It is an important quantity because \( \mathcal{F} \) can be measured ex-

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a state \( \bar{p} \) after one of the constituent quarks is given a
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Three Different Equations for $F$

\[ \frac{\partial}{\partial b} \left( z \frac{\partial}{\partial x} \phi \right) \psi \mid_{z \phi \partial x} \int \frac{\partial}{\partial \psi} = \left( \frac{\partial b}{\partial q} \right)_{R_H} \]

Where $\psi$ is the relativistically correct, mind you.

Note: Normalization requires $F(0) = 1$.

**The LF Equation:**

\[ (x \frac{\partial}{\partial x} \phi (x - \psi) + \frac{\partial}{\partial \psi} \right) \psi \mid_{x \phi \partial x} \int = \left( \frac{\partial b}{\partial q} \right)_{R_H} \]

In the Breit Frame:

\[ \frac{\partial}{\partial b} \left( z \frac{\partial}{\partial x} \phi \right) \psi \mid_{z \phi \partial x} \int = \left( \frac{\partial b}{\partial q} \right)_{R_H} \]

The non-relativistic equation (the following are solutions $\phi$, are solutions to the SE):

\[ (x \frac{\partial}{\partial x} \phi (x - \psi) + \frac{\partial}{\partial \psi} \right) \psi \mid_{x \phi \partial x} \int = \left( \frac{\partial b}{\partial q} \right)_{R_H} \]
The Harmonic Oscillator

The ground state solution to the SE with $V = \frac{1}{2}m \dot{x}^2$ is

$$\psi = \frac{1}{\sqrt{\pi \sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$
Plot of the three F’s
HO Form Factors at High $q$
The Coulombic Potential

\[
\frac{\varepsilon (\varepsilon d^2 \varepsilon + 1)}{1} B = (d) \Psi
\]

Ground state solution: \( \Psi \) •

\[
\frac{d}{\varepsilon} = (d) \nabla \Psi
\]

Potential: \( \nabla \Psi \) •
Coulomb F’s
High q
The NR limit of \( F \)

- In the limit \( \frac{p^2}{m^2} \ll 1 \) then \( F \to F_{nr} \)

- Keeping more terms gives a better \( F_{nr}(q^2) \)

- Work in progress... sorry, no pictures yet.
The Search for an Inverse Transformation

It would be nice to be able to calculate

$$\frac{x - 1}{q(x - 1)x^\lambda} \frac{d}{dx} P \int_1^0 \left( \frac{x}{1 + \frac{1}{x}} \right) dx = \left( \frac{q}{r} \right)$$

More work is being done... might not be possible.

So far:

I've been working on finding a way

It would be nice to be able to calculate given

The Search for an Inverse Transformation
Conclusions: What I’ve gained

- Learned LaTeX
- Learned how to use Mathematica
- Got to work on problems that are unsolved
- Calculated lots of form factors
- Now know what is and how to calculate the form factor
- LF coordinates can simplify relativistic GM