Single Electron Detection and Spectroscopy

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The Search for New Physics

Helium-6
- Experiment investigating weak interactions in nuclei

Project 8
- Neutrino mass experiment

Need: Energy spectra of $\beta$-decay electrons
What is Project 8?

- Measures energies of trapped electrons via understanding of their motion
- Made the first measurement of single-electron cyclotron radiation
Cyclotron Frequency

\[ f_\gamma = \frac{qB}{2\pi \gamma m} \]

\[ \gamma = \left(1 + \frac{K}{m_e c^2}\right) \]
Apparatus: The Cell

Signal
Pre-amplifier
Waveguide
Superconducting magnet
Krypton
Magnetic field
Apparatus: The Magnetic Field

\[ \vec{F} = -\nabla \left( \vec{\mu} \cdot \vec{B} \right) \]
Apparatus: The Waveguide and Radiated Power

- **Larmor Power:**

\[
P(\gamma, \theta) = \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{e^4}{m_e^2 c} B^2 (\gamma^2 - 1) \sin^2 \theta.
\]
Creating a Frequency Spectrum

Figure: Example of Frequency Spectrum
Creating an Energy Spectrum

\[ f_\gamma = \frac{qB}{2\pi\gamma m} \]

**Figure**: Example of Energy Spectrum
Goal and Advantages

- **Task**: Distribution of average magnetic fields $\bar{B}$
  
  \[ f_\gamma = \frac{qB}{2\pi\gamma m} \]

- **Goal**: Energy spectra for trapped electrons

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Field Configuration

\[ \vec{B} = B_0 \left( 1 + \frac{Z^2}{L^2} \right) \hat{z}. \]
Motion of the Trapped Electron
Motion of the Trapped Electron (Contd.)

- Force: directed along $\hat{z}$.
- $z$-motion: $\ddot{z} = z_{\text{max}} \sin \omega t \; \hat{z}$.
- $z_{\text{max}}$: maximum $z$-position attainable by a trapped particle with initial $z$ and $\phi$. 
Maximum Displacement for a Trapped Particle

\[
\frac{\sin^2 \phi}{\sin^2 \phi_{\text{max}}} = \frac{B(z)}{B(z_{\text{max}})}
\]

\[
\sin^2 \phi = \frac{B_0(1 + \frac{z^2}{L^2})}{B_0(1 + \frac{z_{\text{max}}^2}{L^2})}
\]

\[
z_{\text{max}} = \sqrt{\csc^2 \phi (L^2 + z^2) - L^2}
\]
Time-Averaged Magnetic Field

- We need $z_{\text{max}}$ in order to find the $\bar{B}$ experienced by a given trapped electron.
- After the integration dust settles, we have

$$\bar{B} = B_0 \left[ 1 + \frac{z_{\text{max}}^2}{2L^2} \right] \hat{z}.$$
Constructing the Probability Density Function

\[
\frac{dN}{d \cos \phi} = \frac{dN}{d \tilde{B}} \frac{d \tilde{B}}{d \cos \phi}
\]

constant

Solve:

Want:
Constructing the Monte Carlo Simulation

- Populate $z$ uniformly with electrons.
- Give each electron some pitch angle $\phi$.
- Check to see if the electron is trapped.
- Calculate $\bar{B}$ and create $\bar{B}$ histogram.
Monte Carlo Simulation and Analytical Solution

![Graph showing B_bar (T) and counts]

- **Counts**: 90, 92, 94, 96, 98, 100, 102, 104, 106, 108, 110
- **B_bar**: 1.001, 1.002, 1.003, 1.004, 1.005
- **Entries**: 1e+07
- **Mean**: 1.002
- **RMS**: 0.001443

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Off-axis Monte Carlo Simulation

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Off-axis_Monte_Carlo_Simulation}
\caption{B_{\text{bar}}}
\end{figure}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
\textbf{Hist} & \textbf{B_{\text{bar}}} \\
\hline
\textbf{Entries} & 1000000 \\
\textbf{Mean} & 1.002 \\
\textbf{RMS} & 0.001431 \\
\hline
\end{tabular}
\end{table}

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Range of Trapped Angles and $\bar{B}$ Values:

- Electrons with $\phi$ in a particular range remain trapped.
- Maximum $\bar{B}$: determined by truncated quadratic.
- Minimum $\bar{B}$: dependent on the particle’s initial $z$ for $\phi = 90^\circ$. 
(B_bar/B_0) v. cos(theta) (for z=0)

<table>
<thead>
<tr>
<th>cos(theta)</th>
<th>(B_bar/B_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
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<tr>
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<td>1.001</td>
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<tr>
<td>0.06</td>
<td>1.0015</td>
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<td>1.002</td>
</tr>
<tr>
<td>0.1</td>
<td>1.0025</td>
</tr>
</tbody>
</table>

### Parameters

- **Entries**: 1000000
- **Mean x**: 0.0496
- **Mean y**: 1.002
- **RMS x**: 0.02874
- **RMS y**: 0.001486

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