Chiral Forces 2

Plan: Multiple topics addressing different aspects & chiral EFT forces

1. Loose ends from yesterday
   - Story & Nijmegen fits with $m_t$ as free parameter
     ⇒ see 12b-5 notes.
   - (Maybe) go back and step through 12b-11 on calculating $g_A$

2. Nuclear forces from chiral Lagrangians
   - method of unitary transformations

3. NDA, naturalness, and resonance saturation for
   - Lee's fit with Weinberg counting
   - Deltaful vs. Delta to less chiral EFT
   - Renormalization issues with Weinberg counting

4. Building chiral Lagrangians

A lot of stuff ⇒ partial detail is what your appetite for more!
How can we really test both the one-pion and two-pion exchange interactions predicted from chiral symmetry are present in NN scattering?

- The Nijmegen group together with Bia van Kolck and Jim Fiar (in various combinations) used the Nijmegen PWA methods to make fits (leaving only fit to their X-square minimization)
  - Particularly convincing is when they let the mass of the pion be a free parameter in their X-square minimization
  - But fits the couplings to pp and np data.

- In the original work from 1993, they determined the pp coupling constant in each partial wave except for 3S0 (by letting one float in that partial wave) and they agree at 5% level with each other and the extracted value from NN scattering.
  - Extracting the neutral and charged pion masses agreed with experiment within estimated one percent errors.

Subsequent analyses in 1999 and 2003 looked with finer resolution, to look for the direct evidence of two-pion exchange physics:

- Here they found that using the pion mass as a free parameter gave agreement with experiment at 10 percent level.
  - The coupling constants are consistent with those from NN scattering, although there are still sizable uncertainties in both these determinations (We'll see this many times still.)

- Open question: can we do something similar for nuclear structure?

- My dream: have Mπ as a free parameter in an energy density functional with long-range chiral effects (next week)
  - and determine from a fit (say along an isotope chain) a nice fit for NN in this fashion
Let's consider the calculation for $g_{\rho}$. We want to calculate the proton matrix element of the axial vector current:

$$\langle p', s' | A_\mu^{\rho} | p, s \rangle = \bar{u}(p', s') \left[ M_{\rho} G_\rho(q^2) + \delta_{\mu, 1} \frac{2q_\mu}{m_N} G_\pi(q^2) \right] u(p, s)$$

In the limit $q = p' - p \to 0$, we find $g_{\rho} = G_{\rho}(0)$ at $p = p'$.

This is a 3pt function. In general (suppressing many indices):

$$G(t, x, \vec{p}, \vec{p'}) = \sum_{x_0} e^{-i \vec{p} \cdot \vec{x}} e^{-i \vec{p'} \cdot \vec{x}} \langle 0 | \bar{N}(x_0, t) A_\mu^{\rho} (x, \vec{p}) N(0) | 0 \rangle$$

- Use $N(x, t) = e^{i \vec{p} \cdot \vec{x}} e^{i \phi \cdot x} e^{-i H t}$
- Insert $I = \sum_{x_0} \sum_{s, s'} \bar{b}_{s'} \Gamma_{s, s'} b_s$ twice

$$G(t, x, \vec{p}, \vec{p'}) = \sum_{x_0} \sum_{s, s'} e^{-E_{s'}(t)} e^{-E_s(x_0)} \langle 0 | N(x_0) | b_{s'} s' \rangle \langle b_s s | N(0) | 0 \rangle$$

With $x = (t, \vec{x})$ and $t > 0$: $\bar{b}_{s'} \Gamma_{s, s'} b_s$

We want to calculate the 3pt function (without the $A_\mu^{\rho}$ insertion) and divide!
Nuclear forces from chiral Lagrangians (from Eppleman lectures)

There are several paths from the chiral Lagrangians we wrote down yesterday, which had both nucleon and pion fields, to a potential between nucleons (as pion coordinates).

1 S-matrix-based methods
   - Roblot et al., da Rocha et al., Kaiser et al., Aug. 08, ...
   - Calculate the amplitude for S-S' scattering in CHPT (full theory) and require it match in perturbation theory to the Lippmann-Schwinger series
     - Two ways to calculate the same thing: field theory from $\mathcal{L}$ and with a potential $\Rightarrow$ make them agree order by order

2 Hamiltonian-based methods
   - We've already used time-ordered perturbation theory for deriving the OPE potential:

\[
\begin{align*}
\ldots & \quad \pi^- \quad \pi^+ \quad \bar{\pi}^- \quad \bar{\pi}^+ \\
\end{align*}
\]

- Weinberg '90, '91; Ordanetz et al., '92, '94
- Leads to energy dependent potentials that are (at least) inconvenient for $A \geq 2$ calculations.
- The work by Eppleman, Glöckle, and Meissner has used the so-called method of unitary transformations to decouple the $\pi$ part of the Hamiltonian from the nucleon part.

- We look at this further because the use of unitary transformations in analogous ways will be a topic of future lectures.
Schematically, we can view the Hamiltonian as a matrix

\[
\begin{pmatrix}
N \quad N \\
N \quad N
\end{pmatrix}
\]

states with no pions

states with at least one pion

\[
\begin{pmatrix}
N - N \\
N - N
\end{pmatrix}
\]

- We would like to find a unitary transformation such that the transformed Hamiltonian has no off-diagonal sectors connecting the \( N - N \) block.

- By making these sectors non-zero, we have decoupled them.

- The result is an energy-independent inter-nucleon potential with \( V_{NN} \), \( V_{NN}, V_{NN} \), ... parts.

In practice, it is easiest to think about doing this in second quantization:

\[
H_{mn} = \sum_n \frac{1}{n!} \left[ \sum_{\alpha, \beta} (\psi^{\alpha}_n \psi^{\beta}_n) H_{\alpha \beta} + \sum_{\alpha} (\psi^{\alpha}_n \psi^{\alpha}_n) H_{\alpha \alpha} \right]
\]

Now devise a unitary transformation that has the same form:

\[
U = \prod_n \frac{1}{n!} \left[ \sum_{\alpha, \beta} (\psi^{\alpha}_n \psi^{\beta}_n) U_{\alpha \beta} + \sum_{\alpha} (\psi^{\alpha}_n \psi^{\alpha}_n) U_{\alpha \alpha} \right]
\]

- We require the \( U \) coefficients to be such that

\[ U^\dagger U = I \]

but there is still much freedom.
7/19/13
So we require the off-diagonal matrix elements to be zero for the transformed Hamiltonian:

$$H_{nn'} = U^t H_{nn'} U$$

with

$$\langle N_3 N_8 \cdots N_0 \Pi_1 \Pi_2 \cdots \Pi_{m-1} | U^t H_{nn'} U | N_3 N_8 \cdots N_0 \rangle = 0 \quad \text{for} \quad m \neq 1$$

Alternative description

Let us project on nucleus only subspace \(|\phi\rangle\) of \(|\Pi\rangle\) and let \(|\Pi\rangle\)
be the rest of full space with projector \(P\)

$$|\phi\rangle = \eta |\phi\rangle, \quad |\Pi\rangle = \lambda |\Pi\rangle \quad (\eta, \lambda \text{ are} \{0, 1\} \text{ in other context})$$

Find \(U\) such that

$$H = U^t H U = \begin{pmatrix} \eta^2 \eta & 0 \\ 0 & \lambda \end{pmatrix}$$

Okubo says

$$U = \begin{pmatrix} \eta (1 + \eta \eta)^{-1/2} & -\lambda (1 + \lambda \lambda)^{-1/2} \\ \lambda (1 + \lambda \lambda)^{-1/2} & \eta (1 + \eta \eta)^{-1/2} \end{pmatrix}$$

\(A\) satisfies

$$\left( H - \lambda A H (1 + \eta \eta)^{-1} \right) \eta = 0$$

This can be solved perturbatively

$$A = \sum_{n=1}^{\infty} \gamma^n \tilde{A}^n$$

See the Finkelstein review for details.
NDI, naturalness, and resonance saturation

We often speak of "naturalness" in discussing the size of low energy constants (LECs). The idea is that we expect a particular size based on dimensional analysis combined with physical considerations.

Recall from last week the case of a planar EFT with a single momentum scale $\Lambda_6$ in dim-6 theory. (At breakdown scale we call it that because an expansion in $\alpha_s$ fails when $Q \gg \Lambda_6$.) See $\Lambda_6$ in diagram.

For a short-range potential (e.g., hard-sphere) with range $R$,

- For $R \sim \Lambda_6$, $\Lambda \sim 1/R$.
- We found that (again, in dimensional regularization)

$$C_{2i} = \left( \frac{4\pi^2}{m^3} \right)^{\Lambda_{R^2} - 1}$$

$$D_{2i} = b_i \left( \frac{4\pi^3}{m^4} \right)^{\Lambda_{R^2} - 1}$$

with $a_i, b_i \sim O(1)$ (often we say natural is $\frac{1}{3} \leq a_i, b_i \leq 3$).

The $4\pi^3/m^4$ came from relating the $T$-matrix, described by the diagrams, to the scattering amplitude and actual observables.

Often it is just dimensions, because $\Lambda_6 \sim 1/R$ is all there is.

This is called naive dimensional analysis or NDA.

- Not because it is foolish (after all, you have to be smart to figure out the $4\pi^3/m^4$).
- But because it neglects other considerations that might cause a very different estimate (like shallow bound states or symmetries).

Do we do the same thing for constants in the chiral Lagrangian?
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Georgi and Manohar originally argued that a low-energy theory of QCD, that is to say, one describing the physics of hadrons below a scale characterizing spontaneous symmetry breaking, should be analyzed with two dimensional factors:

- pion decay constant $f_\pi \approx 100 \text{ MeV}$
- the chiral symmetry breaking scale $\Lambda_\chi \approx 500-1000 \text{ MeV}$
  \[ \Rightarrow \text{ typical mass of low-lying (non-Goldstone) bound states (eg. \eta \sim 780\text{MeV})} \]

- How should we combine these? George, in Generalized dimensional analysis (Phys. Lett. B318 (1993) 197) says for each term in $\mathcal{L}_{\text{eff}}$:
  1. include an overall factor of $f_\pi^2 \Lambda_\chi^2$
  2. include a factor of $1/f_\pi^2$ for each strongly interacting field
  3. add factors of $\Lambda$ to get $R$ dimension to 4
  \[ \text{(with } \Lambda = 1/5 \text{, } S = 50\text{GeV } \Lambda \text{ is dimensionless, so } R \sim 5N^2) \]

- When applied to nucleon fields $N$ and pion fields $\pi$:

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{dim}} \left( \frac{N^2}{f_\pi^2 \Lambda_\chi^2} \right) \left( \frac{\pi^2}{f_\pi^2 \Lambda_\chi^2} \right)^m \left( \frac{\pi \mathcal{M}_{\pi \pi}}{\Lambda} \right)^m \left( f_\pi^2 \Lambda_\chi^2 \right)^n \]

- If we scale the Lagrangian this way, we expect the dimensionless constant $\mathcal{L}_{\text{dim}}$ to be order unity.

- Georgi's argument is that $f_\pi$ is a universal measure of the amplitude for producing a strongly interacting bound state, so each field gets such a factor. The only other thing happening is at the dimensional scale set by $\Lambda$.

- Let's try it out for some contact terms in the contact potential:

\[ V_{\text{cont}} = C_5 + C_7 (\bar{\phi}_3 \cdot \bar{\phi}_3) + C_4 \bar{\phi}_3^* \bar{\phi}_3 + C_2 (\bar{\phi}_4 \cdot \bar{\phi}_4) \bar{\phi}_3 \cdot \bar{\phi}_3 + i C_6 (\bar{\phi}_4 \cdot \bar{\phi}_3)(\bar{\phi}_3 \cdot \bar{\phi}_4) \]

\[ \bar{\phi}_5 \cdot \bar{\phi}_4 \]

\[ k = \frac{\hbar \cdot \vec{p}}{m} \]
Now we have to step back to the Lagrangian

\[ L_0 = -\frac{1}{2} g S (\bar{N}^T N) (\bar{N} N) - \frac{1}{2} C_T (\bar{N}^T \Delta^* N) (\bar{N}^* \Delta N) \]

\[ \Rightarrow l=2 \text{ for } b^0 \beta, m=0, n=0 \]

\[ \Rightarrow \frac{1}{2} C_S \sim c_{200} \left( \frac{p^a}{p^n A} \right)^2 \quad \cdot \quad f_{\pi} A_x = \frac{c_{200}}{p^n A^2} \quad \text{or} \quad c_{200} \sim p^n A \]

Same for \( C_T \). If we try this with the Egelhaefter et al. potential, we find (varying the cutoff from 500-600 MeV)

\[ \left( \frac{p^2}{p^n A} C_S \right) = -1.08 \ldots -0.95 \quad \left( \frac{p^2}{p^n A} C_T \right) = 0.002 \ldots 0.040 \]

\( C_S \) works!

\( C_T \) coping! But wait. Unnaturally small can signal an (unrecognized) symmetry, Wigner proposed that SU(4) spin-isospin transformations:

\[ SN = i e_{
u} O^{\nu} \gamma^{\nu} N, \quad N = |0\rangle_M, \nu = 0, 1, 2, 3 \]

with \( O^{\nu} = (1, \gamma^5), \gamma^{\nu} = (1, \gamma^5) \), \( e_{\nu} \) a group parameter.

Is an approximate symmetry of the strong interaction.

\( C_S \) term invariant but \( C_T \) breaks \( \Rightarrow C_T \sim 0 \).

That is, the \( C_T \) term would be absent if SU(4) symmetry were exact.

But about NLO? A typical term is \( C_4 \):

\[ \Rightarrow \frac{1}{2} C_4 \left[ (N^T \Delta^* N)^2 + (\bar{N} \Delta N)^2 \right] \Rightarrow l=3, m=0, n=2 \]

\[ \Rightarrow C_4 \sim c_{200} \left( \frac{p^a}{p^n A} \right)^3 \quad \cdot \quad f_{\pi} A_x = \frac{c_{200}}{p^n A^3} \quad \text{and the same} \]

is true for all \( \pi \) or \( \eta \) \( C_4 \) - \( C_7 \). \( \Rightarrow \) look at \( f_{\pi} A_x \) \( C_4 \)

but we also should not be completely naive and account for \( p \) in \( \phi \)

from \( q^2 = (p-p')^2 \) and \( R^2 = (p+p')^2 / 4 \).
The results are:

\[ f^2 \Lambda^2 C_4 = 3.143 \pm 2.665 \]
\[ f^2 \Lambda^2 C_3 = 0.103 \pm 0.381 \]
\[ f^2 \Lambda^2 C_5 = 2.816 \pm 3.110 \]
\[ f^2 \Lambda^2 C_7 = -1.939 \pm 1.181 \]

So we can use this estimate for terms in higher order not calculated.

- But this doesn't tell us precise quantitative values or the sign & coupling.
- Let's use non-Goldstone boson exchange as a model for the short-distance physics unresolved in chiral EFT.
- If this is correct, it should be encoded in the coefficients of contact terms:

\[ \sum_{\text{all}} \frac{g^2}{m^2} \left( \frac{1}{m^2} \right) \left( \frac{1}{m^2} \right) \left( \frac{1}{m^2} \right) \]  

\[ \sum_{\text{structure}} \]  

Let's try phenomenological boson exchange models \( \Rightarrow \) tells us \( g's \) & \( m's \) (See Egelstaff et al, 2009)

\[ \Rightarrow \] see the slides.

Comments:
- The agreement is quite impressive!
- The assumption that most of the constants are given by virtual exchanges is called "resonance saturation."
- Does the agreement validate boson exchange or is it just an alternative way to parametrize short-distance physics?
Delta ful vs. Deltaless EFT (based on H. Krebs)

- Our "standard" Deltaless EFT supposes that
  \[ Q \sim m_\pi \ll \Delta = m_\Delta - m_N = 300 \text{ MeV} \]

- Herrmann, Holstein, Kambar 1996 proposed "small scale expansion":
  \[ Q \sim m_\pi \ll \Delta \ll \Lambda \] to organize \( Q \) expansion (decide what diagrams in each order)

- If we think of the LECs \( Q \) as encoding \( \Delta \) delta contribution in going to Deltaless:
  \[ F_{\Delta} \rightarrow 3 \mu_\pi / B \] (in large \( N_c \))

\[ \Rightarrow C_3 = -2 C_1 = C_3(\Delta) = \frac{4 \mu_\pi}{9 B_{\Delta}} \]

- So we expect that including \( Q \) & \( \Delta \) will lead to more natural LECs, better convergence, higher breakdown scale,

* To see pictures

- Achim will have more to say about 3NF and \( \Delta \)'s.
Renormalization Issue

If we consider iterating the leading order (LO) potential

\[ V^{(0)} = C_S + C_\Delta \bar{\phi} \phi + V^{(0)} \]

in Weinberg counting using the Lippmann-Schwinger (LS) equation:

\[ T = V + V \sqrt{\frac{-1}{H_0}} T = V + V \sqrt{-H_0} V + V \sqrt{-H_0} V \sqrt{-H_0} V \ldots \]

then we need to cutoff the potential, e.g.

\[ V(p^2 \rightarrow p^2) \Rightarrow e^{i \phi^2} V(p^2 \rightarrow p^2) e^{-i \phi^2} \]

here non-local but we heard of a different local regulator by Alex.

Ideally observe, but non-local.

- For an EFT we want the results to be insensitive to how we regulate and with what \( \Lambda \) we use.

- But strong cutoff dependence and divergences are found with \( V^{\text{LO}} \) in \( S \)-channel where the tensor force is attractive.

\( V^{\text{LO}} \) is very singular \( \sim \frac{1}{p^2} \) and worse when iterated.

- Nagga et al. found that \( S \)-wave can take \( \Lambda \approx 0 \) with stable results.

- \( p_0, \bar{p}_0, \bar{p}_0, \bar{p}_0 \) (attractive tensor) need counterterms even at LO to absorb \( \Lambda \) dependence.

Practical solution is keep \( \Lambda \) breakdown, aspiring study for formal solution.
Building chiral Lagrangians

- Here we follow the accessible but incomplete treatment in the review of Machleidt and Engler.
  - includes references to details
  - more rigorous discussion with mathematical detail in E. Epelbaum's review

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{NN}} + \mathcal{L}_{\text{MN}} + \mathcal{L}_{\text{NN}} \]

- Think in terms of building blocks, combined in all possible ways to generate \( \mathcal{L}'s \) of \( \pi^+, \pi^0, \pi^\pm \) and making Lorentz invariance by contracting indices.

For \( \mathcal{L}_{\text{NN}} \) the building block is an SU(2) matrix \( U \) made up of the pion fields.

- Perhaps easiest to start with adding a scalar meson \( \sigma \)

\[ \sigma = (\bar{\pi}, \pi) = (\pi, \pi, \pi, \pi, \pi, \pi) \quad \text{(real vector)} \]

with SU(2) \( \times \) SU(2) chiral symmetry \( \Rightarrow \text{SO}(4) \) rotation of this vector. A potential \( V(\sigma^2) = \lambda \sigma^2 \Rightarrow \sigma^2 + i \pi^2 = |V_e|^2 \) is minimum.

- But we don't want the \( \sigma \) as a low-energy degree of freedom \( \Rightarrow \) take its mass to infinity (so \( \sigma \) acts in minimum) \[ \begin{align*}
\text{fixed constraint } & \sigma^2 + \pi^2 = |V_e|^2 \Rightarrow \sigma = \sqrt{\pi^2 - \pi^2} \text{ eliminated} \\
\\end{align*} \]

- Under a vector transformation \( \pi \to e^V \pi' = \pi + \delta' \times \pi \) linear rotation \( (\text{rotation of}) \)

- But axial vector is non-linear from constraint \( \pi \to e^{i \theta} \pi = \pi + \delta' \times \pi \)

Write \( U = (\sigma + i \pi \cdot \pi') / f_\pi \quad \sigma = \sqrt{\pi^2 - \pi^2} \text{ as 2x2 matrix} \)

Check \( U^* U = \frac{1}{f_\pi^2} (\sigma^2 - i \pi \cdot \pi') (\sigma + i \pi \cdot \pi') = \frac{1}{f_\pi^2} (\sigma^2 + \pi^2) = 1 \quad \text{agree} \)
Under a global chiral transformation

\[ U \Rightarrow U' = g_L U g_R^+ \Rightarrow U^+ = g_R U g_L^+ \]

\[ \text{SU}(2)_L \Rightarrow \text{SU}(2)_R \]

So \( \text{tr}(U'U) \Rightarrow \text{tr}(g_R U g_L^+ U^+) = \text{tr}(g_R U g_L U) = \text{tr}(U) \)

\[ \Rightarrow \text{invariant but } \text{tr}(U^+U) \not\propto I \Rightarrow \text{ limit is trivial} \]

\[ \Rightarrow \text{need derivatives. Also need mass term} \]

\[ \text{two derivatives on } \pi_	ext{m}^\pm \]

\[ \pi^m = \frac{p^m}{4} - \text{tr} \left[ \frac{\lambda}{2} \pi^m U^+ + m^2 (U + U^+) \right] \]

\[ \text{chosen to get isosync} \]

\[ \text{and mass term correct} \]

\[ \text{breaks chiral symmetry in some way that mass terms in QCD do.} \]

- \( U \) can be parameterized many different ways, e.g.,

\[ U = e^{i \frac{\pi^m}{F_m}} \text{ (check } U^+U = I \text{ trivially)} \]

- Find \( \pi^m \) poles by expanding

\[ U = 1 + \frac{\pi^m}{F_m} - \frac{i \lambda}{F_m} \pi^m - \frac{i \lambda}{F_m} (\pi^m)^2 + \frac{8 \lambda^2}{8 F_m^2} \pi^m + \ldots \]

- all powers of \( \pi^m \) field related \( \Rightarrow \) chiral symmetry,

- \( \lambda \) reflects different parameterizations, but related by redefinitions of \( \pi^m \); "field redefinitions" \( \Rightarrow \) equivalence theorems say observables (S-matrix elements) are unchanged

- [basically no change of variables in path integral where you show that the Jacobian and new external source terms do not contribute to the S-matrix, see Furnstahl et al. for finite density]
\[ E_{\pi}^{\text{3G}} = \frac{1}{5} \delta_{\mu \nu} \partial_\mu \partial_\nu - \frac{1}{5} m_{\pi}^2 \partial_\mu \partial_\nu + \frac{1}{2 \sqrt{\alpha}} \left( \frac{\partial_\mu \partial_\nu}{\partial_\rho \partial_\sigma} \right) \cdot \partial_\rho \partial_\sigma \]

\[ \text{\(\pi-\pi\) scattering), } \rightarrow \frac{\alpha}{\sqrt{2}} \left( \partial_\mu \partial_\nu \right) \cdot \partial_\rho \partial_\sigma + \text{terms} \]

\[ \text{See Gasser-Leutwyler.} \]

For \( \pi N \), start with relativistic \( \pi N \) formulation [Gasser et al.] and then reduce to nonrel. with heavy baryon expansion \( \ln \frac{m}{\Lambda} \)

Building blocks are \( J_\mu \rightarrow D_\mu \) and \( u_\mu \equiv m \) combine in Lorentz contracted ways (with other symmetries obeyed)

\[ Y_{\text{3G}}^{\text{1G}} = \frac{1}{i} (i \gamma_\mu D_\mu - M N + \frac{3}{2} \frac{g_N}{g_p} \gamma_\mu u_\mu) \]

\[ u_\mu = \frac{1}{2} (\delta_{3G} - \delta_{3P}) = -\frac{1}{2} \gamma_\mu \partial_\mu \]

\[ D_\mu = \partial_\mu + \gamma_\mu \gamma_5 \]

\[ \gamma_\mu = \frac{1}{2} \left[ \gamma_\mu, \gamma_5 \right] \rightarrow \frac{i}{4 \pi F} \left( \gamma_\mu \partial_\mu - \frac{1}{2} \partial_\mu \right) + \mathcal{O}(\Lambda^2) \]

and \( \zeta = \sqrt{u} \) (sometimes \( u \))

\[ Y_{\text{3G}}^{\text{1G}} = \frac{1}{i} (i \gamma_\mu D_\mu - M N + \frac{3}{2} \frac{g_N}{g_p} \gamma_\mu u_\mu) - \frac{3}{2} \frac{g_N}{g_p} \gamma_\mu u_\mu \]

\[ \text{Need to make non-relativistic so loop expansion associated with GFT expansion (recall v formula: } L \rightarrow \frac{v}{L}) \]

\[ \text{but do } \mathcal{I} \rightarrow M_N \mathcal{I} \text{ in numerator and } M_N / N \lambda \chi \rightarrow e^{i \mathcal{I} \delta t} \]

\[ \text{Use heavy baryon formalism } \rightarrow \text{ gives } \frac{1}{m} \text{ expansion.} \]
Nuclear momentum: $p^µ = M v^µ + l^µ \ll$ small residual momentum

Four velocity of nucleon (big lumbering thing with pions bouncing off)

\[ e^{-i m v \cdot x} (N + h) \]

Post time dependence out \quad \Rightarrow \quad \text{small components}

\[ N = e^{-i m v \cdot x} \text{projection operators} \]

Insert and use $\nu^\mu = (1, \delta) \Rightarrow e^{-i m t}$, leaves $N$ upper only.

\[ y_{\text{upper}}^\mu = N (i D^\mu - \frac{m}{8} \delta^\mu_0 \delta^0_0) \]

\[ = N (i \not\partial - \not\pi \not\pi \not\pi - \not\delta) (3 \text{ or more}) \]

At dimension 3, write down all terms with covariant derivatives.

After Heavy bag expansion reduction:

\[ y_{\text{upper}}^\mu = N \left[ \frac{2 m^2}{16 \pi^2} (\not\partial + \not\pi \not\pi) \right] \]

\[ = N \left[ \frac{4 m^2}{16 \pi^2} \not\not\partial - \frac{2 m^2}{16 \pi^2} \not\pi \not\pi \not\pi + \frac{1}{3} \left( c_4 + \frac{1}{4} m^2 \right) \not\not\partial \not\not\partial + \frac{1}{3} \left( c_3 - \frac{5}{3} m^2 \right) \not\not\partial \not\not\partial + \frac{3}{2} m^2 \not\not\partial \not\not\partial \not\not\partial \right] + \ldots \]

Get $\omega_{\text{NN}}$ contact terms (no pions) from conventional constraints.