Emergent Frontiers in Quantum Materials: High Temperature superconductivity and Topological Phases

Jiun-Haw Chu
University of Washington
The nature of the problem in Condensed Matter Physics

Consider a small piece (mm size) of metal.

- \( \sim 10^{20} \) valence electrons and atoms
- They are all mutually interacting via electromagnetic forces and Pauli exclusion principles
- The goal of condensed matter physics is to understand and ultimately control the emergent collective behavior

Problem: How do we solve a Schrodinger Equation with \( \sim 10^{20} \) degrees of freedom?

\[
\mathcal{H} = - \sum_{j}^{N_e} \frac{\hbar^2}{2m} \nabla_j^2 - \sum_{\alpha}^{N_i} \frac{\hbar^2}{2M_\alpha} \nabla_\alpha^2 \\
- \sum_{j}^{N_e} \sum_{\alpha}^{N_i} \frac{Z_\alpha e^2}{|\vec{r}_j - \vec{R}_\alpha|} + \sum_{j \ll k}^{N_e} \frac{e^2}{|\vec{r}_j - \vec{r}_k|} + \sum_{\alpha \ll \beta}^{N_j} \frac{Z_\alpha Z_\beta e^2}{|\vec{R}_\alpha - \vec{r}_\beta|}.
\]
Diverse behavior emerging from a simple equation

Diverse intriguing physical phenomena arising from the collective behaviors of electrons and atoms in REAL MATERIALS.

Intriguing physical phenomena:

Superconductivity, Charge/Spin density wave, Ferromagnetism, Anti-ferromagnetism, Ferroelectricity, Antiferroelectricity, Band Insulator Mott Insulator, Anderson Insulator, Heavy Fermion, High temperature superconductor Frustrated magnet, Spin ice, Spin liquid, Integer/Fractional Quantum Hall Effect Quantum Spin Hall effect, Topological Insulator, Topological superconductor, Topological semimetal, Dirac Fermion, Weyl Fermion, Majorana Fermion, ....
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- Band Insulator Mott Insulator, Anderson Insulator,
- Heavy Fermion, High temperature superconductor
- Frustrated magnet, Spin ice, Spin liquid,
- Integer/Fractional Quantum Hall Effect
- Quantum Spin Hall effect, Topological Insulator,
- Topological superconductor, Topological semimetal,
- Dirac Fermion, Weyl Fermion, Majorana Fermion, ....

• The task seems obvious – we just need to solve the equation and make the prediction.
• Problem – there is almost no exact solution beyond two particles. Even with powerful computer it’s hard to solve numerically.
• Just to give you an idea, you need to diagonalize a $10^{18} \times 10^{18}$ matrix if you just simply consider 32 electrons occupying 8x8 lattice sites with a single quantum state.
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Physicist built powerful phenomenological models to describe the emergent behaviors observed in experiment

Landau Fermi Liquid theory

BCS Theory of superconductivity

Landau theory of symmetry breaking phase transition

......
Quantum Materials: beyond the standard model

New materials and phenomenon challenge the old paradigm, they also bring in new concept, such as quantum critical point, topology,

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A quick comparison to the situations in high energy physics

• There are good “Standard Models”
  • Landau Fermi Liquid theory
  • BCS Theory of superconductivity
  • Landau theory for symmetry breaking
  • Wilson-Fisher theory of criticality
  • ...

• The Theory of Everything is known

\[ \mathcal{H} = -\sum_j \frac{\hbar^2}{2m} \nabla_j^2 - \sum_\alpha \frac{\hbar^2}{2M_\alpha} \nabla_\alpha^2 \\
- \sum_j \sum_\alpha \frac{Z_\alpha e^2}{|\vec{r}_j - \vec{R}_\alpha|} + \sum_{j<k} \frac{e^2}{|\vec{r}_j - \vec{r}_k|} + \sum_{\alpha<\beta} \frac{Z_\alpha Z_\beta e^2}{|\vec{R}_\alpha - \vec{r}_\beta|} \]

• Experiments for physics beyond standard model are abundant

Non-Fermi liquid behavior in high temperature superconductors

Topological phases that cannot be classified by broken symmetry
How do we make this happen?

The art of crystal growth:
Learn to become an alchemist

Hydrogen torch station $T \sim 3000K$

Tube and box furnaces $T \sim 1500K$
How do we make this happen?

And then we take it down to low temperatures and high magnetic field

Applying elastic strain field

PPMS Dynacool T \sim 2K \quad B \sim 14T

Janis flow cryo system 500K \sim 80K (LN_2) or 10K (LHe)
• High temperature superconductivity

• Topological phases
A short introduction of superconductivity
Bardeen-Cooper-Schrieffer theory of superconductivity

Electrons bind into Cooper pairs by phonon interactions.

Cooper pairs condensate into macroscopic wave function.

The phonon set the energy scale of this phenomenon, therefore set the critical temperature.
A short history of superconductivity

- 1911 Kamerlingh Onnes discovered the first superconductor Hg ~ 4K

- 1957 Bardeen – Cooper – Schrieffer theory of superconductivity

- 1986 Cuprates (Tc ~ 100K )

- 2008 Iron based (Tc ~ 50K)
High temperature superconductors: the big question

The parent stoichiometric compound is almost always non-superconducting. Optimal superconductivity needs to be induced by chemical substitutions.

In the T-x phase diagram, superconductivity is always interweaved by various lines: phase boundary, cross-over....

The big question in high Tc: what are these lines? Are they phase boundaries? Symmetry breaking phase transitions? What symmetry is broken?

Some of these phases are well studied but some others are poorly understood, eg. Electronic nematic phase.
Broken rotational symmetry in solid crystals: **Electronic nematic phase**

**Nematic liquid crystal phase**, long molecules spontaneously breaks full rotational symmetry.

**Electronic nematic phase**, spontaneous electronic order breaks discrete rotational symmetry.

**Electrical ferro-quadrupole order**

Example of electron nematic phase in real space
Broken rotational symmetry in solid crystals: 

Electronic nematic phase

Soft condensed matter

Nematic liquid crystal phase, long molecules spontaneously breaks full rotational symmetry.

Strongly correlated electronic system

Electronic nematic phase, spontaneous electronic order breaks discrete rotational symmetry

Example of electron nematic phase in momentum space
The signature of an electronic nematic phase: Divergent elastoresistance

\[ R = \frac{\rho L}{A} \Rightarrow \frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta L}{L} - \frac{\Delta A}{A} \]

However, not all materials behave the same way, and it’s related to the idea of
Broken symmetries
High temperature superconductors: big question

If nematic fluctuations are responsible for superconducting pairing, can we design new superconductors by deliberately create a nematic quantum critical point?

Fe pnictides:
• High temperature superconductivity

• Topological phases
What is a topological phase

• A major goal of condensed matter physics is to discover and understand new state of matter.
• New states of matter often can be characterized by the symmetries they break.

Ferromagnet

Antiferromagnet

Superconductor

Time reversal
Rotational

Time reversal
Rotational

Gauge symmetry

Translational
What is a topological phase

More precisely, topology studies properties that are preserved under continuous deformations, including stretching and bending, but not tearing or gluing. -- Wikipedia

$g$ is an integer topological invariant that can be expressed in terms of the gaussian curvature $\kappa$ that characterizes the local radii of curvature

$$\kappa = \frac{1}{r_1 r_2}$$

Gauss Bonnet Theorem:

$$\int_S \kappa dA = 4\pi (1 - g)$$
First Topological phase – Quantum Hall Effect

For theoretical discoveries of topological phase transitions and topological phases of matter

Quantized Hall conductivity:

\[ J_y = \sigma_{xy} E_x \]

\[ \sigma_{xy} = n \frac{e^2}{h} \]

Integer accurate to $10^{-9}$

TKNN number = Chern number

\[ n = \frac{1}{2\pi} \int_{BZ} d^2 k F(k) = \frac{1}{2\pi} \oint_C A \cdot dk \]

Thouless, Kohmoto, Nightingale and den Nijs 82
Quantum Hall effect without time reversal symmetry breaking: Quantum Spin Hall Effect

Energy gaps in graphene:

\[ H = v_F \sigma \cdot p + V \]

\[ E(p) = \pm \sqrt{v_F^2 p^2 + \Delta^2} \]

Periodic Magnetic Field with no net flux \((Haldane \text{ PRL '88})\)

\[ V = \Delta_{\text{Haldane}} \sigma^z \tau^z \]

Broken Time Reversal Symmetry
Quantized Hall Effect \(\sigma_{xy} = \text{sgn} \Delta \frac{e^2}{h}\)

Intrinsic Spin Orbit Potential

\[ V = \Delta_{SO} \sigma^z \tau^z S^z \]

Respects ALL symmetries
Quantum Spin-Hall Effect
What is the Topological invariant

Inversion (P) Symmetry: determined by Parity of occupied 2D Bloch states

\[ P \left| \psi_n (\Lambda_a) \right\rangle = \xi_n (\Lambda_a) \left| \psi_n (\Lambda_a) \right\rangle \]

\[ \xi_n (\Lambda_a) = \pm 1 \]

In a special gauge: \[ \delta (\Lambda_a) = \prod_n \xi_n (\Lambda_a) \]

2D topological insulator with larger gap

Theory: Bernevig, Hughes and Zhang, Science '06
Expt: König, Wiedmann, Brune, Roth, Buhmann, Molenkamp, Qi, Zhang Science 2007

WTe2: David Cobden et. al.
3D Topological Insulator

Each of the time reversal invariant planes in the 3D Brillouin zone is characterized by a 2D invariant.

Weak Topological Invariants (vector):

\[ (-1)^{ν_i} = \prod_{a=1}^{4} \delta(Λ_a) \quad \text{plane} \]

\[ G_v = \frac{2π}{a} (ν_1, ν_2, ν_3) \]

“mod 2” reciprocal lattice vector indexes lattice planes for layered 2D QSHI

Strong Topological Invariant (scalar)

\[ (-1)^{ν_o} = \prod_{a=1}^{8} \delta(Λ_a) \]

Example: Bi\(_{1-x}\)Sb\(_x\), Bi\(_2\)Se\(_3\)
Surface states of TI

In the interface between a topological insulator and an ordinary insulator (or vacuum), there are energy states connecting between the conduction/valence bands of the two insulators.
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Surface states of Hong Kong

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Ordinary Insulator

Topological Insulator
2D topological insulator: Quantum spin Hall effect

Electrons in conventional materials

Electron transport in 2D topological insulator
2D topological insulator: Quantum spin Hall effect

HgTe quantum well theoretically proposed by Bernevig, Hughes and Zhang
Realized by Molenkamp

Monolayer WTe$_2$ discovered by David Cobden@UW

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Q: How do we control topological phase of matter?

Topological properties by definition is robust against deformation?

A: By proper choice of material, one can control the electronic topology by small mechanical deformation of the crystal lattice.