Lecture QCD

will develop and work with EFT of QCD

Comedown to QCD and it's symmetry important throughout the lecture

⇒ brief introduction to the theory of strong interactions

**Quantum Chromodynamics**

\[
\mathbb{D}_{\text{QCD}} = \bar{\Psi}_i \left( i \gamma^m D_m \right) \Psi_j - m_i \delta_{ij} \Psi_j - \frac{1}{4} F_{\mu \nu}^a F^{\mu \nu a}
\]

\[
\partial_\mu A^a_\mu \Psi_j = \partial_\mu A^a_\mu \Psi_j + g f_{abc} A^b_\mu A^c_\mu
\]

Input: quark masses and \( g \)

<table>
<thead>
<tr>
<th>Quark</th>
<th>Charge</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u ) (up)</td>
<td>( +\frac{2}{3} )</td>
<td>2-3 TeV</td>
</tr>
<tr>
<td>( d ) (down)</td>
<td>( -\frac{1}{3} )</td>
<td>4-6 TeV</td>
</tr>
<tr>
<td>( c ) (charm)</td>
<td>( +\frac{2}{3} )</td>
<td>1.3 GeV</td>
</tr>
<tr>
<td>( s ) (strange)</td>
<td>( -\frac{1}{3} )</td>
<td>100 MeV</td>
</tr>
<tr>
<td>( t ) (top)</td>
<td>( +\frac{2}{3} )</td>
<td>170 GeV</td>
</tr>
<tr>
<td>( b ) (bottom)</td>
<td>( -\frac{1}{3} )</td>
<td>4.5 GeV</td>
</tr>
</tbody>
</table>

Comparison with QED

QED

\( e^+ e^- \rightarrow \text{quark} \)

QCD

\( e^+ e^- \rightarrow \text{quark} \)

\( 1 \) photon \( \rightarrow \) 8 gluons (massless)

\( \text{gauge group} \rightarrow SU(3) \)
Forces between quarks mediated by massless gluons

\[ g^2 \frac{1}{4\pi} = \alpha_s \text{ strong coupling} \]

Compared to \( \frac{e^2}{4\pi} = \frac{1}{137} \text{ in QED} \)

**Running coupling**

**QED:**
- Electric charge: scattering microscopically effective charge at short distance
- High momentum scale \( Q \)

- Coupling strength changes with scale "running coupling" (\( \Rightarrow \) key concept for nuclear forces)

**QCD:**
- Color anti-screening leads to

\[ \frac{1}{\alpha_s(Q)} = \frac{33 - 2N_f}{6\pi} \log \frac{Q}{\Lambda_{\text{QCD}}} \]

\( \Lambda_{\text{QCD}} = \text{scale of QCD} \)

\( \Lambda_{\text{QCD}} \approx 200-400 \text{ MeV} \)

\( \Rightarrow \) asymptotic freedom

\( \Rightarrow \) input to QCD: mass, \( \Lambda_{\text{QCD}} \) instead of \( g \)

For chiral limit: mass \( \to 0 \), \( \Lambda_{\text{QCD}} \to \infty \), QCD is only scale
QCD is nonperturbative at low energies \[ \Rightarrow \text{EFT for nuclear forces} \]
leads to 1) Confinement and 2) chiral symmetry breaking
- quarks cannot be isolated, confined to color singlet (colored) hadrons
- energy to separate \( q \bar{q} \): \( E = \frac{1}{r} \)
  - string tension \([r] \Rightarrow E \text{ is sufficient to break} q \bar{q} \text{ flux as} r \text{ increases, new} q \bar{q} \text{ pair created}
- degree of freedom at low energies are hadrons

masses of hadrons
- bosons: mesons \( \pi, \rho \ldots \) form as hadron of \( u \bar{d} \) quarks
- fermions: baryons \( N, D \ldots \)

\( m_{\text{hadron}} \sim 1 \text{ GeV}, \text{ except for light} \pi, K \)

\[ \Lambda_{\text{QCD}} \gg m_u, m_d \Rightarrow \text{can treat} \Lambda_{\text{QCD}} \text{ as standard} QCD \text{ kilogram} \]

QCD symmetry of quarks \[ \Rightarrow \text{symmetry in hadron spectrum} \]
- \( m_u = m_d \Rightarrow u \bar{d} \) quark form isospin multiplets
  \( |u> = |\text{isospin} 0> = |T_3 = \frac{1}{2} \rangle \)
  \( |d> = |\text{isospin} 0> = |T_3 = -\frac{1}{2} \rangle \)
- isospin operator \[ I^3 = \frac{1}{2} \] with Pauli matrix, \( T_3 \).
Isospin symmetry (approximate symmetry because \( m_u \neq m_d \))

clearly seen in hadron spectrum

Baryons:

\[ \text{Nucleon} \quad N \left( \frac{1}{2}^- \right) \]
\[ 940 \text{ MeV} \]

\[ |n> = |T=1/2, M_T = -1/2> \quad \text{dud} \]
\[ |p> = |T=1/2, M_T = +1/2> \quad \text{umd} \]

Isospin doublet is nontrivial part (\( S=0 \)) of baryon octet

\[ \Delta \left( 3^+ \right) \]
\[ 1232 \text{ MeV} \]
\[ \text{ddd, udd, uuu} \text{ or \( \Delta^+ \)} \]
\[ |T=3/2, M_T = -3/2, -1/2, 1/2, 3/2> \]

Isospin octet of baryon decuplet

Simple constituent quark mass model:

\[ m_N = 3 \cdot m_{\text{constituent}} - \text{diquark ud \( S=0 \) binding} \]
\[ B_{\text{diquark}} = 300 \text{ MeV} \]

Mesons:

Pseudoscalar:

\[ \pi \left( 0^- \right) \]
\[ 140 \text{ MeV} \]
\[ |T=0, M_T = 0> \]

Vector mesons:

\[ g \left( 1^- \right) \]
\[ 770 \text{ MeV} \]
\[ |g> \]

Meson masses:

\[ m_{\pi} \approx 2 \cdot m_{\text{constituent}} = 800 \text{ MeV} \quad \text{O.K.} \]

but \( m_{\pi} = 140 \text{ MeV} \ll 2 \cdot m_{\text{constituent}} - B_{\text{diquark}} = 500 \text{ MeV} \]
\[ Z_q = \bar{u} i D u + \bar{d} i D d = \bar{u}_L i D u_L + \bar{u}_R i D u_R + \bar{d}_L i D d_L + \bar{d}_R i D d_R \]

Squarks decomposed into left- and right-handed quarks

\[ \Rightarrow Z_{QCD} \text{ is symmetric under independent rotations in } u_d \text{ space of } L-R \text{- handed quarks} \]

Symmetry: \( \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_Y \times \text{U}(1)_A \)

\[ = \text{SU}(2)_{L-R} \times \text{U}(1)_Y \times \text{U}(1)_A \]

- Vector
- Axial
- Isospin
- Chiral
- Baryon number symmetry

\[ \text{SU}(2)_{\text{isospin}} \text{ is present in hadron spectrum} \]

\[ \text{SU}(2)_{\text{axial}} \text{ implies degenerate parity partners} \]

\[
\text{e.g., for the nucleon } N(\frac{1}{2}^+) \text{ and } N(\frac{3}{2}^-) \\
\]

\[
m_N^{\frac{1}{2}^+} = 940 \text{ MeV} \quad m_N^{\frac{3}{2}^-} = 1535 \text{ MeV} \\
\]

\[ \Rightarrow \text{Chiral symmetry is spontaneously broken in the QCD ground state vacuum} \]

In addition, \( \text{SU}(2)_A \) is explicitly broken by \( m_u, m_d \neq 0 \) mixes, \( L, N \)

\[ Z_{QCD}^{u, d} = -\bar{u}_L m_u u_L - \bar{d}_L m_d d_L - \bar{u}_R m_u u_R - \bar{d}_R m_d d_R \]
Spontaneous symmetry breaking

Effective potential is symmetric $\Rightarrow$ broken rotational symmetry

- Physical ground state breaks symmetry
- Low-energy excitations in original symmetry direction cost very little energy $E \propto k^2$
- For low momenta $k = \frac{l}{\lambda}$
- Long wavelengths $\lambda$
  $\Rightarrow$ Spontaneous symmetry breaking leads to massless Goldstone bosons

Light pions are Goldstone bosons of chiral symmetry breaking

Gell-Mann-Oakes-Renner relation $m_\pi^2 \sim m_q$

Finite pion mass due to explicit chiral symmetry breaking

Other examples: $S\bar{S}$ and Goldstone bosons

- Symmetry
- Broken symmetry
- Goldstone boson
- $\phi$
- Crystal translations
- Magnon rotations

$\phi \overleftrightarrow{\phi}$

In addition to the light pions, chiral symmetry breaking is responsible for the dynamical mass generation of mesons $\sim 300\text{ MeV}$ to $\sim 1\text{ GeV}$

QCD phase diagram

- At high temperature and density $= $ high momenta $\Rightarrow$ asymptotic freedom
- Transition to deconfinement and chiral symmetry restoration
- Quarks and gluons become free of their confinement into hadrons
- Energetic QCD at zero chemical potential for $T \gtrsim 170\text{ GeV} \sim 10^{12}\text{ K}$

We will focus on the low $T$, low baryon density region of the QCD phase diagrams $\Rightarrow$ degrees of freedom: nucleons and pions (and $\Delta$s)
Units

We will work in units with \( \hbar = c = 1 \)

Use \( \hbar c = 197.327 \text{ MeV\cdot fm} \) to convert \( \text{MeV} \rightarrow \text{fm^{-1}} \)

\[ \text{e.g. pion mass } m_{\pi} = 140 \text{ MeV} = \frac{140 \text{ MeV}}{\hbar c} = 0.7 \text{ fm}^{-1} \]

Useful to remember
\[ \frac{\hbar c^2}{m} = \frac{\hbar e^2}{m c^2} = 41.4 \text{ MeV fm}^2 \]

Naive dimensional analysis and naturalness

Example: Radius \( r \) and energy \( E \) of hydrogen-like atom \( \text{He}^+_2 \)

Reduced mass \( \mu = \frac{m_e m_n}{m_e + m_n} \)

What can \( r \) and \( E \) depend on? \( \mu \) relevant quantities

\[ \mu \]

Reduced mass \( \mu \)

\[ \text{ Coulomb potential } V(r) = -\frac{k e^2}{r} \]

\[ \text{Quantization } \hbar \]

\[ r \sim \frac{\hbar^2}{k_e^2 \cdot \mu} \quad \text{and} \quad E \sim \frac{-k e^2}{r} = \frac{(k e^2)^2 m_e}{\hbar^2} \]

\[ \mu = \frac{m_e}{2} \text{ Bohr radius} \]

Quantum: \( r = \frac{a_0}{2} \)

\[ \text{Quantum constant } \frac{1}{2} \]

So constant -1 = 1

NDA often allows one to estimate the answer and scaling law up to an overall factor that is usually of \( O(1) \) \( \Rightarrow \text{ naturalness} \)