Lecture F26: Nuclear forces and electroweak interactions

1. Review of nuclear structure: Simple shell model

\[ ^{10}\text{B} \]

- np for N=5 nucleus, \( T=0 \) (demonstration)
- \( J=1 \) or \((\text{p}_{\frac{3}{2}})^2\) with \( j=\frac{3}{2} \) aligned
- \( 3^+, 1^+ \) lowest states

\[ ^{13}\text{C} \]
- \( \sigma \rightarrow \frac{1}{2}^- \) ground state

Excited state
- \( J=2 \)
- \( \pi \rightarrow \frac{3}{2}^- \)

2. Electroweak currents

3. Example: Axial-vector weak current \( \rightarrow \) neutral current interaction

- e.g. V-A interaction

4. Electromagnetic currents
- Charged current interactions
- e.g. \( Z \) decay

5. Weak force and vacuum anomaly

Next week
Electroweak interactions

- astrophysical reactions: stellar evolution, reactions in sun, supernovae, nucleosynthesis
- fundamental symmetries: supersymmetry, fermion $\beta$-decay $\rightarrow Vud$ and CKM matrix

$\beta$ decay $\rightarrow$ neutrino mass
neutrinoless $\beta$ decay $\rightarrow$ Majorana nature of $\nu$
+ lepton number violation

- nuclear structure and electroweak transitions, resonances
- systematic calculations with theoretical uncertainties needed

- chiral EFT: coupling to electroweak probes $\rightarrow$ minimal subtraction for e.m.

![Diagram of weak and strong forces]

lepton
weak
strong

at low energies
write as current-current interaction, e.g. in case of weak interactions
\[
L_{\text{int}} = \frac{g^2 F}{12} \int d^4 x \left( J^\mu(r) \tilde{J}^\mu(r) \right)
\]
leptonic
nuclear
current

Terminology:

- One-body current (corrections)
- Two-body currents
- Three-body currents (very small)

Also called meson-exchange currents (MEC)
expand currents consistently in chiral EFT to $A^u$

Why do $2^+$ (and high-body) currents occur? take generic strong interaction process and consider electromagnetic coupling

from $\Delta$

intermediate states involve charged $N, \pi, \rho$ can couple to these!

Example: axial-vector weak interactions $\gamma^\mu T^i = g_A \bar{\Psi} \gamma^\mu \tau^i \Psi$ $i = 3 \rightarrow$ neutral $i = 2 \rightarrow$ charged

Axial-vector currents based on same symmetry as chiral symmetry currents $\rightarrow \pi, \rho, \omega$

see this e.g. at the level of $\pi \nu \rightarrow \pi \nu \nu$ exchange

Goldberger-Treiman relation $g_{\pi NN} f_\pi = g_A m_N$

\[ T\bar{N} \text{ coupling} \leftrightarrow \text{axial coupling from weak interactions} \]

diagrammatically

\[ \sim g_A \frac{\bar{\Psi} \gamma^\mu \tau^i \Psi}{2m} \leftrightarrow g_A \frac{\bar{\Psi} \gamma^\mu \tau^i \Psi}{2m} \]
Single-nucleon current to order $Q^2$

\[ j_{\mu}^{(\text{G-T})} \propto (0^+ \gamma_\mu) = -i \left( g_A (p^2) \vec{\sigma} \cdot \vec{p} - g_V (p^2) \frac{\vec{p} \cdot \vec{\sigma}}{2m} \right) + i (g_{1\nu} + g_{1\nu}) \frac{\vec{p} \cdot \vec{n}}{2m} - g_{1\nu} \frac{\vec{p}}{2m} \]

\[ j_{\mu}^{(Fm)} = \bar{u} (p) \gamma^\mu (\gamma^5 \theta (p^2) - \frac{g_A}{2m} \frac{\vec{p} \cdot \vec{\sigma}}{2m} + \frac{g_V (p^2) \vec{p} \cdot \vec{\theta}}{2m}) \]

Fermi operator

P-dependence of couplings, due to loop corrections and pion propagators, to $Q^2$

\[ g_{A\nu} (p^2) = g_{A\nu} \left( 1 - 2 \frac{p^2}{\Lambda_{\text{A}}^2} \right) \]
\[ g_{1\nu} = 1, \quad g_A = 1.27 \]
\[ \Lambda_{\nu} = 850 \text{MeV}, \quad \Lambda_{\text{A}} = 1040 \text{MeV} \]

\[ g_{p\nu} (p^2) = \frac{2g_{\pi\nu}}{p^2 + m_{\pi}^2} - 4g_A (p^2) \frac{m}{\Lambda_{\text{A}}^2} \]
\[ g_{\pi\nu} = 13.05 \]

\[ g_{m} = \frac{m_p - m_n}{2} = 3.70 \]

Example: nuclear $\beta$ decay $\rightarrow$ very low momentum transfer (low $Q^2$-values!)

\[ f = \frac{2\pi^3 \theta_n^2}{m_e^2 G_F^2 \nu \left[ (1 + \Delta^0) \left( g_{A\nu} \right)^2 \nu^2 + (1 + \Delta^A) \frac{g_A^2 \nu^2}{4 \Delta A} M^2 \right]} \]

Comparative half-life

includes dependence on $B$ of daughter

and max electron energy $\nu_f$
Two-body axial-vector weak currents

\[ M_{V}^{2} \sim \left| \langle \mathbf{i} \mid \Sigma_{n=1}^{A} \mathbf{t}_{n}^{\pm} \mathbf{t}_{n}^{\pm} \mathbf{f}^{\pm} \rangle \right|^{2} \]

\[ M_{A}^{2} \sim \left| \langle \mathbf{i} \mid \Sigma_{n=1}^{A} \mathbf{t}_{n}^{\pm} \mathbf{a}^{\pm} \mathbf{a}^{\pm} \mathbf{f}^{\pm} \rangle \right|^{2} \]

Driven with diagrammatic rule

\[ \text{ vertex at } Q^{3} \rightarrow 3N \text{ form } \]

\[ \text{ long range, short range } \]

\[ \Rightarrow \]

\[ \text{ given by } c_{i} \text{ by } c_{D} \]

\[ \Rightarrow c_{i} \text{ relates } \pi N, N N, 3N, \text{ axial-vector weak currents} \]

\[ c_{D} \text{ relates } 3N, \text{ axial-vector weak currents, and } T_{i} \text{-production} \]

\[ \text{ can use this to fit } 3N \text{ form to } ^{3}H \text{ } \beta\text{-decay!} \]

\[ \vec{j}^{(3)i} = \sum_{m,n=1}^{A} \vec{j}^{(i)mn} \] with many structures, for details see Pask et al. 