TALENT/INT Course on Nuclear Forces
Exercises and Discussion Questions M2

[Last revised on July 8, 2013 at 09:40:35.]

Monday 2: Chiral EFT 1; chiral symmetry in NN phase shifts, QCD 2

We have again grouped all of the two-minute and discussion questions toward the beginning. But remember to spend only about an hour working on questions and then try some of the other problems as well. When you need a break, go back and try another question!

1. Two-minute and discussion questions:

   (a) What is the difference between explicit and spontaneous breaking of chiral symmetry? Give an analogy for each type of breaking for rotational symmetry (e.g., for a spin system and ferromagnetism).

   (b) Are the unnaturally large NN scattering lengths because of chiral symmetry? How do you know?

   (c) Do pions interact weakly at low energies because their coupling constants are small? If not, what is the explanation?

   (d) Should the $SU(3)$ color gauge “symmetry” of QCD be required in the low-energy chiral EFT?

   (e) Is the short-range part of the chiral NN potential scheme dependent? What about the long-range part?

   (f) Why does iterating all parts of the NN potential in the Lippmann-Schwinger equation require us to keep the chiral EFT cutoff finite (as opposed to being able to take it to infinity)?

   (g) In Weinberg counting we iterate the NN potential to all orders in the Lippmann-Schwinger equation. Do you expect high-order contributions to be perturbative or nonperturbative?

   (h) Compare numerical values of $\epsilon \sim e$ with the typical Q/Λ_b to justify the counting rules for isospin-symmetry-breaking corrections.

   (i) How might you look for direct evidence of chiral symmetry (e.g., pions) in nuclear structure calculations?

   (j) What are the different types of error in a lattice QCD calculation?

   (k) How do you choose a lattice QCD source for the calculation of a mass? Is the choice unique?

   (l) Why is a potential extracted from an LQCD calculation not unique? Give a “source” of non-uniqueness (there is a hint here!).

2. NN scattering partial-wave phase shifts.

   (a) Review of NN phase shift discussion from last week (remember NN-online.org!).

      i. Explain the general properties of the two S-wave phase shifts (behavior at low energy, with increasing energy, and then becoming negative).

      ii. The $^1\mathrm{D}_2$ phase shift stays positive at high energy, unlike the S-waves. Assuming this is because of the angular momentum barrier, estimate the range of the repulsive core in a local potential using the energy where the $^1\mathrm{S}_0$ phase crosses 0.

      iii. How could we explain why the $^3\mathrm{P}_0$, $^3\mathrm{P}_1$, and $^3\mathrm{P}_2$ phase shifts have such different behavior?

   (b) What is the highest partial wave with an LEC at LO, NLO, N^2LO, and N^3LO in Weinberg counting? What does that say about the higher partial waves?

   (c) Why do we look at high partial waves (that is, with large orbital angular momentum) to isolate the effects of long-range pion exchange? Can you estimate at what partial wave the NN scattering phase shifts should be dominated by one-pion exchange.

   (d) Use NN-online in combination with the `OPE_NN_scattering.nb` VPA notebook for general L to explore the last part explicitly. That is, compare the actual phase shift in the higher (uncoupled) partial waves to what is predicted from one-pion exchange. It seems to work for the triplet states but fail for the singlet states. Can you figure out why?

3. The low-energy Hamiltonian is given by $H = T + V_{\mathrm{NN}} + V_{3\mathrm{N}} + \cdots$, where all $V_{A\mathrm{N}}$ have dimension of energy in coordinate space. Use dimensional analysis in the Fourier transformation to show that the Fourier transforms of 3N vs. NN interactions scale as:

$$\widetilde{V}_{3\mathrm{N}} \sim Q^3\, \widetilde{V}_{\mathrm{NN}}\,.$$

4. We discussed the power counting formula for ν that predicts the order Q^ν of any diagram:

$$\nu = -4 + 2N + 2L + \sum_i (d_i + n_i/2 - 2) \geqslant 0\,.$$

   Here N is the number of external nucleons, L is the number of loop integrals, and i sums over each vertex, with n_i the number of nucleon lines in or out of the vertex and d_i the number of derivatives or factors of m_π.

   (a) Why is it important that $\nu \geqslant 0$?

   (b) Try this out on the diagrams entering the NN potential at NLO and N^2LO (see the slide from the lecture), each one in your group picking a different diagram.

   (c) What does the formula imply for how suppressed four-body forces should be relative to the leading two-body forces? Check this explicitly for the first diagram for 4N forces on the slide from the lecture.

5. Why are the NLO and N^{2}LO cutoff bands in the NN phase shifts similar (see the slide from the lecture)? This was also observed in the QMC calculations with chiral EFT interactions presented in Alex Gezerlis' lecture.

6. If we look at the pion parts of $\mathcal{L}^{(0)}$ and $\mathcal{L}^{(1)}$ from the chiral EFT Lagrangian,

$$\mathcal{L}^{(0)} = \frac{1}{2}\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - \frac{1}{2} m_\pi^2 \boldsymbol{\pi}^2 + N^\dagger \left[i\partial_0 + \frac{g_A}{2f_\pi} \boldsymbol{\tau} \vec{\sigma} \cdot \vec{\nabla} \boldsymbol{\pi} - \frac{1}{4f_\pi^2} \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \dot{\boldsymbol{\pi}}) \right] N + \dots , \tag{1}$$

$$\begin{aligned}\mathcal{L}^{(1)} = N^\dagger \Big[& 4c_1 m_\pi^2 - \frac{2c_1}{f_\pi^2} m_\pi^2 \boldsymbol{\pi}^2 + \frac{c_2}{f_\pi^2} \dot{\boldsymbol{\pi}}^2 + \frac{c_3}{f_\pi^2} (\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}) \\ & - \frac{c_4}{2f_\pi^2} \epsilon_{ijk}\, \epsilon_{abc}\, \sigma_i \tau_a (\nabla_j\, \pi_b)(\nabla_k\, \pi_c) \Big] N - \frac{D}{4f_\pi} (N^\dagger N)(N^\dagger \vec{\sigma} \boldsymbol{\tau} N) \cdot \vec{\nabla} \boldsymbol{\pi} + \dots \end{aligned} \tag{2}$$

we see that interaction terms involving pion (π) fields always have at least one derivative or powers of the pion mass.

(a) Why is this?

(b) What is the consequence for the contribution of such a term for low-energy pions? [Hint: Think of a gradient as acting on a plane wave for the pion.]

(c) Notation: What is the difference between f_π and F_π? [Warning! There are several conventions in the literature for the value of the pion decay constant.]

7. What is special about how the pion couples to (i.e., interacts with) nucleons that makes an EFT expansion possible, as opposed to how a heavier meson (e.g., ω or ρ) might couple? [Hint: This is really the same as the last question, asked a different way!]

8. [Problem from E. Epelbaum lectures.] In the Henley-Miller classification, class-III interactions have the isospin dependence

$$V_{III} = \alpha_{III}(\tau_1^3 + \tau_2^3)\,.$$

(a) Show that class-III two-nucleon forces do not lead to isospin mixing in the two-nucleon system. That is, show that they commute with the operator $\mathbf{T}^2$.

(b) Does this still hold true for systems with three and more nucleons?

9. Renormalization controversy. Renormalization in an EFT removes sensitivity to unresolved short-distance physics, order-by-order; for a cutoff EFT this sensitivity is manifested as dependence on the cutoff parameter.

(a) At each order in the EFT expansion, is it necessary that cutoff dependence be removed completely, or is it sufficient that the residual dependence is comparable to the truncation error at that order? (Note: the latter is closer to what is done in numerical analysis where there is always some residual dependence on the mesh).

(b) In the latter case, how do you really know you have successfully renormalized? (That is, what are the signatures?)

10. Follow the arguments for the infrared enhancement in Weinberg counting, discussed, e.g., in Sect. 4.1 of E. Epelbaum, Nuclear forces from chiral EFT – a primer, arXiv:1001.3229.

11. Looking up one of the reviews on chiral EFT for nuclear forces (e.g., the review of R. Machleidt and D. Entem, Phys. Rep. **503**, 1 (2011), which has many explicit formulas on the different-order contributions), which operator structures (central, spin-orbit, tensor) are generated from the NLO two-pion exchange?