Shimming of a Magnet for Calibration of NMR Probes

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Outline

- Background
  - The muon anomaly
  - The g-2 Experiment
  - NMR

- Design
  - Helmholtz coils producing a gradient

- Results

- Future work
The Muon anomaly

- The magnetic moment of a particle:
  \[ \hat{\mu} = g\mu_B \hat{s} \quad \mu_B = \frac{e\hbar}{2mc} \]

- A $g$ factor of 2 is expected for point-like fermions

- There is a contribution to $g$ from interactions with virtual fields

The Muon anomaly:
\[ a_\mu = \frac{g\mu^{-2}}{2} \]
The Muon anomaly

1) Electromagnetic interaction

2) Strong interaction

3) Weak interaction

4) New physics – supersymmetry?
The Muon g-2 Experiment

Goal: measure the anomalous magnetic moment of the muon to the precision of 0.14 ppm

1) Collect polarized muons
2) Precession in \((g - 2)\) storage ring
3) Measure arrival time and energy of positron from muon decay
The Muon g-2 Experiment

Determining the anomaly:

\[ a_\mu = \frac{\omega_a / \omega_p}{\mu_\mu / \mu_p - \omega_a / \omega_p} \]

- \( \mu_\mu / \mu_p = 3.183345137(85) \)
  - From muonium hyperfine level measurements

- \( \omega_a \): Difference frequency
  - \( \omega_a = \omega_s - \omega_c = a_\mu \left( \frac{e}{m_\mu c} \right) B \)
  - From detection of positrons

- \( \omega_p \): Larmor frequency of free protons
  - Measured with 400 NMR probes
A moment precessing about a strong field $\vec{H}_0$ may be flipped with the addition of a weak field $\vec{H}_1$ rotating with a frequency close to resonance.

Use a $\pi/2$ pulse in NMR.
NMR Time Constants

- $T_1$: spin-lattice relaxation
  - Time until magnetization reaches thermal equilibrium value (z-direction)

- $T_2^*$: spin-spin relaxation
  - due to magnetization parallel to rf field and field inhomogeneities
Shimming the magnet

- Shimming: removing inhomogeneities in the field
- Passive shims
  - Iron pieces on the yoke and in gaps
  - Pole face alignment
  - Edge and wedge shims
- Active shims
  - Control of superconductor current
  - Surface correction coils
  - Dipole and gap correction loops
CENPA test magnet
The best signals were around 3.8 ms (until amplitude reaches 1/e)
Helmholtz coils

$h$ (distance between the coils) should be equal to $R$ (the radius of a coil) for maximum uniformity.

Using the Biot-Savart Law, the field at midpoint between coils:

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 nI}{R}$$
Helmholtz coils

A linear gradient is created between the coils
Helmholtz coils
Helmholtz coils
Results – varying the current

- Current: 0 A
  Length: 0.144 ms

- Current: 0.35 A
  Length: 1.064 ms

- Current: 0.45 A
  Length: 7.864 ms

- Current: 0.55 A
  Length: 0.584 ms

The best signals found were 9.14 ms at 0.87 A
Results – 2-probe coils

Maximizing both probes at the same current does not work, but one may be used for calibration. Long signals are repeatable.

After switching the probe positions
Extracting the Frequency

Zero-crossing method

Fourier Transform method
Repairing Probes

- Diagnose problems with old probes using a vector impedance meter
- Repair broken circuitry or determine that the sample has leaked

![Diagram of working probe resonances with impedance and frequency data. The graph shows peak impedances at specific frequencies, with labels for LC circuit maxima and minima, and a marked frequency of 61.74 MHz at 50 Ω.]
Future Work

- Test the temperature dependence of the NMR probes
- Diagnose problems with old probes
- Re-design and rebuild 400 probes
- Test and calibrate the probes using the coils
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References


Questions?
NMR

A system with angular momentum $\mathbf{j}$, magnetic moment $\mathbf{m}$, and gyromagnetic ratio $\gamma$:

$$\frac{d\mathbf{j}}{dt} = \mathbf{m} \times \mathbf{B}_0 \quad \mathbf{m} = \gamma \mathbf{j}$$

$$\frac{d\mathbf{m}(t)}{dt} = \gamma \mathbf{m}(t) \times \mathbf{B}_0$$

Add a perpendicular field $\mathbf{B}_1(t)$ rotating about $\mathbf{B}_0$ with angular velocity $\omega$

$$\frac{d\mathbf{m}(t)}{dt} = \gamma \mathbf{m}(t) \times [\mathbf{B}_0 + \mathbf{B}_1(t)]$$

Set $\Delta \omega = \omega - \omega_0$ and move to the rotating frame

$$\left(\frac{d\mathbf{m}(t)}{dt}\right)_{\text{rel}} = \mathbf{m}(t) \times [\Delta \omega - \omega_1]$$

Resonance condition: $\Delta \omega \ll \omega_1$

The moment can be flipped with a small rotating field $\mathbf{B}_1$
How NMR works – Quantum Mechanics

The state vector of the spin system:

\[ |\psi(t)\rangle = a_+(t)|+\rangle + a_-(t)|-\rangle \]

The Hamiltonian:

\[ H(t) = -\mathbf{M} \cdot \mathbf{B}(t) = -\gamma \mathbf{S} \cdot [\mathbf{B}_0 + \mathbf{B}_1(t)] \]

From the spin matrices:

\[ H = \frac{\hbar}{2} \begin{pmatrix} \omega_0 & \omega_1 e^{-i\omega t} \\ \omega_1 e^{i\omega t} & \omega_0 \end{pmatrix} \]

Define functions \( b_+(t) = e^{i\omega t/2} a_+(t) \) and \( b_-(t) = e^{-i\omega t/2} a_-(t) \) and set up the Schrodinger equation

\[
\begin{cases} 
  i \frac{d}{dt} b_+(t) = -\frac{\Delta \omega}{2} b_+(t) + \frac{\omega_1}{2} b_-(t) \\
  i \frac{d}{dt} b_-(t) = \frac{\omega_1}{2} b_+(t) + \frac{\Delta \omega}{2} b_-(t) 
\end{cases}
\]

The Hamiltonian is now time-independent (we are in the rotating frame)

\[ i\hbar \frac{d}{dt} |\tilde{\psi}(t)\rangle = \tilde{H} |\tilde{\psi}(t)\rangle \]

Find the transition probability

\[ P_{+\rightarrow-}(t) = |\langle - |\psi(t)\rangle|^2 = |\langle - |\tilde{\psi}(t)\rangle|^2 \]

Where the initial condition is \( |\psi(0)\rangle = |+\rangle \)

Rabi's formula:

\[ P_{+\rightarrow-}(t) = \frac{\omega_1^2}{\omega_1^2 - (\Delta \omega)^2} \sin \left[ \sqrt{\omega_1^2 + (\Delta \omega)^2} \frac{t}{2} \right] \]

Same resonance condition as classical mechanics: \( \Delta \omega \ll \omega_1 \)
NMR Time Constants

- $T_1$: spin-lattice relaxation
  - Time until magnetization reaches thermal equilibrium value (z-direction)
  - $\frac{dn}{dt} = \left(\frac{1}{T_1}\right)(n_0 - n)$
  - $n$: surplus population at lower levels
  - $n_0$: number at equilibrium

- $T_2$: spin-spin relaxation
  - due to magnetization parallel to rf field if the field is perfectly homogeneous

- $T_2^*$: spin-spin relaxation combined with field inhomogeneities
  - due to magnetization parallel to rf field and field inhomogeneities
  - $T_2^* = \frac{1}{2} g(\nu)$

  $g(\nu)$: shape factor of the absorption line of energy from the magnetic field
Circuit Resonances

Working Probe Resonances

LC circuit resonances close-up

Phase shift